

## Remédiation 3

exo 2 :

$$1) \sum_{k=1}^n k(k+1) = \sum_{k=1}^n k^2 + \sum_{k=1}^n k = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)[2n+1+3]}{6} = \frac{n(n+1)(n+2)}{3}$$

2) fait en classe

$$3) \sum_{k=0}^n \frac{3^{k+1}}{2^{2k}} = \sum_{k=0}^n \frac{3^k}{4^k} + \frac{1}{4^k} = \sum_{k=0}^n \left(\frac{3}{4}\right)^k + \sum_{k=0}^n \left(\frac{1}{4}\right)^k$$
$$= \frac{1 - \left(\frac{3}{4}\right)^{n+1}}{1 - \frac{3}{4}} + \frac{1 - \left(\frac{1}{4}\right)^{n+1}}{1 - \frac{1}{4}}$$
$$= 4\left(1 - \frac{3^{n+1}}{4^{n+1}}\right) + \frac{4}{3}\left(1 - \frac{1}{4^{n+1}}\right)$$
$$= 4 + \frac{4}{3} - \frac{3^{n+1}}{4^n} + \frac{1}{3} = \frac{16}{3} - \frac{3^{n+1}}{4^n} + \frac{1}{3}$$

$$4) \sum_{k=1}^n k(n-k) = n \sum_{k=1}^n k - \sum_{k=1}^n k^2 = n \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6}$$
$$= \frac{n(n+1)(3n - (2n+1))}{6} = \frac{(n-1)n(n+1)}{6}$$

exo 3

1) sera corrigé en classe

$$2) \sum_{k=-2}^7 -2 = -2 \times (7 - (-2) + 1) = -20$$

$$3) \sum_{k=1}^{n-1} (k+1)^2 = \sum_{k=1}^{n-1} k^2 + 2 \sum_{k=1}^{n-1} k + \sum_{k=1}^{n-1} 1$$
$$= \frac{(n-1)n(2(n-1)+1)}{6} + 2 \frac{(n-1)n}{2} + n-1$$
$$= \frac{n(n-1)(2n-1)}{6} + n^2 - 1$$
$$= \frac{n-1}{6} \times (n(2n-1) + 6(n+1)) = \frac{(n-1)(2n^2+5n+6)}{6}$$

$$4) \sum_{k=0}^n 2^{-3k} = \sum_{k=0}^n \left(\frac{1}{2^3}\right)^k = \sum_{k=0}^n \left(\frac{1}{8}\right)^k = \frac{1 - \left(\frac{1}{8}\right)^{n+1}}{1 - \frac{1}{8}}$$

$$= \frac{8}{7} \left(1 - \frac{1}{8^{n+1}}\right) = \frac{8 - \frac{1}{8^n}}{7}$$

$$5) \sum_{k=0}^n \sqrt{3}^k = \frac{1 - \sqrt{3}^{n+1}}{1 - \sqrt{3}} = \frac{(1 - \sqrt{3}^{n+1})(1 + \sqrt{3})}{1^2 - \sqrt{3}^2}$$

$$= -\frac{1}{2} (1 - \sqrt{3}^{n+1} + \sqrt{3} - \sqrt{3}^{n+2}) = \frac{\sqrt{3}^{n+1} + \sqrt{3}^{n+2} - 1 - \sqrt{3}}{2}$$

$$6) \sum_{k=0}^n (n-k)^2 = \sum_{k=0}^n (n^2 - 2kn + k^2)$$

$$= \sum_{k=0}^n n^2 - 2n \sum_{k=0}^n k + \sum_{k=0}^n k^2$$

$$= n^2 \times (n+1) - 2n \times \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$