

→ Cette semaine

reprise mercredi 14h (207)

DS Vendredi 8h (151)

- étude des fonctions

- récurrence

- sommes + binôme

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demain + jeudi

Rapports: sommes arithmétiques

$\forall n \in \mathbb{N}$

$$\rightarrow \sum_{k=0}^n k = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

(n) arithmétique (n - m + 1) termes

$$\sum_{k=m}^n u_k = u_m + u_{m+1} + \dots + u_{n-1} + u_n$$

$$= \frac{u_m + u_n}{2} (n - m + 1)$$

Applications

$$A = \sum_{k=0}^n (2k+1)$$

ex: $n=3$

$$A = 1 + 3 + 5 + 7 = 16$$

$$= (n+1)^2 = 4 \times 4$$

$$[m \ m] : m - m + 1$$

I

$$\begin{aligned}
 A &= \sum_{k=0}^m (2k+1) \\
 &= 2 \cdot \sum_{k=0}^m k + \sum_{k=0}^m 1 \quad \begin{array}{l} k=0 \quad k=1 \quad \dots \quad k=m \end{array} \\
 &= 2 \cdot \frac{m(m+1)}{2} + (m+1) \quad \begin{array}{l} \underbrace{1+1+\dots+1}_{(n+1) \text{ terms}} \end{array} \\
 &= (m+1) \left[\frac{2m}{2} + 1 \right] \\
 &= (m+1)^2
 \end{aligned}$$

II

$$\begin{aligned}
 A &= \sum_{k=0}^m (2k+1) = \frac{u_0 + u_m}{2} \times (m - 0 + 1) \\
 &= \frac{(2 \times 0 + 1) + (2m+1)}{2} (m+1) \\
 &= \frac{2m+2}{2} (m+1) \\
 &= (m+1)^2
 \end{aligned}$$

$$B = \sum_{k=\underline{n+1}}^{2n} k = (n+1) + (n+2) + \dots + (2n-1) + 2n$$

$$\sum_{k=0}^n k = \frac{n(n+1)}{2} = (0+1) + 2 + \dots$$

$$B = \frac{u_{n+1} + u_{2n}}{2} \times (2n - (n+1) + 1)$$

$$= \frac{n+1 + 2n}{2} \times (n)$$

$$= \frac{n(3n+1)}{2}$$

Charles

$$\sum_{k=n}^n u_k = u_n + u_{n+1} + \dots + u_n$$

$$\sum_{k=1}^n u_k = u_1 + u_2 + \dots + u_{n-1} + [u_n + \dots + u_n]$$

$$\sum_{k=1}^n u_k = \sum_{k=1}^n u_k - \sum_{k=1}^{n-1} u_k$$

$$\begin{aligned}
 B &= \sum_{k=m+1}^{2m} k = \sum_{k=1}^{2m} k - \sum_{k=1}^m k \\
 &= \frac{2m(2m+1)}{2} - \frac{m(m+1)}{2} \\
 &= \frac{m}{2} [2(2m+1) - (m+1)] \\
 &= \frac{m \cdot (3m+1)}{2}
 \end{aligned}$$

$$B' = \sum_{k=m+1}^{3m} (2k+1)$$

approches : x formule moyenne M

* Relation de Charles C

$$A_m = \sum_{b=0}^m (2b+1) = (m+1)^2 \quad \checkmark$$

M:

$$B' = \frac{(2(n+1)+1) + 2 \cdot 3n+1}{2} [3n - (n+1) + 1]$$

$$= \frac{1}{2} (8n + \cancel{2}) 2n$$

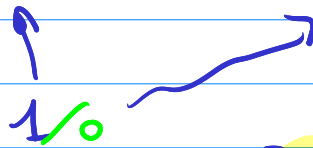
$$= (4n + \cancel{1}) 2n$$

$$= 8n^2 + \cancel{4n}$$



CM

$$B' : \sum_{n+1}^{3n} \rightsquigarrow 3n$$



$$\sum_{1/n}^{3n} = \sum_{1/n}^n + \sum_{n+1}^{3n}$$

$$\sum_{n+1}^{3n} = \sum_{1/n}^{3n} - \sum_{1/n}^n$$

$$B' = \sum_{k=1/n}^{3n} (2k+1) - \sum_{k=1/n}^n (2k+1)$$

$$= A_{3n} - A_n$$

$$= (3n+1)^2 - (n+1)^2$$

$$= 2n \cdot (4n+2) \quad \checkmark$$

$$C = \sum_{k=0}^n (n-2k).$$

$$= \overset{k=0}{n} + \overset{k=1}{(n-2)} + \overset{k=2}{\dots} + \overset{k=n-1}{-n+2} + \overset{k=n}{-n}$$

$$= 0$$

$$= \frac{n_0 + n_n}{2} \times (n+1)$$

$$= 0$$

Sommes géométriques (important)

$$\sum_{k=0}^n q^k = 1 + q + q^2 + \dots + q^n$$

$$= \begin{cases} \frac{1 - q^{n+1}}{1 - q} & q \neq 1 \\ \frac{q^{n+1} - 1}{q - 1} & q = 1 \end{cases}$$

(q^k = 1)

$$\frac{a-b}{c-d} = \frac{b-a}{d-c}$$

Generalisation

$$\sum_{k=m}^n a q^k = \underbrace{u_m + u_{m+1} + \dots + u_n}_{\text{pas dans la somme}} \quad (u_{m+1} \dots u_n)$$

$$= \frac{u_m - u_{m+1}}{1 - q}$$

$$A = \sum_{j=1}^{n+1} 3^j = \sum_{k=1}^{n+1} 3^k$$

$$A' = \sum_{k=0}^n 3^k = \frac{1 - 3^{n+1}}{1 - 3} = \frac{3^{n+1} - 1}{2}$$

$$A'' = \sum_{k=1}^{n+1} 3^k = \frac{3^{n+1} - 3}{3 - 1} = \frac{3}{2} (3^n - 1)$$

$$A = A'' (n \leftarrow n+1)$$

$$= \frac{3}{2} (3^{n+1} - 1)$$

$$\begin{aligned}
 A &= 3 + 9 + 27 + \dots + 3^n + 3^{n+1} \\
 &= 3 (1 + 3 + 9 + \dots + 3^{n-1} + 3^n) \\
 &= 3 \sum_{k=0}^n 3^k = 3 \frac{3^{n+1} - 1}{3 - 1} \\
 &= \frac{3}{2} (3^{n+1} - 1)
 \end{aligned}$$

convertissons le rg pour pouvoir appliquer la formule de réf

$$B = \sum_{k=2}^n \frac{2^{k+1}}{4^k}$$

$$\begin{aligned} a^k b^k &= (ab)^k \\ a^m a^n &= a^{m+n} \end{aligned}$$

$$\left\{ \sum \frac{a_k}{b_k} \neq \sum \frac{a_k}{b_k} \right\}$$

$$= 2 \sum_{k=2}^n \left(\frac{1}{2}\right)^k$$

$$q = \frac{1}{2}$$

$$= 2 \times \frac{1/2^2 - 1/2^{n+1}}{1 - 1/2}$$

$$\begin{aligned} \frac{2^{k+1}}{4^k} &= \frac{2 \times 2^k}{4^k} \\ &= 2 \times \left(\frac{2}{4}\right)^k \\ &= 2 \times \left(\frac{1}{2}\right)^k = \frac{2}{2^k} \end{aligned}$$

$$= 2 \times \frac{1/2^2 (1 - 1/2^{n+1})}{1/2}$$

$$= 1 - \frac{1}{2^{n+1}} \quad \checkmark \checkmark$$

Vérif: nb de termes $2 \leadsto n$

$$\left[(n) - 2 + 1 \right] = n - 1$$

$$\sum_{n=1}^m \text{geom} = 1^{\text{er terme}} \times \frac{1 - q}{1 - q} \quad \text{nb termes}$$

$$= a q^m \cdot \frac{1 - q^{n-m+1}}{1 - q}$$

$$\text{ici: } \frac{2^3}{4^2} \cdot \frac{1 - 1/2^{n-1}}{1 - 1/2} = \frac{1}{2} \left(1 - 1/2^{n-1} \right)$$

La formule du binôme

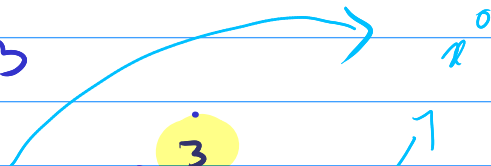
$$(a+b)^2 = a^2 + 2ab + b^2$$

$n=2$

3 termes,

$$(1+x)^2 = 1 + 2x + x^2$$

$$(x+1)^2 = x^2 + 2x + 1$$

$n=3$ 

$$(1+x)^3 = 1 + 3x + 3x^2 + x^3$$

$$(x+1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(a+b)^3 = 1a^3b^0 + 3a^2b + 3ab^2 + 1a^0b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3$$

4 termes.

$$11^3 = (10+1)^3 = (1+10)^3 \quad x=10$$

$$= 1^3 + 3 \cdot 1^2 \cdot 10 + 3 \cdot 1 \cdot 10^2 + 10^3$$

$$= 1 + 3 \times 10 + 3 \times 100 + 1000$$

$$= 1331$$

Coeff binomiaux : triangle de Pascal

$n=3$ →

$n \backslash k$	0	1	2	3	4	5	6
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	
6	1	6	15	20	15	6	1

Annotations: $\binom{n}{k}$ (ordonnée n , abscisse k)

$$(1,01)^3 = (1+x)^3$$

$$x = \frac{1}{10}$$

$$= 1 + 3 \times \frac{1}{10} + \frac{3 \times 1}{10^2} + \frac{1}{10^3}$$

$$= 1,331 + 0,03 + 0,001$$

$$(1,01)^3 = 1 + \frac{3}{100} + \frac{3}{10000} + \frac{1}{1000000}$$

$$= 1,030301$$

$\times 2 \quad \times 4 \quad \times 8$

$$(1,02)^3 = 1 + 3 \times 0,02 + 3 \times (0,02)^2 + (0,02)^3$$

$$= 1 + 0,06 + 0,0012 + 0,000008$$

$$= 1,061208$$

$$(1001)^5 = (x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

$$= (1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

$$x = 1000$$

$$x^2 = 1,000,000$$

$$x^3 = 1,000,000,000$$

⋮

$$\begin{array}{l} \swarrow 10^{15} \quad \swarrow 5 \cdot 10^{12} \quad \swarrow 10 \times 10^9 = 10^{10} \quad \swarrow 10 \times 10^6 \quad \swarrow 500 \quad \swarrow \\ 10^{15} \quad 5 \cdot 10^{12} \quad 10 \times 10^9 = 10^{10} \quad 10 \times 10^6 \quad 500 \end{array}$$

$$(1001)^5 = 1 \ 005 \ 010 \ 010 \ 005 \ 001$$

$$(\sqrt{3} + 1)^4 = (x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$$

$$= 9 + 4 \times 3\sqrt{3} + 6 \times 3 + 4\sqrt{3} + 1$$

$$x = \sqrt{3} \quad \nearrow \times \sqrt{3}$$

$$x^2 = 3 \quad \nearrow \sqrt{3}$$

$$x^3 = 3\sqrt{3} \quad \nearrow$$

$$x^4 = 9 \quad \nearrow$$

$$= (9 + 6 \times 3 + 1) + (4 \times 3 + 4)\sqrt{3}$$

$$= 28 + 16\sqrt{3}$$

$$(\sqrt{2} - 1)^5$$

$$(\sqrt{2}-1)^5 = (x-1)^5$$

$$(1+x)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

$$(1-x)^5 = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$$

$(-x)^2 = x^2$ $(-x)^3 = -x^3$

$$(x+1)^5 = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$$

$$(x-1)^5 = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

$(-1)^5$

$$(2+a)^4 = 2^4 + 4 \cdot 2^3 a + 6 \dots$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(m+n)^4 = m^4 + 4m^3n + 6m^2n^2 + 4mn^3 + n^4$$

$\forall n \in \mathbb{N}$

$$(-1)^n = \begin{cases} +1 & \text{si } n \text{ pair} \\ -1 & \text{impair} \end{cases}$$

$$(-x)^n = (-1)^n \cdot x^n$$

$$= \begin{cases} x^n & n \text{ pair} \\ -x^n & \text{impair} \end{cases}$$

Demain:

* Terminer ces calculs

de forme du binôme

* (apprendre à l'écrite

$$\rightarrow (a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

* Relation de Charles

$$\sum_{k=0}^{n+1} a_k = \sum_{k=0}^n a_k + a_{n+1}$$