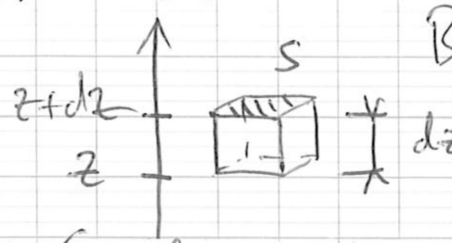
ATS 2022

1. 

Bilan des forces :

$$\left| \begin{array}{l} d\vec{F}(z+dz) = -P(z+dz).S.\vec{u_z} \\ d\vec{F}(z) = +P(z).S.\vec{u_z} \end{array} \right.$$

$\hookrightarrow$ Forces de pression

(les forces de pression latérales se compensent)

$$d\vec{P} = dm.\vec{g} = -\rho S dz g \vec{u_z}$$

PFS puis projection :

$$-P(z+dz)\cancel{S} + P(z)\cancel{S} = -\rho \cancel{S} dz g$$

$$+dP(z) = -\rho dz g$$

$$+\frac{dP}{dz} = -\rho g \quad (1)$$

2. GP : $PV = nRT \Rightarrow V = \frac{nRT_0}{P}$ (isotherme)

$\cancel{P = \frac{nRT}{V} = \frac{nRT_0}{V}}$ ~~(isotherme)~~

~~[illegible] $\Rightarrow$ nRT0~~ $\rho(z) = \frac{dm}{dV} = \frac{m}{V} = \frac{m}{nRT_0}P = \frac{MP(z)}{RT_0}$

3. (1) $\Rightarrow \frac{dP}{dz} = -\frac{Mg}{RT_0}P$

$$\frac{dP}{dz} + \frac{Mg}{RT_0}P = 0$$

D'où : $P(z) = A\exp\left(-\frac{z}{H}\right)$ avec $H = \frac{RT_0}{Mg}$

CL : $P(0) = P_0$

$\Rightarrow$

$$P(z) = P_0 \exp\left(-\frac{z}{H}\right).$$

4. $H = \frac{8,3 \times (273+15)}{29.10^{-3} \times 9,8} \simeq \frac{290}{29.10^{-3}} \simeq 10^4 \text{ m}$

$\simeq$ <u>10 km</u>

5. Doc 2 | : $z_{lim} \simeq 8$ km
Doc 3 |
Hypothèse "isotherme" est la moins bien vérifiée !

6. $Q = \frac{4}{3}\pi R^3 \rho$.

7. Plans de symétries :
$(M, \vec{e_r}, \vec{e_\theta})$
$(M, \vec{e_r}, \vec{e_\varphi})$
Concl : $\vec{E} = E(r,\theta,\varphi)\vec{e_r}$
Invariances suivant θ et φ.
Concl : $\vec{E} = E(r)\vec{e_r}$

8. Th. de Gauss : $\oiint \vec{E}.\vec{dS} = \frac{Q_{int}}{\varepsilon_0}$

Dans la boule : $E(r).\cancel{4\pi} r^{\cancel{2}} = \frac{1}{\varepsilon_0}\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}\rho$

$E(r) = \frac{\rho r}{3\varepsilon_0}$ ($\nearrow$ en r)

A l'extérieur : $E(r)\cancel{4\pi} r^2 = \frac{\cancel{4\pi} R^3\rho}{3\varepsilon_0}$

$E(r) = \frac{\rho R^3}{3\varepsilon_0 r^2}$ ($\searrow$ en $\frac{1}{r^2}$)

D'où : $g(r) = \frac{GM_T}{r^2}$

9. $g(r) = \frac{GM_T}{r^2}$

Au sol : $g(0) = \frac{GM_T}{R_T^2}$

A l'altitude z : $g(z) = \frac{GM_T}{(R_T+z)^2} = g_0 \frac{R_T^2}{(R_T+z)^2}$

10. $g(z) = g_0 \dfrac{1}{\left(1+\dfrac{z}{R_T}\right)^2} \simeq g_0\left(1 - 2\dfrac{z}{R_T}\right)$

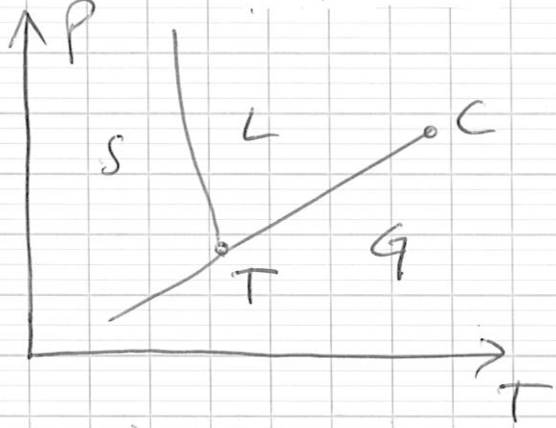
$g(z) \simeq g_0\left(1 - 2\dfrac{z}{R_T}\right)$

11. Pour $z = 6{,}4$ km, la diminution est de :

$2\dfrac{z}{R_T} = 2 \times \dfrac{6{,}4}{6400} = 0{,}2\%$.

Document 1 : $\dfrac{9{,}807 - 9{,}787}{9{,}807} \simeq \dfrac{0{,}02}{10} \simeq 0{,}2\%$

12.



13. Doc 4 b)

$\Rightarrow T_{eau} = 98°C$

Doc 4 a)

$T_{eau} = 97°C \Rightarrow P_{vapeur\ saturante} = \underline{0{,}95 \text{ bar}}$.

Or : Doc 1

$h = 2400$ m $\Rightarrow$ Pression $\underline{0{,}75 \text{ bar}}$.

A l'envers

$h = 2400$ m $\Rightarrow$ Pression : $0{,}75$ bar

$\Rightarrow T_{LV} = 92°C$

$\Rightarrow$ Temps cuisson $3{,}6$ min

14 – Rapide ⇒ Adiabatique.
Brutale

15. Transformation | Adiabatique + Réversible = Isentropique
| gaz parfait.

$$PV^\gamma = cte \Rightarrow P\left(\frac{nRT}{P}\right)^\gamma = cte$$

$$\Rightarrow P^{1-\gamma}T^\gamma = cte$$

16 –

$$\ln\left(P^{1-\gamma}T^\gamma\right) = \ln cte$$

$$\left((1-\gamma)\ln P + \gamma \ln T\right) = \ln cte$$

$$\Rightarrow d\left(\qquad \right) = 0$$

$$\Rightarrow (1-\gamma)\frac{dP}{P} + \gamma\frac{dT}{T} = 0$$

17. $$dT = \frac{\gamma-1}{\gamma}\frac{T}{P}dP$$

Or $PV = nRT$

donc : $$\frac{T}{P} = \frac{V}{nR} = \frac{\frac{V}{m}}{\frac{n}{m}R} = \frac{M}{\rho R}$$

$$dT = \frac{\gamma-1}{\gamma}\frac{M}{\rho R}dP$$

$$\boxed{\frac{dT}{dz} = \frac{\gamma-1}{\gamma}\frac{M}{\rho R}\frac{dP}{dz}}$$

18 – Avec $\frac{dP}{dz} = -\rho g$ $\quad \frac{dT}{dz} = \frac{1-\gamma}{\gamma}\frac{M}{\cancel{\rho} R}\cancel{\rho} g$

$$\frac{dT}{dz} = \frac{1-\gamma}{\gamma}\frac{Mg}{R}$$

19 – $\frac{dT}{dz} = \frac{1-1,4}{1,4} \times \frac{29.10^{-3} \times 9,8}{8,3}$

$\simeq -\frac{0,4}{1,4} \times 30.10^{-3} \simeq -10^{-2}\ K/m$

$\simeq -10\ K/km$

20 – Système = Eau

$\Delta H = Q_{eau} = -m_{vap}.h_{vap}$.

D'où :

$Q_{air} = -Q_{eau} = \boxed{m_{vap}.h_{vap}}$

21 – $\Delta H = m_{air}\,c_p\,\Delta T$

d'où :

$\Delta T = \frac{\Delta H}{m_{air}\,c_p} = \frac{Q_{air}}{m_{air}\,c_p} = \frac{m_{vap}.h_{vap}}{m_{air}.c_p}$

$$\boxed{\Delta T = \frac{m_{vap}}{m_{air}}.\frac{h_{vap}}{c_p}}$$

22 – 20°C / 70% $\Rightarrow \frac{m_{vap}}{m_{air}} = 11\ g.kg^{-1}$
$= 11.10^{-3}$.

23 – $\Delta T = 11.10^{-3} \times \frac{2,3.10^6}{10^3}$.

$\simeq 10^{-2} \times \frac{2,5.10^6}{10^3} \simeq 25$ °C ou K.

24. $\Delta T > 0$ car liquéfaction exothermique !

25. $\frac{dT}{dz} \simeq -10\,K/km$ – $h = 1km$

$\Rightarrow T_B = 20 - 10 = \underline{\underline{10\,°C}}$

5

26. $T_C = 20°C$. Pas d'effet de foehn

27. $\Delta T = 25°C + 20°C = 45°C$

28. $\downarrow \vec{e_z}$ $\quad E_p = -mgz$

29. Conservation d'Em.

$$E_p + E_C = cte = E_{p_0} + E_{c_0}$$

$$-\cancel{m}gz + \frac{1}{2}\cancel{m}v^2 = \qquad + 0.$$

$$\frac{1}{2}v^2 = \qquad g z$$

$$v = \sqrt{2gh}$$

30. $v = \sqrt{2 \times 10 \times 1000} = \sqrt{2} \times \sqrt{10^4}$

$= 1,4 . 10^2 = 140 \text{ m.s}^{-1}$

$\times 3,6 \simeq 500 \text{ km.h}^{-1}$

Trop élevé ! Frottements !

31. $\vec{P} + \vec{f} = m\vec{a}$

$$mg - \alpha v^2 = m\frac{dv}{dt}$$

$$\frac{dv}{dt} + \frac{\alpha}{m}v^2 = g$$

32. $\frac{dv}{dt} = 0$

$$\frac{\alpha}{m}v_{lim}^2 = g \qquad v_{lim} = \sqrt{\frac{mg}{\alpha}}.$$

34. $\frac{dv}{dt} = g - \frac{\alpha}{m} v^2$

$\frac{v_{i+1} - v_i}{dt} = g - \frac{\alpha}{m} v_i^2$

$v_{i+1} = dt\left(g - \frac{alpha}{m} v_i \text{^} 2\right) + v_i$

37. $v_i = + \frac{dz}{dt} = \frac{z_{i+1} - z_i}{dt}$

$z_{i+1} = v_i . dt + z_i$

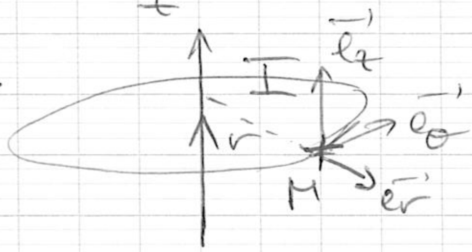
35. $v_{lim} = 40\ m/s \times 3{,}6 \rightarrow 130\ km/h \simeq$ OK

$40\ m/s \times 75\ \% = 30\ m/s$

$\rightarrow t = 4\ s$

$\rightarrow z = \underline{\underline{70\ m}}$

36 -



Plan de symétrie $(M, \vec{e_r}, \vec{e_z})$

$\Rightarrow \vec{B} \perp$ Plan de symétrie

$\Rightarrow \vec{B} = B . \vec{e_\theta}$

Invariance par | translation suivant z
| rotation suivant θ

$\Rightarrow \underline{\vec{B} = B(r) \vec{e_\theta}}$

37. Th. d'Ampère :

$\oint \vec{B} . \vec{dl} = \mu_0 I_e$

Contour d'Ampère : Cercle de rayon r.

$B . 2\pi r = \mu_0 I$.

$B = \frac{\mu_0 I}{2\pi r}$ $\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r} \vec{e_\theta}}$

38. $I_{max} = 50\ kA$

B à une distance $d = 10\ m$.

$$B_{max} = \frac{\mu_0 I_{max}}{2\pi d} = \frac{2\cancel{4\pi}\cdot 10^{-7}\times 50\cdot 10^{3}}{\cancel{2\pi}\times 10}$$

$$= 10\cdot 10^{-7}\cdot 10^{3} = \underline{10^{-3}\ T}$$

$$\cancel{\frac{d\phi}{dt}}\quad \phi_{max} = B_{max}\times S = 10^{-3}\times 10\ (m^2)$$
$$= 10^{-2}\ \text{Weber}.$$

$$|e| = \frac{d\phi_{max}}{dt}$$
$$= \frac{\phi_{max}}{\Delta t} = \frac{10^{-2}}{\underline{2\cdot 10^{-6}}} =$$

Croissance du courant.

$$= 0{,}5\cdot 10^{4} \simeq \underline{5\ kV}$$

39. $$I = \iint \vec{j}\cdot\vec{dS} = j\pi R^2$$

$$j = \frac{I}{\pi R^2} \qquad \vec{j} = \frac{I}{\pi R^2}\vec{e_z}$$

40. $$\vec{j} = \sigma\vec{E} \Rightarrow \vec{E} = \frac{\vec{j}}{\sigma}$$

$$(P_j = \vec{j}\cdot\vec{E} = \sigma\vec{E}\cdot\vec{E} = \sigma E^2)$$

$$P_j = \vec{j}\cdot\vec{E} = \frac{j^2}{\sigma} = \frac{I^2}{(\pi R^2)^2\sigma}$$

41. $$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{r P_j}{\lambda}$$

$$r\frac{dT}{dr} = -\frac{r^2 P_j}{2\lambda} + cte_1$$

$$\frac{dT}{dr} = -\frac{r P_j}{2\lambda} + \frac{cte_1}{r}$$

$$T(r) = -\frac{r^2 \rho \dot{\jmath}}{4\lambda} + cte_1 \ln(r) + cte_2$$

$$cte_1 = 0$$

$$T(r) = -\frac{r^2 \rho \dot{\jmath}}{4\lambda} + cte_2$$

$$CL : T(0) = T_0 \Rightarrow cte_2 = T_0.$$

$$T(r) = T_0 - \frac{r^2 \rho \dot{\jmath}}{4\lambda}$$

42. $-\lambda \, \overrightarrow{grad}\, T = \overrightarrow{j_{th}} = \overrightarrow{j_{th}}(R)$

$$-\lambda \frac{dT}{dr} = h(T - T_{ext}).$$

$$+\cancel{\lambda}\frac{r \rho \dot{\jmath}}{2\cancel{\lambda}} = h\left(T_0 - \frac{r^2 \rho \dot{\jmath}}{4\lambda} - T_{ext}\right).$$

$$T_0 - \frac{r^2 \rho \dot{\jmath}}{4\lambda} - T_{ext} = +\frac{r \rho \dot{\jmath}}{2h}$$

$$T_0 = T_{ext} + \frac{R^2 \rho \dot{\jmath}}{4\lambda} + \frac{R \rho \dot{\jmath}}{2h}$$

43. $T_0 \simeq T_{ext} + \frac{R \rho \dot{\jmath}}{2h}$

si $\frac{R^2 \cancel{\rho \dot{\jmath}}}{2 \cdot 4\lambda} \ll \frac{R \cancel{\rho \dot{\jmath}}}{2h}$

$$2\frac{R}{\lambda} \ll \frac{1}{h}$$

$$h \ll \frac{2\lambda}{R}$$

ou $1 \ll \frac{4\lambda}{Rh}$

$$\frac{Rh}{2 \cdot 4\lambda} \ll 1$$

Ici : $\frac{R\lambda}{2h} = \frac{\cancel{2}.10^{-3} \times 10}{\cancel{2} \times 400} = \frac{10^{-2}}{4.10^2}$

$= 0{,}25.10^{-4}$

$= 2{,}5.10^{-5}$ ok

44. $T_0 = T_{ext} + \frac{P_J R}{2h}$

$$P_J = \frac{I^2}{(\pi R^2)^2 \gamma}$$

$$T_0 = T_{ext} + \frac{I^2 \cancel{R}}{\pi^2 R^{\cancel{4}\,3} \gamma \times 2h}$$

$$T_0 - T_{ext} = \frac{I^2}{\pi^2 R^3 \gamma \times 2h}$$

$$2(T_0 - T_{ext})\pi^2 R^3 \gamma h = I^2$$

$$I = \sqrt{2\pi^2 R^3 \gamma h (T_0 - T_{ext})}$$

$= 320$ A ?!

45. $p_A + \rho g z_A + \frac{1}{2}\rho v_A^2 = p_B + \rho g z_B + \frac{1}{2}\rho v_B^2$

| Fluide incompressible parfait
| Regime permanent
| Sur 1 ligne de courant.

46. A : lac ~~Noir (Bas)~~ Blanc (Haut)
B : lac ~~Blanc (Haut)~~ Noir (Bas)

$p_A + \rho g z_A + \frac{1}{2}\rho v_A^2 = p_B + \rho g z_B + \frac{1}{2}\rho v_B^2$

$\cancel{p_{atm}} + \rho g z_A + 0 = \cancel{p_{atm}} + \rho g z_B + \frac{1}{2}\rho v_B^2$

$$\frac{1}{2}\cancel{\rho} v_B^2 = \cancel{\rho} g(z_A - z_B)$$

$$v_B^2 = 2g(z_A - z_B)$$

$$v_B = \sqrt{2g(z_A - z_B)} = \sqrt{2gh}$$

$$v_B \simeq \sqrt{2 \times 10 \times 100} \simeq \sqrt{2000}$$

$$\simeq \sqrt{0{,}2 \times 10^4} \simeq 0{,}5 \times 10^2 \simeq \underline{50\ m.s^{-1}}$$

47. $Q = v_B S = v_B . \pi \frac{D^2}{4}$

$$\simeq 50 \times \pi \times \frac{2^2}{4} \simeq 50\pi \simeq \underline{150\ m^3.s}$$

$$Q = \frac{\Delta V}{\Delta t}$$

$$\Delta t = \frac{\Delta V}{Q} = \frac{3.10^6}{150} = \frac{300.10^4}{150} = \underline{2.10^4\ s}$$

48. $D_v\left(\cancel{P_A} + \rho g z_A + \cancel{\frac{1}{2}\rho v_A^2}\right) = \left(\cancel{P_B} + \rho g z_B + \cancel{\frac{1}{2}\rho v_B^2}\right) D_v + P$

$$D_v \rho g h = P.$$

49. $P = 60 \times 1000 \times 10 \times 100$

$$= 6.10^7\ W$$

$$= \underline{60\ MW}$$

50. Rendement !