Fiche n° 9. Notation arccos, arcsin, arctan

Réponses

9.1 a) .... $\boxed{\dfrac{\pi}{6}}$

9.1 b) .... $\boxed{2}$

9.1 c) .... $\boxed{\dfrac{\pi}{4}}$

9.1 d) .... $\boxed{\dfrac{\pi}{6}}$

9.1 e) .... $\boxed{\dfrac{\pi}{4}}$

9.1 f) .... $\boxed{\dfrac{\pi}{3}}$

9.2 a) .... $\boxed{0}$

9.2 b) .... $\boxed{\dfrac{\pi}{2}}$

9.2 c) .... $\boxed{\dfrac{\pi}{2}}$

9.2 d) .... $\boxed{\dfrac{\pi}{3}}$

9.2 e) .... $\boxed{0}$

9.2 f) .... $\boxed{-\dfrac{\pi}{3}}$

9.3 a) .... $\boxed{1}$

9.3 b) .... $\boxed{\pi-2}$

9.3 c) .... $\boxed{3}$

9.3 d) .... $\boxed{\dfrac{\pi}{17}}$

9.3 e) .... $\boxed{3-\pi}$

9.3 f) .... $\boxed{-\dfrac{\pi}{7}}$

9.4 a) .... $\boxed{1}$

9.4 b) .... $\boxed{0}$

9.4 c) .... $\boxed{\left\{\dfrac{\pi}{2}+k\pi,\ k\in\mathbb{Z}\right\}}$

9.4 d) .... $\boxed{\begin{array}{l}\left\{\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z}\right\}\\ \cup\left\{\frac{2\pi}{3}+2k\pi,\ k\in\mathbb{Z}\right\}\end{array}}$

9.4 e) .... $\boxed{\begin{array}{l}\left\{\frac{1}{3}+2k\pi,\ k\in\mathbb{Z}\right\}\\ \cup\left\{\pi-\frac{1}{3}+2k\pi,\ k\in\mathbb{Z}\right\}\end{array}}$

9.4 f) .... $\boxed{1}$

9.5 a) .... $\boxed{x\longmapsto\dfrac{2}{1+4x^2}}$

9.5 b) .... $\boxed{x\longmapsto 0}$

9.5 c) .... $\boxed{x\longmapsto\dfrac{1}{1+x^2}}$

Corrigés

9.1 b) On calcule : $\dfrac{\arcsin\left(\frac{\sqrt{3}}{2}\right)}{\arccos\left(\frac{\sqrt{3}}{2}\right)}=\dfrac{\frac{\pi}{3}}{\frac{\pi}{6}}=2.$

9.1 c) On remarque que $\arccos\left(\dfrac{1}{\sqrt{2}}\right)=\arccos\left(\dfrac{\sqrt{2}}{\sqrt{2}\times\sqrt{2}}\right)=\arccos\left(\dfrac{\sqrt{2}}{2}\right)=\dfrac{\pi}{4}.$

9.1 d) On remarque que $\arctan\left(\dfrac{\sqrt{3}}{3}\right)=\arctan\left(\dfrac{1}{\sqrt{3}}\right)=\arctan\left(\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=\dfrac{\pi}{6}.$

9.1 f) On remarque que $\arccos\left(\dfrac{1}{3}+\dfrac{1}{6}\right)=\arccos\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}.$

9.3 a) $1\in\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right]$ donc $\arcsin(\sin(1))=1$

9.3 b) $\sin(2)=\sin(\pi-2)$ et $\pi-2\in\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right]$ donc $\arcsin(\sin(2))=\pi-2$

9.3 c) $3\in[0;\pi]$ donc $\arccos(\cos(3))=3$

9.3 d) $\cos\left(-\frac{\pi}{17}\right) = \cos\left(\frac{\pi}{17}\right)$ et $\frac{\pi}{17} \in [0;\pi]$ donc $\arccos\left(\cos\left(-\frac{\pi}{17}\right)\right) = \frac{\pi}{17}$

9.3 e) $\tan(3) = \tan(3-\pi)$ et $3-\pi \in \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$ donc $\arctan(\tan(3)) = 3-\pi$

9.3 f) $\tan\left(-\frac{8\pi}{7}\right) = \tan\left(-\frac{\pi}{7}\right)$ et $-\frac{\pi}{7} \in \left]-\frac{\pi}{2};\frac{\pi}{2}\right[$ donc $\arctan\left(\tan\left(-\frac{8\pi}{7}\right)\right) = -\frac{\pi}{7}$

9.4 b) Soit $x \in [-1,1]$. Alors $\cos(\arccos(x)) = 0 \Leftrightarrow \exists k \in \mathbb{Z},\ \arccos(x) = \frac{\pi}{2} + k\pi$. Mais comme arccos est à valeurs dans $[0,\pi]$, $\cos(\arccos(x)) = 0 \Leftrightarrow \arccos(x) = \frac{\pi}{2} \Leftrightarrow x = 0$.

9.4 c) Soit $x \in \mathbb{R}$. Alors $\arccos(\cos(x)) = 0 \Leftrightarrow \cos(x) = 1 \Leftrightarrow x \in \{2k\pi,\ k \in \mathbb{Z}\}$.

9.4 d) Soit $x \in \mathbb{R}$. Alors

$$\arcsin(\sin(x)) = \frac{\pi}{3} \Leftrightarrow \sin(x) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \Leftrightarrow x \in \left\{\frac{\pi}{3} + 2k\pi,\ k \in \mathbb{Z}\right\} \cup \left\{\frac{2\pi}{3} + 2k\pi,\ k \in \mathbb{Z}\right\}$$

9.4 e) Ici, pas besoin de connaître $\sin\left(\frac{1}{3}\right)$! Soit $x \in \mathbb{R}$. Alors

$$\arcsin(\sin(x)) = \frac{1}{3} \Leftrightarrow \sin(x) = \sin\left(\frac{1}{3}\right) \Leftrightarrow x \in \left\{\frac{1}{3} + 2k\pi,\ k \in \mathbb{Z}\right\} \cup \left\{\pi - \frac{1}{3} + 2k\pi,\ k \in \mathbb{Z}\right\}$$

9.4 f) Soit $x \in \mathbb{R}$. Alors $\tan(\arctan(x)) = 1 \Leftrightarrow \exists k \in \mathbb{Z},\ \arctan(x) = \frac{\pi}{4} + k\pi \Leftrightarrow x = 1$.