NOM :

PRENOM :

Question 1 (/2 pts). Énoncer la formule de symétrie des coefficients binomiaux.

$\forall n \in \mathbb{N}, \forall k \in [\![0,n]\!], \quad \binom{n}{k} = \binom{n}{n-k}$

Question 2 (/2 pts). Développer rapidement $\left(a+\frac{1}{a}\right)^4$.

$$\left(a+\frac{1}{a}\right)^4 = 1a^0\left(\frac{1}{a}\right)^4 + 4a^1\left(\frac{1}{a}\right)^3 + 6a^2\left(\frac{1}{a}\right)^2 + 4a^3\left(\frac{1}{a}\right)^1 + 1a^4\left(\frac{1}{a}\right)^0$$
$$= \frac{1}{a^4} + \frac{4}{a^2} + 6 + 4a^2 + a^4$$

Question 3 (/3 pts). Calculer $S = \sum_{k=0}^{n} \binom{n}{k} 2^{3k+1} 3^{n-k}$ et $T = \sum_{k=0}^{n} 2^k 3^{n-k}$.

$$S = \sum_{k=0}^{n} \binom{n}{k} (2^3)^k \times 2^1 \times 3^{n-k} = 2\sum_{k=0}^{n} \binom{n}{k} 8^k 3^{n-k} = 2(8+3)^n = 2\times 11^n$$

$$T = \sum_{k=0}^{n} 2^k 3^{n-k} = \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k \times 3^n = 3^n \sum_{k=0}^{n} \left(\frac{2}{3}\right)^k = 3^n \times \frac{1-\left(\frac{2}{3}\right)^{n+1}}{1-\frac{2}{3}}$$
$$= 3^n \times \frac{1}{\frac{1}{3}} \times \left(1-\left(\frac{2}{3}\right)^{n+1}\right) = 3^{n+1}\left(1-\frac{2^{n+1}}{3^{n+1}}\right) = 3^{n+1} - 2^{n+1}$$

Question 4 (/3 pts). Calculer $S = \sum_{k=1}^{n} \binom{n+1}{k} 2^{k-n-1}$.

$$S = \sum_{k=0}^{n+1} \binom{n+1}{k} 2^{k-n-1} - \binom{n+1}{0} 2^{0-n-1} - \binom{n+1}{n+1} 2^{n+1-n-1}$$
$$= \sum_{k=0}^{n+1} \binom{n+1}{k} \left(\frac{1}{2}\right)^{n+1-k} 1^k - 2^{-n-1} - 1$$
$$= \left(1+\frac{1}{2}\right)^{n+1} - 2^{-n-1} - 1$$
$$= \left(\frac{3}{2}\right)^{n+1} - \left(\frac{1}{2}\right)^{n+1} - 1$$