Fiche n° 8. Trigonométrie

Réponses

8.1 a) .... $\boxed{-\dfrac{\sqrt{2}}{2}}$

8.1 b) .... $\boxed{1}$

8.1 c) .... $\boxed{1}$

8.1 d) .... $\boxed{-1}$

8.1 e) .... $\boxed{\dfrac{\sqrt{3}}{2}}$

8.1 f) .... $\boxed{-\dfrac{1}{2}}$

8.2 a) .... $\boxed{0}$

8.2 b) .... $\boxed{0}$

8.2 c) .... $\boxed{-1-\sqrt{3}}$

8.2 d) .... $\boxed{1}$

8.2 e) .... $\boxed{-\dfrac{1}{2}}$

8.3 a) .... $\boxed{>0}$

8.3 b) .... $\boxed{<0}$

8.3 c) .... $\boxed{<0}$

8.3 d) .... $\boxed{<0}$

8.3 e) .... $\boxed{>0}$

8.3 f) .... $\boxed{>0}$

8.4 a) .... $\boxed{0}$

8.4 b) .... $\boxed{-\sin x}$

8.4 c) .... $\boxed{2\cos x}$

8.4 d) .... $\boxed{-2\cos x}$

8.5 a) .... $\boxed{\dfrac{\sqrt{2+\sqrt{2}}}{2}}$

8.5 b) .... $\boxed{\dfrac{\sqrt{2-\sqrt{2}}}{2}}$

8.5 c) .... $\boxed{\sqrt{2}-1}$

8.6 a) .... $\boxed{\tan x}$

8.6 b) .... $\boxed{-\dfrac{1}{\cos(x)}}$

8.7 a) .... $\boxed{\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}}$

8.7 a) .... $\boxed{\left\{-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right\}}$

8.7 a) .... $\boxed{\left\{\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{-\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{-2\pi}{3}, \dfrac{-\pi}{3}\right\}}$

8.7 b) .... $\boxed{\left\{\dfrac{4\pi}{3}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{3}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 c) .... $\boxed{\left\{\dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

8.7 c) .... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}\right\}}$

8.7 c) .... $\boxed{\left\{\dfrac{7\pi}{6}+2k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{6}+2k\pi, k\in\mathbb{Z}\right\}}$

8.7 d) .... $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}}$

8.7 d) .... $\boxed{\left\{-\dfrac{3\pi}{4}, \dfrac{\pi}{4}\right\}}$

8.7 d) .... $\boxed{\left\{\dfrac{\pi}{4}+k\pi, k\in\mathbb{Z}\right\}}$

8.7 e) .... $\boxed{\left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}\dfrac{7\pi}{4}\right\}}$

8.7 e) .... $\boxed{\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}}$

8.7 e) .... $\boxed{\left\{\dfrac{\pi}{4}+k\dfrac{\pi}{2}, k\in\mathbb{Z}\right\}}$

8.7 f) .... $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

8.7 f) .... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

8.7 f) .... $\boxed{\left\{\dfrac{\pi}{6}+k\pi, k\in\mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{6}+k\pi, k\in\mathbb{Z}\right\}}$

8.8 a) .... $\boxed{\left[0, \dfrac{3\pi}{4}\right] \cup \left[\dfrac{5\pi}{4}, 2\pi\right]}$

8.8 a) .... $\boxed{\left[-\dfrac{3\pi}{4}, \dfrac{3\pi}{4}\right]}$

8.8 b) .... $\boxed{\left[\frac{\pi}{3}, \frac{5\pi}{3}\right]}$

8.8 b) .... $\boxed{\left[-\pi, -\frac{\pi}{3}\right] \cup \left[\frac{\pi}{3}, \pi\right]}$

8.8 c) .... $\boxed{\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]}$

8.8 c) .... $\boxed{\left[-\pi, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]}$

8.8 d) .... $\boxed{\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \frac{7\pi}{6}\right] \cup \left[\frac{11\pi}{6}, 2\pi\right]}$

8.8 d) .... $\boxed{\left[-\pi, -\frac{5\pi}{6}\right] \cup \left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]}$

8.8 e) .... $\boxed{\left[\frac{\pi}{4}, \frac{\pi}{2}\right[\cup \left[\frac{5\pi}{4}, \frac{3\pi}{2}\right[}$

8.8 e) .... $\boxed{\left[-\frac{3\pi}{4}, -\frac{\pi}{2}\right[\cup \left[\frac{\pi}{4}, \frac{\pi}{2}\right[}$

8.8 f) .... $\boxed{\left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]}$

8.8 f) .... $\boxed{\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]}$

Corrigés

8.5 a) On a $\cos\left(\frac{\pi}{4}\right) = 2\cos^2(\frac{\pi}{8}) - 1$ donc $\cos^2(\frac{\pi}{8}) = \frac{\frac{\sqrt{2}}{2}+1}{2} = \frac{\sqrt{2}+2}{4}$.

De plus, $\cos(\frac{\pi}{8}) \geqslant 0$ donc $\cos(\frac{\pi}{8}) = \frac{\sqrt{2+\sqrt{2}}}{2}$.

8.5 b) On a $\sin^2(\frac{\pi}{8}) = 1 - \cos^2(\frac{\pi}{8}) = \frac{2-\sqrt{2}}{4}$ et $\sin(\frac{\pi}{8}) \geqslant 0$ donc $\sin(\frac{\pi}{8}) = \frac{\sqrt{2-\sqrt{2}}}{2}$.

8.5 c) $\tan\left(\frac{\pi}{8}\right) = \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} = \sqrt{\frac{(2-\sqrt{2})^2}{4-2}} = \frac{2-\sqrt{2}}{\sqrt{2}} = \sqrt{2}-1$

8.6 a) On a $\cos(2x) = 1 - 2\sin^2(x)$ donc $\frac{1-\cos(2x)}{\sin(2x)} = \frac{2\sin^2(x)}{2\sin x\cos(x)} = \tan(x)$.

8.6 b) $\frac{\cos(2x)}{\cos(x)} - \frac{\sin(2x)}{\sin(x)} = \frac{\cos^2(x)-\sin^2(x)}{\cos(x)} - \frac{2\sin(x)\cos(x)}{\sin(x)} = \frac{\cos^2(x)-\sin^2(x)}{\cos(x)} - \frac{2\cos^2(x)}{\cos(x)} = -\frac{\cos^2(x)+\sin^2(x)}{\cos(x)}$

8.7 e) Cela revient à résoudre « $\cos(x) = \frac{\sqrt{2}}{2}$ ou $\cos(x) = -\frac{\sqrt{2}}{2}$ ».

8.8 d) Cela revient à résoudre $-\frac{1}{2} \leqslant \sin(x) \leqslant \frac{1}{2}$.

8.8 f) Si $x \in [0, 2\pi]$, alors $t = x - \frac{\pi}{4} \in \left[-\frac{\pi}{4}, 2\pi - \frac{\pi}{4}\right]$. On résout donc $\cos(t) \geqslant 0$ pour $t \in \left[-\frac{\pi}{4}, 2\pi - \frac{\pi}{4}\right]$ ce qui donne $t \in \left[-\frac{\pi}{4}, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, \frac{7\pi}{4}\right]$ et donc $x \in \left[0, \frac{3\pi}{4}\right] \cup \left[\frac{7\pi}{4}, 2\pi\right]$.
