Fiche n° 8. Trigonométrie

Réponses

8.1 a) .... $\boxed{0}$

8.1 b) .... $\boxed{0}$

8.1 c) .... $\boxed{-1-\sqrt{3}}$

8.1 d) .... $\boxed{-\dfrac{1}{2}}$

8.2 a) .... $\boxed{0}$

8.2 b) .... $\boxed{-\sin x}$

8.2 c) .... $\boxed{2\cos x}$

8.2 d) .... $\boxed{-2\cos x}$

8.3 a) .... $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 b) .... $\boxed{\dfrac{\sqrt{6}+\sqrt{2}}{4}}$

8.3 c) .... $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 d) .... $\boxed{\dfrac{\sqrt{3}-1}{\sqrt{3}+1}}$

8.4 a) .... $\boxed{-\sin x}$

8.4 b) .... $\boxed{\dfrac{1}{\cos x}}$

8.4 c) .... $\boxed{0}$

8.4 d) .... $\boxed{4\cos^3 x - 3\cos x}$

8.5 a) .... $\boxed{\dfrac{\sqrt{2+\sqrt{2}}}{2}}$

8.5 b) .... $\boxed{\dfrac{\sqrt{2-\sqrt{2}}}{2}}$

8.6 a) .... $\boxed{\tan x}$

8.6 b) .... $\boxed{2}$

8.6 c) .... $\boxed{8\cos^4 x - 8\cos^2 x + 1}$

8.7 a) .... $\boxed{\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}}$

8.7 a) .... $\boxed{\left\{-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right\}}$

8.7 a) .... $\boxed{\left\{\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{-\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{-2\pi}{3}, \dfrac{-\pi}{3}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{4\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 c) .... $\boxed{\left\{\dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

8.7 c) .... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}\right\}}$

8.7 c) .... $\boxed{\left\{\dfrac{7\pi}{6}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{6}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 d) .... $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}}$

8.7 d) .... $\boxed{\left\{-\dfrac{3\pi}{4}, \dfrac{\pi}{4}\right\}}$

8.7 d) .... $\boxed{\left\{\dfrac{\pi}{4}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 e) .... $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4} \dfrac{7\pi}{4}\right\}}$

8.7 e) .... $\boxed{\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}}$

8.7 e) .... $\boxed{\left\{\dfrac{\pi}{4}+k\dfrac{\pi}{2}, k\in\mathbb{Z}\right\}}$

8.7 f) .... $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

8.7 f) .... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

8.7 f) .... $\boxed{\left\{\dfrac{\pi}{6}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{6}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 g) .... $\boxed{\left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}}$

8.7 g) .... $\boxed{\left\{-\dfrac{11\pi}{12}, -\dfrac{\pi}{12}, \dfrac{\pi}{12}, \dfrac{11\pi}{12}\right\}}$

8.7 g) .... $\boxed{\left\{\dfrac{\pi}{12}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{12}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 h) .... $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{3\pi}{2}\right\}}$

8.7 h) .... $\boxed{\left\{-\dfrac{\pi}{2}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

8.7 h) .... $\left\{\frac{\pi}{6} + k\frac{2\pi}{3}, k \in \mathbb{Z}\right\}$

8.7 i) .... $\left\{\frac{\pi}{7}, \frac{13\pi}{7}\right\}$

8.7 i) .... $\left\{-\frac{\pi}{7}, \frac{\pi}{7}\right\}$

8.7 i) ...... $\left\{\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{7} + 2k\pi, k \in \mathbb{Z}\right\}$

8.7 j) .... $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

8.7 j) .... $\left\{\frac{5\pi}{14}, \frac{9\pi}{14}\right\}$

8.7 j) ...... $\left\{\frac{5\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\frac{9\pi}{14} + 2k\pi, k \in \mathbb{Z}\right\}$

8.8 a) .... $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, 2\pi\right]$

8.8 a) .... $\left[-\frac{3\pi}{4}, \frac{3\pi}{4}\right]$

8.8 b) .... $\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]$

8.8 b) .... $\left[-\pi, -\frac{\pi}{3}\right] \cup \left[\frac{\pi}{3}, \pi\right]$

8.8 c) .... $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$

8.8 c) .... $\left[-\pi, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

8.8 d) .... $\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]$

8.8 d) .... $\left[-\pi, -\frac{5\pi}{6}\right] \cup \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

8.8 e) .... $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[$

8.8 e) .... $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[\cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[$

8.8 f) ..... $\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right] \cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[\cup \left]\frac{3\pi}{2}, \frac{7\pi}{4}\right]$

8.8 f) .... $\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[\cup \left]-\frac{\pi}{2}, -\frac{\pi}{4}\right] \cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left]\frac{\pi}{2}, \frac{3\pi}{4}\right]$

8.8 g) .... $\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$

8.8 g) .... $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$

8.8 h) .... $\left[0, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \frac{11\pi}{8}\right] \cup \left[\frac{15\pi}{8}, 2\pi\right]$

8.8 h) .... $\left[-\pi, -\frac{5\pi}{8}\right] \cup \left[-\frac{\pi}{8}, \frac{3\pi}{8}\right] \cup \left[\frac{7\pi}{8}, \pi\right]$

Corrigés

8.3 b) On peut utiliser $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ puis les formules d'addition.

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8.4 b) On a

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{\sin 2x \cos x - \cos 2x \sin x}{\sin x \cos x} = \frac{\sin(2x - x)}{\sin x \cos x} = \frac{1}{\cos x}.$$

On peut aussi faire cette simplification à l'aide des formules de duplication :

$$\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{2\sin x \cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x} = \frac{1}{\cos x}$$

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8.4 d) On calcule

$$\cos(3x) = \cos(2x + x) = \cos(2x)\cos x - \sin(2x)\sin x = (2\cos^2 x - 1)\cos x - 2\cos x \sin^2 x$$
$$= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) = 4\cos^3 x - 3\cos x.$$

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8.5 a) On a $\cos\frac{\pi}{4} = 2\cos^2\frac{\pi}{8} - 1$ donc $\cos^2\frac{\pi}{8} = \frac{\frac{\sqrt{2}}{2} + 1}{2} = \frac{\sqrt{2} + 2}{4}$. De plus, $\cos\frac{\pi}{8} \geqslant 0$ donc $\cos\frac{\pi}{8} = \frac{\sqrt{2 + \sqrt{2}}}{2}$.

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8.5 b) On a $\sin^2\frac{\pi}{8} = 1 - \cos^2\frac{\pi}{8} = \frac{2 - \sqrt{2}}{4}$ et $\sin\frac{\pi}{8} \geqslant 0$ donc $\sin\frac{\pi}{8} = \frac{\sqrt{2 - \sqrt{2}}}{2}$.

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8.6 a) On a $\cos(2x) = 1 - 2\sin^2 x$ donc $\dfrac{1-\cos(2x)}{\sin(2x)} = \dfrac{2\sin^2 x}{2\sin x \cos x} = \tan x$.

8.6 b) On a $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = \dfrac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} = \dfrac{\sin(3x-x)}{\sin x \cos x} = \dfrac{\sin(2x)}{\sin x} = \dfrac{2\sin x \cos x}{\sin x \cos x} = 2$.

8.6 c) On a $\cos(4x) = 2\cos^2(2x) - 1 = 2(2\cos^2 x - 1)^2 - 1 = 8\cos^4 x - 8\cos^2 x + 1$.

8.7 e) Cela revient à résoudre « $\cos x = \dfrac{\sqrt{2}}{2}$ ou $\cos x = -\dfrac{\sqrt{2}}{2}$ ».

8.7 g) Si on résout avec $x \in [0, 2\pi]$, alors $t = 2x \in [0, 4\pi]$.

Or, dans $[0, 4\pi]$, on a $\cos t = \dfrac{\sqrt{3}}{2}$ pour $t \in \left\{\dfrac{\pi}{6}, \dfrac{11\pi}{6}, \dfrac{13\pi}{6}, \dfrac{23\pi}{6}\right\}$ et donc pour $x \in \left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}$.

8.7 h) $\sin x$ est solution de l'équation de degré 2 : $2t^2 + t - 1 = 0$ dont les solutions sont $t = -1$ et $t = \dfrac{1}{2}$. Ainsi, les x solutions sont les x tels que $\sin x = -1$ ou $\sin x = \dfrac{1}{2}$.

8.7 j) On a $\cos\dfrac{\pi}{7} = \sin\left(\dfrac{\pi}{2} - \dfrac{\pi}{7}\right) = \sin\dfrac{5\pi}{14}$. Finalement, on résout $\sin x = \sin\dfrac{5\pi}{14}$.

8.8 d) Cela revient à résoudre $-\dfrac{1}{2} \leqslant \sin x \leqslant \dfrac{1}{2}$.

8.8 f) On résout « $\tan x \geqslant 1$ ou $\tan x \leqslant -1$ ».

8.8 g) Si $x \in [0, 2\pi]$, alors $t = x - \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}\right]$. On résout donc $\cos t \geqslant 0$ pour $t \in \left[-\dfrac{\pi}{4}, 2\pi - \dfrac{\pi}{4}\right]$ ce qui donne $t \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right]$ et donc $x \in \left[0, \dfrac{3\pi}{4}\right] \cup \left[\dfrac{7\pi}{4}, 2\pi\right]$.

8.8 h) Si $x \in [0, 2\pi]$, alors $t = 2x - \dfrac{\pi}{4} \in \left[-\dfrac{\pi}{4}, 4\pi - \dfrac{\pi}{4}\right]$. On résout donc $\cos t \geqslant 0$ pour $t \in \left[-\dfrac{\pi}{4}, 4\pi - \dfrac{\pi}{4}\right]$ ce qui donne $t \in \left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right] \cup \left[\dfrac{3\pi}{2}, \dfrac{5\pi}{2}\right] \cup \left[\dfrac{7\pi}{2}, \dfrac{15\pi}{4}\right]$ puis $x \in \left[0, \dfrac{3\pi}{8}\right] \cup \left[\dfrac{7\pi}{8}, \dfrac{11\pi}{8}\right] \cup \left[\dfrac{15\pi}{8}, 2\pi\right]$.
