TD - ENTRAINEMENT - Corrigé

1BCPST 2 — Feuille d'exercice — Année 2023- 2024

Exercice 1 : *(30 secondes)* Simplifier au maximum ces fractions :

1) $\frac{1}{30}-\frac{1}{45}$ **2)** $5\cdot\frac{8}{40}-\frac{4}{13}$ **3)** $\frac{5}{24}+\frac{5}{12}$ **4)** $\frac{32}{192}$ **5)** $\frac{3\pi}{10}-\frac{15\pi}{6}$ **6)** $\frac{135}{90}-2$ **7)** $\left(\frac{4}{5}\right)^2-\frac{1}{3}$

8) $\frac{\frac{1}{5}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}}$ **9)** $\frac{390}{325}$ **10)** $\frac{57}{4}+\frac{25}{6}$ **11)** $\frac{8}{3}\times\left(\frac{2}{13}-5\right)$ **12)** $\frac{\frac{1}{2}-2}{\frac{1}{4}+\frac{1}{2}}$ **13)** $\frac{\frac{9}{14}}{\frac{78}{21}}$ **14)** $\frac{15}{77}-\frac{8}{33}$

15) $\frac{15^2-25^2}{80^2}$ **16)** $\frac{38}{72}+\frac{14}{49}$ **17)** $\frac{1}{25}-\frac{1}{16}+\frac{1}{20}$ **18)** $\frac{\frac{1}{2}-\frac{1}{6}+\frac{1}{3}}{4}$ **19)** $\frac{1}{5-\frac{1}{3}}+\frac{1}{6-\frac{1}{6}}$

20) $\frac{\frac{\frac{1}{2}+\frac{1}{3}}{3}+\frac{\frac{1}{2}+\frac{1}{3}}{2}}{10}$ **21)** $\frac{49^2-1}{4800}$ **22)** $\frac{x^2+4x+4}{x^2+2x}$

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Réponses : **1)** $\frac{1}{30}-\frac{1}{45}=\frac{1}{3\times15}-\frac{1}{3\times15}=\frac{1}{15}\left(\frac{1}{2}-\frac{1}{3}\right)=\frac{1}{15}\times\frac{3-2}{6}=\frac{1}{15\times6}=\boxed{\frac{1}{90}}$

2) $5\frac{8}{40}-\frac{4}{13}=5\times\frac{8}{5\times8}-\frac{4}{13}=1-\frac{4}{13}=\frac{13-4}{13}=\boxed{\frac{9}{13}}$

3) $\frac{5}{24}+\frac{5}{12}=\frac{5}{12}+\frac{5}{2\times12}=\frac{5}{12}\left(1+\frac{1}{2}\right)=\frac{5}{12}\times\frac{3}{2}=\frac{5\times3}{4\times3\times2}=\boxed{\frac{5}{8}}$.

4) $\frac{32}{192}=\frac{2\times16}{2\times96}=\frac{16}{96}=\frac{2\times8}{2\times48}=\frac{8}{48}=\frac{8}{8\times6}=\boxed{\frac{1}{6}}$

5) $\frac{3\pi}{10}-\frac{15\pi}{6}=\frac{3\pi}{5\times2}-\frac{5\times3\pi}{2\times3}=\frac{3\pi}{5\times2}-\frac{5\times\pi}{2}=\frac{\pi}{2}\left(\frac{3}{5}-5\right)=\frac{\pi}{2}\times\frac{3-25}{5}=\frac{\pi\times22}{2\times5}=\boxed{\frac{11\pi}{5}}$

6) $\frac{135}{90}-2=\frac{5\times27}{5\times18}-2=\frac{27}{18}-2=\frac{9\times3}{9\times2}-2=\frac{3}{2}-2=\frac{3-4}{2}=\boxed{-\frac{1}{2}}$

7) $\left(\frac{4}{5}\right)^2-\frac{1}{3}=\frac{16}{25}-\frac{1}{3}=\frac{3\times16-25}{25\times3}=\frac{48-25}{75}=\boxed{\frac{23}{75}}$ ne se simplifie pas plus car 23 est premier.

8) $\frac{\frac{1}{5}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}}=\frac{2\times3\times5}{2\times3\times5}\times\frac{\frac{1}{5}-\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}}=\frac{2\times3-3\times5}{3\times5-2\times5}=\frac{6-15}{15-10}=\boxed{\frac{-9}{5}}$

9) $\frac{390}{325}=\frac{5\times78}{5\times65}=\frac{78}{65}=\frac{2\times39}{5\times13}=\frac{2\times3\times13}{5\times13}=\boxed{\frac{6}{5}}$

10) $\frac{57}{4}+\frac{25}{6}=\frac{1}{2}\times\left(\frac{57}{2}+\frac{25}{3}\right)=\frac{1}{2}\times\frac{3\times57+2\times25}{6}=\frac{171+50}{12}=\boxed{\frac{221}{12}}$

221 ne se divise pas par 2 ni par 3.

11) $\frac{8}{3}\times\left(\frac{2}{13}-5\right)=\frac{8}{3}\times\frac{2-5\times13}{13}=\frac{8}{3}\times\frac{2-65}{13}=\frac{8}{3}\times\frac{-63}{13}=\frac{8}{3}\times\frac{-3\times21}{13}=\frac{-8\times21}{13}=\boxed{-\frac{168}{13}}$

12) $\frac{\frac{1}{2}-2}{\frac{1}{4}+\frac{1}{2}}=\frac{4}{4}\times\frac{\frac{1}{2}-2}{\frac{1}{4}+\frac{1}{2}}=\frac{4\left(\frac{1}{2}-2\right)}{4\left(\frac{1}{4}+\frac{1}{2}\right)}=\frac{2-8}{1+2}=\frac{-6}{3}=\boxed{-2}$.

13) $\frac{\frac{9}{14}}{\frac{78}{21}}=\frac{9\times21}{2\times14}=\frac{3\times3\times3\times7}{2\times39\times2\times7}=\frac{3\times3\times3}{2\times3\times13\times2}=\frac{3\times3}{4\times13}=\boxed{\frac{9}{52}}$

14) $\frac{15}{77}-\frac{8}{33}=\frac{15}{7\times11}-\frac{8}{3\times11}=\frac{1}{11}\left(\frac{15}{7}-\frac{8}{3}\right)=\frac{1}{11}\times\frac{15\times3-8\times7}{21}=\frac{1}{11}\frac{45-56}{21}=\frac{1}{11}\times\frac{-11}{21}=\boxed{\frac{-1}{21}}$

15) $\frac{15^2-25^2}{80^2}=\frac{(15-25)(15+25)}{80\times80}=\frac{-10\times40}{8\times10\times2\times40}=\boxed{\frac{-1}{16}}$

16) $\frac{38}{72}+\frac{14}{49}=\frac{2\times19}{2\times36}+\frac{2\times7}{7\times7}=\frac{19}{36}+\frac{2}{7}=\frac{7\times19+2\times36}{7\times36}=\frac{133-72}{7\times36}=\boxed{\frac{205}{252}}$

ne se simplifie plus car $205=5\times41$ où 41 est premier.

17) $\frac{1}{25}-\frac{1}{16}+\frac{1}{20}=\frac{1}{5^2}-\frac{1}{4^2}+\frac{1}{4\times 5}=\frac{4^2-5^2+4\times 5}{4^2\times 5^2}=\frac{16-25+20}{4^2\times 5^2}=\frac{11}{20^2}=\boxed{\frac{11}{400}}$

18) $\frac{\frac{1}{2}-\frac{1}{6}+\frac{1}{3}}{4}=\frac{1}{4}\times\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{3}\right)=\frac{1}{4}\times\frac{3-1+2}{6}=\frac{4}{24}=\boxed{\frac{1}{6}}$

19) $\frac{1}{5-\frac{1}{3}}+\frac{1}{6-\frac{1}{6}}=\frac{3}{3\times\left(5-\frac{1}{3}\right)}+\frac{6}{6\times\left(6-\frac{1}{6}\right)}=\frac{3}{15-1}+\frac{6}{36-1}=\frac{3}{14}+\frac{6}{35}=\frac{3}{7}\times\left(\frac{1}{2}+\frac{2}{5}\right)=\frac{3}{7}\times\frac{5+4}{10}=$
$\frac{3\times 9}{7\times 10}=\boxed{\frac{27}{70}}$

20) $\frac{\frac{\frac{1}{2}+\frac{1}{3}}{3}+\frac{\frac{1}{2}+\frac{1}{3}}{2}}{10}=\frac{\left(\frac{1}{2}+\frac{1}{3}\right)\left(\frac{1}{3}+\frac{1}{2}\right)}{10}=\frac{1}{10}\left(\frac{1}{3}+\frac{1}{2}\right)^2=\frac{1}{10}\left(\frac{2+3}{6}\right)^2=\frac{5^2}{10\times 6^2}=\frac{5}{2\times 36}=\boxed{\frac{5}{72}}$

21) $\frac{49^2-1}{4800}=\frac{(49-1)\times(49+1)}{48\times 100}=\frac{48\times 50}{48\times 2\times 50}=\boxed{\frac{1}{2}}$

22) $\frac{x^2+4x+4}{x^2+2x}=\frac{(x+2)^2}{x(x+2)}=\boxed{\frac{x+2}{x}=1+\frac{2}{x}}$

<u>Exercice 2</u> : *(30 secondes)* Puissances : simplifier ces expression au maximum :

1) $\dfrac{2^7-2^5}{2^{10}}$ **2)** $\dfrac{4^9}{2^{15}}$ **3)** $\dfrac{6^4-15^4}{3^6}$ **4)** $\dfrac{(2^2)^{-2}}{(2^{-2})^2}$ **5)** $3^{-5}\times(3^6+3^7-3^5)$ **6)** $\dfrac{27^{-4}}{3^{-13}}$

7) $7^{101}+(-7)^{101}+2^{102}-(-2)^{102}$ **8)** $\dfrac{1}{128}-\dfrac{1}{512}$ **9)** $\sqrt[3]{\left(\dfrac{5}{2}\right)^3-\left(\dfrac{7}{2}\right)^2}$ **10)** $\dfrac{\sqrt{24}}{2\sqrt{2}}$

11) $\sqrt{\dfrac{4^2-5^2}{8^2-10^2}}$ **12)** $\dfrac{\sqrt{6}-\sqrt{15}}{\sqrt{5}-\sqrt{2}}$ **13)** $\dfrac{\sqrt{1+\sqrt{2}}}{\sqrt{1-\sqrt{2}}}$ **14)** $\sqrt[5]{\dfrac{-5^5-32}{15^5+6^5}}$ **15)** $\dfrac{11^4+(-11)^5}{110^3}$ **16)** $\sqrt[5]{\sqrt{4^{15}}}$

17) $\dfrac{-6^{-4}+6^{-3}}{6^{-2}-(-6)^{-3}}$

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Réponses : **1)** $\dfrac{2^7-2^5}{2^{10}}=\dfrac{2^5(2^2-1)}{2^{10}}=\dfrac{4-1}{2^{10-5}}=\dfrac{3}{2^5}=\boxed{\dfrac{3}{32}}$

2) $\dfrac{4^9}{2^{15}}=\dfrac{(2^2)^9}{2^{15}}=\dfrac{2^{2\times9}}{2^{15}}=\dfrac{2^{18}}{2^{15}}=2^{18-15}=2^3=\boxed{8}$

3) $\dfrac{6^4-15^4}{3^6}=\dfrac{3^4\times2^4-3^4\times5^4}{3^6}=\dfrac{3^4\times(2^4-5^4)}{3^6}=\dfrac{16-625}{3^{6-4}}=\dfrac{609}{3^2}=\dfrac{3\times203}{3\times3}=\boxed{\dfrac{203}{3}}$

4) $\dfrac{(2^2)^{-2}}{(2^{-2})^2}=\dfrac{2^{2\times(-2)}}{2^{-2\times2}}=\dfrac{2^{-4}}{2^{-4}}=\boxed{1}$

5) $3^{-5}\times(3^6+3^7-3^5)=3^{6-5}+3^{7-5}-3^{5-5}=3^1+3^2-3^0=3+9-1=\boxed{11}$

6) $\dfrac{27^{-4}}{3^{-13}}=\dfrac{(3^3)^{-4}}{3^{-13}}=\dfrac{3^{3\times(-4)}}{3^{-13}}=\dfrac{3^{-12}}{3^{-13}}=3^{-12-(-13)}=3^1=\boxed{3}$

7) $7^{101}+(-7)^{101}+2^{102}-(-2)^{102}\ =\ 7^{101}+(-1)^{101}\times7^{101}+2^{102}-(-1)^{102}\times2^{102}$
$=7^{101}+(-1)\times7^{101}+2^{102}-1\times2^{102}=7^{101}-7^{101}+2^{102}-2^{102}=\boxed{0}$

On rappelle que $(-1)^n=1$ si n est pair et $(-1)^n=-1$ si n est impaire.

8) $\dfrac{1}{128}-\dfrac{1}{512}=\dfrac{1}{2^7}-\dfrac{1}{2^9}=\dfrac{2^2-1}{2^9}=\dfrac{4-1}{512}=\boxed{\dfrac{3}{512}}$

9) $\sqrt[3]{\left(\dfrac{5}{2}\right)^3-\left(\dfrac{7}{2}\right)^2}=\sqrt[3]{\dfrac{5^3}{2^3}-\dfrac{7^2}{2^2}}=\sqrt[3]{\dfrac{5^3-2\times7^2}{2^3}}=\sqrt[3]{\dfrac{125-98}{8}}=\sqrt[3]{\dfrac{27}{8}}=\sqrt[3]{\left(\dfrac{3}{2}\right)^3}=\boxed{\dfrac{3}{2}}$

10) $\dfrac{\sqrt{24}}{2\sqrt{2}}=\dfrac{\sqrt{8\times3}}{2\sqrt{2}}=\dfrac{\sqrt{8}\times\sqrt{3}}{2\sqrt{2}}=\dfrac{\sqrt{2\times4}\times\sqrt{3}}{2\sqrt{2}}=\dfrac{\sqrt{2}\times\sqrt{4}\times\sqrt{3}}{2\sqrt{2}}=\dfrac{\sqrt{2}\times2\times\sqrt{3}}{2\sqrt{2}}=\boxed{\sqrt{3}}$

11) $\sqrt{\dfrac{4^2-5^2}{8^2-10^2}}=\sqrt{\dfrac{4^2-5^2}{2^2\times4^2-2^2\times5^2}}=\sqrt{\dfrac{4^2-5^2}{2^2\times(4^2-5^2)}}=\sqrt{\dfrac{1}{2^2}}=\dfrac{\sqrt{1}}{\sqrt{2^2}}=\boxed{\dfrac{1}{2}}$

12) $\dfrac{\sqrt{6}-\sqrt{15}}{\sqrt{5}-\sqrt{2}}=\dfrac{\sqrt{3\times2}-\sqrt{3\times5}}{\sqrt{5}-\sqrt{2}}=\dfrac{\sqrt{3}\times\sqrt{2}-\sqrt{3}\times\sqrt{5}}{\sqrt{5}-\sqrt{2}}=\dfrac{\sqrt{3}\times(\sqrt{2}-\sqrt{5})}{\sqrt{5}-\sqrt{2})}=\dfrac{-\sqrt{3}\times(\sqrt{5}-\sqrt{2})}{\sqrt{5}-\sqrt{2}}$
$=\boxed{-\sqrt{3}}$

13) $\dfrac{\sqrt{1+\sqrt{2}}}{\sqrt{\sqrt{2}-1}}=\sqrt{\dfrac{1+\sqrt{2}}{1-\sqrt{2}}}=\sqrt{\dfrac{1+\sqrt{2}}{\sqrt{2}-1}\times\dfrac{1+\sqrt{2}}{\sqrt{2}+1}}=\sqrt{\dfrac{(1+\sqrt{2})^2}{(\sqrt{2}-1)(\sqrt{2}+1)}}=\sqrt{\dfrac{(1+\sqrt{2})^2}{\sqrt{2}^2-1^2}}=\dfrac{1+\sqrt{2}}{\sqrt{1}}$
$=\boxed{1+\sqrt{2}}$

14) $\sqrt[5]{\dfrac{-5^5-32}{15^5+6^5}}=\sqrt[5]{-\dfrac{5^5+2^5}{3^5\times5^5+3^5\times2^5}}=\sqrt[5]{-\dfrac{5^5+2^5}{3^5\times(5^5+2^5)}}=\sqrt[5]{\dfrac{1}{-3^5}}=\sqrt[5]{\dfrac{1}{(-3)^5}}=\dfrac{\sqrt[5]{1}}{\sqrt[5]{(-3)^5}}=\dfrac{1}{-3}=$
$\boxed{-\dfrac{1}{3}}$

15) $\dfrac{11^4+(-11)^5}{110^3}=\dfrac{11^4+(-1)^5\times11^5}{110^3}=\dfrac{11^4-\times11^5}{110^3}=\dfrac{11^4\times(1-11^1)}{11^3\times10^3}=\dfrac{11^{4-3}\times(1-11)}{10^3}=\dfrac{11\times(-10)}{10^3}$

$$= \frac{-11}{10^2} = \boxed{\frac{-11}{100}}$$

16) $\sqrt[5]{\sqrt{4^{15}}} = ((4^{15})^{1/2})^{1/5} = 4^{15\times\frac{1}{2}\times\frac{1}{5}} = 4^{15/10} = 4^{3/2} = \sqrt{4}^3 = 2^3 = \boxed{8}$

17) $\dfrac{-6^{-4}+6^{-3}}{6^{-2}-(-6)^{-3}} = \dfrac{(-1)^{-4}\times 6^{-4}+6^{-3}}{6^{-2}-(-1)^{-3}\times 6^{-3}} = \dfrac{6^{-4}+6^{-3}}{6^{-2}+6^{-3}} = \dfrac{6^{-4}\times(1-6^{-3-(-4)})}{6^{-4}\times(6^{-2-(-4)}+6^{-3-(-4)})}$

$$= \frac{1-6^1}{6^2-6^1} = \frac{1-6}{36-6} = \frac{-5}{30} = \boxed{\frac{-1}{6}}$$

<u>Exercice 3</u> : *(30 secondes)* Exprimer ces nombres sous la forme de combinaisons simples de $\ln p$ avec p des un nombres premiers (2,3,5, etc..) :

1) $\ln 6 - \ln 2$ **2)** $\ln \dfrac{\sqrt{2}}{2}$ **3)** $\ln 36 - \ln 2 - \ln 3$ **4)** $\ln\sqrt{18} - \ln\sqrt{2}$ **5)** $\ln(3e) + \ln\dfrac{e}{3}$ **6)** $\ln\dfrac{1}{42} + \ln 48$

7) $\ln 2^{50} - \ln 6^{100} + \ln 3^{100}$ **8)** $\ln\dfrac{1}{27^9} - \ln\dfrac{1}{9^8}$ **9)** $\ln\sqrt[3]{10} + \ln\dfrac{\sqrt{5}}{2}$ **10)** $\ln 8000$

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Réponses : **1)** $\ln 6 - \ln 2 = \ln\dfrac{6}{2} = \boxed{\ln 3}$

2) $\ln\dfrac{\sqrt{2}}{2} = \ln\dfrac{1}{\sqrt{2}} = -\ln\sqrt{2} = \boxed{\dfrac{-1}{2}\ln 2}$

3) $\ln 36 - \ln 2 - \ln 3 = \ln\dfrac{36}{2\times 3} = \ln\dfrac{36}{6} = \ln 6 = \boxed{\ln 2 + \ln 3}$

4) $\ln\sqrt{18} - \ln\sqrt{2} = \ln\dfrac{\sqrt{18}}{\sqrt{2}} = \ln\sqrt{\dfrac{18}{2}} = \ln\sqrt{9} = \boxed{\ln 3}$

5) $\ln(3e) + \ln\dfrac{e}{3} = \ln 3 + \ln e + \ln e - \ln 3 = 2\ln 2 = \boxed{2}$

6) $\ln\dfrac{1}{42} + \ln 48 = \ln\dfrac{48}{42} = \ln\dfrac{6\times 8}{6\times 7} = \ln\dfrac{8}{7} = \ln 8 - \ln 7 = \boxed{3\ln 2 - \ln 7}$

7) $\ln 2^{50} - \ln 6^{100} + \ln 3^{100} = 50\ln 2 - 100\ln 6 + 100\ln 3 = 50\ln 2 - 100(\ln 3 + \ln 2) + 100\ln 3 = 50\ln 2 - 100\ln 2$
$= \boxed{-50\ln 2}$

8) $\ln\dfrac{1}{27^9} - \ln\dfrac{1}{9^8} = -\ln 27^9 - (-\ln 9^8) = -9\ln 27 + 8\ln 9 = -9\ln 3^3 + 8\ln 3^2 = -9\times 3\ln 3 + 2\times 8\ln 3$
$= -27\ln 3 + 16\ln 3 = \boxed{-11\ln 3}$

9) $\ln\sqrt[3]{10} + ln\dfrac{\sqrt{5}}{2} = \dfrac{1}{3}\ln 10 + \ln\sqrt{5} - \ln 2 = \dfrac{1}{3}\ln 2 + \dfrac{1}{3}\ln 5 + \dfrac{1}{2}\ln 5 - \ln 2 = \boxed{\dfrac{5}{6}\ln 5 - \dfrac{2}{3}\ln 2}$

10) $\ln 8000 = \ln(8\times 1000) = \ln 8 + \ln 1000 = \ln 2^3 + \ln 10^3 = 3\ln 2 + 3\ln 10 = 3\ln 2 + 3\ln(2\times 5)$
$= 3\ln 2 + 3\ln 2 + 3\ln 5 = \boxed{6\ln 2 + 3\ln 5}$

Exercice 4 : *(30 secondes)* Factoriser et simplifier au maximum ces expressions :

1) $\dfrac{1}{x^2}-\dfrac{1}{x^2(x+1)}-\dfrac{1}{(x+1)^2}$ **2)** $\dfrac{1}{x^2-4x-5}+\dfrac{1}{x^2-5x}$

3) $1+\dfrac{1}{2}-\dfrac{1}{x+1}-\dfrac{1}{x+2}$

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Réponses : **1)** $\dfrac{1}{x^2}-\dfrac{1}{x^2(x+1)}-\dfrac{1}{(x+1)^2}=\dfrac{(x+1)^2}{x^2(x+1)^2}-\dfrac{x+1}{x^2(x+1)^2}-\dfrac{x^2}{x^2(x+1)^2}=\dfrac{(x+1)^2-x-1-x^2}{x^2(x+1)^2}$
$=\dfrac{x^2+2x+1-x-1-x^2}{x^2(x+1)^2}=\dfrac{x}{x^2(x+1)^2}=\boxed{\dfrac{1}{x(x+1)^2}}$

2) $\dfrac{1}{x^2-4x-5}+\dfrac{1}{x^2-5x}=\dfrac{1}{(x-5)(x+1)}+\dfrac{1}{x(x-5)}=\dfrac{1}{x-5}\left(\dfrac{1}{x+1}-\dfrac{1}{x}\right)=\dfrac{1}{x-5}\times\dfrac{x+x+1}{x(x+1)}=$
$\boxed{\dfrac{2x+1}{x(x+1)(x-5)}}$

3) $1+\dfrac{1}{2}-\dfrac{1}{x+1}-\dfrac{1}{x+2}=\dfrac{3}{2}-\dfrac{x+2+x+1}{(x+1)(x+2)}=\dfrac{3(x+1)(x+2)-2\times(2x+3)}{(x+1)(x+2)}=\dfrac{3(x^2+3x+2)-4x-6}{(x+1)(x+2)}=$
$\dfrac{3x^2+9x+6-4x-6}{(x+1)(x+2)}=\boxed{\dfrac{x(3x+5)}{(x+1)(x+2)}}$

Exercice 5 : *(30 secondes)* Prouver ces égalités en partant du terme de gauche :

1) $\dfrac{1-x^2}{1+x}+1=-x$ **2)** $\dfrac{1-x^5}{1-y^5}=\dfrac{x^5-1}{y^5-1}$ **3)** $\dfrac{x^2+2x+3}{x+3}=x+2$ **4)** $\dfrac{1}{1+e^{-x}}=\dfrac{e^x}{1+e^x}$

5) $\dfrac{1}{\sqrt{x}-\sqrt{x+1}}=\sqrt{x}+\sqrt{x+1}$

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Réponses : **1)** $\dfrac{1-x^2}{1+x}-1=\dfrac{(1-x)(1+x)}{1+x}-1=1-x-1=\boxed{-x}$

2) $\dfrac{1-x^5}{1-y^5}=\dfrac{-1}{-1}\times\dfrac{1-x^5}{1-y^5}=\dfrac{-(1-x^5)}{-(1-y^5)}=\dfrac{-1+x^5}{-1+y^5}=\boxed{\dfrac{x^5-1}{y^5-1}}$

3) $\dfrac{x^2+2x+3}{x+3}=\dfrac{(x+2)(x+3)}{x+3}=\boxed{x+2}$

car x^2+2x+3 est un polynôme de degré 2 dont les racines $x_1=-2$ et $x_2=-3$ permettent de le factoriser :
$x^2+2x+3=(x-x_1)(x-x_2)=(x-(-2))(x-(-3))=(x+2)(x+3)$

Exercice 6 : *(30 secondes)* Dérivées : calculer les dérivées de ces fonctions. Mettre le résultat sous la forme la plus factorisée possible.

1) $f(x)=\dfrac{x}{1+x^2}$ **2)** $f(x)=xe^{1+x^2}$ **3)** $f(x)=x\cos^2 x+\sin^2 x$ **4)** $f(x)=\ln\sqrt{\dfrac{1+x}{1-x}}$

5) $f(x)=\cos^3 x\cos(3x)$ **6)** $f(x)=\dfrac{x-1}{\sqrt{x+1}}$ **7)** $f(x)=\sqrt[3]{(x^2+1)^4}$

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Réponses : **1)** $f'(x)=\dfrac{1\times(1+x^2)-x\times 2x}{(1+x^2)^2}=\dfrac{1+x^2-2x^2}{(1+x^2)^2}=\dfrac{1-x^2}{(1+x^2)^2}=\boxed{\dfrac{(1+x)(1-x)}{(1+x^2)^2}}$

2) $f'(x)=1\times e^{1+x^2}+x\times 2x\times e^{1+x^2}=\boxed{(1+2x^2)e^{1+x^2}}$

3) $f'(x)=1\times\cos^2 x+x\times 2\times -\sin x\times\cos x+2\times\cos x\times\sin x=\cos x(\cos x-2x\sin x+2\sin x)=\boxed{\cos x(\cos x+2(1-x)\sin x)}$

4) $f(x)=\ln\sqrt{\dfrac{1+x}{1-x}}=\dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)=\dfrac{1}{2}(\ln(1+x)-\ln(1-x))$

Donc :$f'(x)=\dfrac{1}{2}\left(\dfrac{1}{1+x}-\dfrac{1}{1-x}\right)=\dfrac{1}{2}\times\dfrac{1-x-(1+x)}{(1+x)(1-x)}=\boxed{\dfrac{1}{(1+x)(1-x)}}=\dfrac{1}{1-x^2}$

5) $f'(x)=3(-\sin x)\cos^2 x\cos(3x)-3\cos^3 x\sin(3x)=-3\cos^2 x(\sin x\cos(3x)+\cos x\sin(3x))$
$=-3\cos x\sin(3x+x)=\boxed{-3\cos x\sin(4x)}$

6) $f(x)=\dfrac{x-1}{\sqrt{x+1}}=(x-1)(x+1)^{\frac{-1}{2}}$ donc

$f'(x)=(x+1)^{\frac{-1}{2}}-\dfrac{1}{2}(x-1)(x+1)^{\frac{-3}{2}}=(x+1)^{\frac{-1}{2}}\left(1-\dfrac{x-1}{2}(x+1)^{-1}\right)=(x+1)^{\frac{-1}{2}}\left(1-\dfrac{x-1}{2(x+1)}\right)$

$=(x+1)^{\frac{-1}{2}}\times\dfrac{2(x+1)-(x-1)}{2(x+1)}=(x+1)^{\frac{-1}{2}}\dfrac{2x+2-x+1}{2(x+1)}=\boxed{\dfrac{x+3}{2(x+1)\sqrt{x+1}}=\dfrac{x+3}{2(x+1)^{3/2}}}$

7) $f(x)=\sqrt[3]{(x^2+1)^4}=(x^2+1)^{4/3}$ donc $f'(x)=\dfrac{4}{3}\times 2x\times(x^2+1)^{4/3-1}=\dfrac{4}{3}\times 2x\times(x^2+1)^{1/3}=\boxed{\dfrac{8x}{3}\sqrt[3]{x^2+1}}$

[En construction] A. Plouviez