Remédiation 1

exo 5 :

1) $(x+y)\left(x^2 + \frac{1}{xy} + xy\right) = x^3 + \frac{x}{xy} + x^2y + yx^2 + \frac{y}{xy} + xy^2 = x^3 + \frac{1}{y} + 2x^2y + \frac{1}{x} + xy^2$

2) $\left(\sqrt{x} - y + \frac{\sqrt{x}}{2y}\right)\left(\sqrt{x}\,y - 2 + \frac{1}{y}\right) = \sqrt{x}^2 y - 2\sqrt{x} + \frac{\sqrt{x}}{y} - \sqrt{x}\,y^2 + 2y - \frac{y}{y}$

$+ \frac{\sqrt{x}^2 y}{2y} - \frac{2\sqrt{x}}{2y} + \frac{\sqrt{x}}{2y^2}$

$= xy - 2\sqrt{x} - \sqrt{x}\,y^2 + 2y - 1 + \frac{x}{2} + \frac{\sqrt{x}}{2y^2}$

3) $\left(2x - \frac{y}{2}\right)^2 = 4x^2 - 2 \times 2a \times \frac{y}{2} + \frac{y^2}{4} = 4x^2 - 2xy + \frac{y^2}{4}$

4) $\left(\frac{1}{2} + \sqrt{x}\right)^2 - \left(x + \frac{1}{2\sqrt{x}}\right)^2 = \frac{1}{4} + 2 \times \frac{1}{2} \times \sqrt{x} + \sqrt{x}^2 - \left(x^2 + 2x\frac{1}{2\sqrt{x}} + \frac{1}{4\sqrt{x}^2}\right)$

$= \frac{1}{4} + \sqrt{x} + x - x^2 - \sqrt{x} - \frac{1}{4x} = \frac{1}{4} + x - x^2 - \frac{1}{4x}$

5) $\left(xy + \frac{1}{xy} + 1\right)^2 = \left(xy + \frac{1}{xy} + 1\right)\left(xy + \frac{1}{xy} + 1\right)$

$= x^2y^2 + \frac{xy}{xy} + xy + \frac{xy}{xy} + \frac{1}{x^2y^2} + \frac{1}{xy} + xy + \frac{1}{xy} + 1$

$= x^2y^2 + 2xy + \frac{2}{xy} + 3 + \frac{1}{x^2y^2}$

exo 6 :

1) $\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)abc = \frac{abc}{a} + \frac{abc}{b} + \frac{abc}{c} = bc + ac + ab$

2) $a \times \frac{\frac{1}{a} + 1}{\frac{1}{a} + 2} = \frac{\frac{a}{a} + a}{\frac{1}{a} + 2} = \frac{1 + a}{\frac{1}{a} + 2} = \frac{a(1+a)}{\frac{a}{a} + 2a} = \frac{a^2 + a}{2a + 1}$

3) $\left(\frac{2}{a} + \frac{a}{2} + 2a\right)^2 = \left(\frac{2}{a} + \frac{a}{2} + 2a\right)\left(\frac{2}{a} + \frac{a}{2} + 2a\right)$

$= \frac{4}{a^2} + \frac{2a}{2a} + \frac{4a}{a} + \frac{2a}{2a} + \frac{a^2}{4} + \frac{2a^2}{2} + \frac{4a}{a} + \frac{2a^2}{2} + 4a^2$

$= \frac{4}{a^2} + 1 + 4 + 2 + 4 + a^2\left(\frac{1}{4} + 1 + 1 + 4\right)$

$= \frac{4}{a^2} + 11 + \frac{25}{4}a^2$

4) $\left(\frac{a}{b} + \frac{b}{a}\right) \times \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{a}{b} \times \frac{1}{a} - \frac{a}{b} \times \frac{1}{b} + \frac{b}{a} \times \frac{1}{a} - \frac{b}{a} \times \frac{1}{b}$

$= \frac{1}{b} - \frac{a}{b^2} + \frac{b}{a^2} - \frac{1}{a}$