

Fiche n° 10. Dérivation

Réponses

10.1 a) $6x^2 + 2x - 11$

10.1 b) $5x^4 - 6x^2 + 4x - 15$

10.1 c) $(2x^2 - 2x + 10) \exp(2x)$

10.1 d) $(6x - 1) \ln(x - 2) + \frac{3x^2 - x}{x - 2}$

10.2 a) $5(x^2 - 5x)^4(2x - 5)$

10.2 b) $4(2x^3 + 4x - 1)(3x^2 + 2)$

10.2 c) $8 \cos^2(x) - 6 \cos(x) \sin(x) - 4$

10.2 d) $-3(3 \cos(x) - \sin(x))^2(3 \sin(x) + \cos(x))$

10.3 a) $\frac{2x}{x^2 + 1}$

10.3 b) $\frac{1}{x \ln(x)}$

10.3 c) $(-2x^2 + 3x - 1) \exp(x^2 + x)$

10.3 d) $6 \cos(2x) \exp(3 \sin(2x))$

10.3 e) $\frac{\cos(x)}{2\sqrt{\sin(x)}}$

10.3 f) $\frac{\cos(\sqrt{x})}{2\sqrt{x}}$

10.4 a) $\frac{(2x + 3)(2 \sin(x) + 3) - (x^2 + 3x) \times 2 \cos(x)}{(2 \sin(x) + 3)^2}$

10.4 b) $\frac{2 - 3x}{2\sqrt{x}(3x + 2)^2}$

10.4 c) $-2 \frac{(x^2 + 1) \sin(2x + 1) + x \cos(2x + 1)}{(x^2 + 1)^2}$

10.4 d) $\frac{(4x + 3) \ln(x) - 2x - 3}{(\ln(x))^2}$

10.5 a) $2x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$

10.5 b) $\frac{9}{(9 - x^2)\sqrt{9 - x^2}}$

10.5 c) $\frac{1}{1 - x^2}$

10.5 d) $\frac{x \cos(x) - \sin(x)}{x \sin(x)}$

10.6 a) $\frac{10x - 5}{(3 - x)^2(2 + x)^2}$

10.6 b) $\frac{2}{x + 1} \left(x + \frac{1 + \sqrt{3}}{2}\right) \left(x + \frac{1 - \sqrt{3}}{2}\right)$

10.6 c) $\frac{2x^2 + 2x + 5}{(x + 2)(x - 1)^2}$

10.6 d) $\frac{x^2}{(x + 1)^2}$

10.6 e) $\frac{2}{x(1 - \ln(x))^2}$

Corrigés

10.1 a) On calcule : $f'(x) = (2x + 3)(2x - 5) + (x^2 + 3x + 2) \times 2 = 6x^2 + 2x - 11$.

10.1 b) On calcule : $f'(x) = (3x^2 + 3)(x^2 - 5) + (x^3 + 3x + 2) \times 2x = 5x^4 - 6x^2 + 4x - 15$.

10.1 c) On calcule : $f'(x) = (2x - 2) \exp(2x) + (x^2 - 2x + 6) \times 2 \exp(2x) = (2x^2 - 2x + 10) \exp(2x)$.

10.1 d) On calcule : $f'(x) = (6x - 1) \ln(x - 2) + (3x^2 - x) \times \frac{1}{x - 2} = (6x - 1) \ln(x - 2) + \frac{3x^2 - x}{x - 2}$.

10.2 a) On calcule : $f'(x) = 5(x^2 - 5x)^4(2x - 5)$.

Fiche n° 16. Systèmes linéaires

Réponses

16.1 a) $\{(3, 1)\}$

16.1 b) $\{(7, 2)\}$

16.1 c) $\left\{\left(\frac{1}{3}, \frac{2}{3}\right)\right\}$

16.1 d) $\left\{\left(\frac{\sqrt{2}}{3}, \frac{\sqrt{2}}{2}\right)\right\}$

16.2 a) $\left\{\left(1 - \frac{a}{4}, \frac{-1}{2} + \frac{3}{8}a\right)\right\}$

16.2 b) $(2, -3)$

16.2 c) $\left\{\left(\frac{1}{13}a + \frac{5}{13}a^2, \frac{2}{13}a - \frac{3}{13}a^2\right)\right\}$

16.2 d) $(a - 2a^2, a + a^2)$

16.3 a) $\{(1 + z, -z, z); z \in \mathbb{R}\}$

16.3 b) $\{(1, y, 3 + 2y); y \in \mathbb{R}\}$

16.3 c) $\left\{\left(\frac{13}{6} - \frac{5}{3}z, -\frac{1}{3} + \frac{4}{3}z, z\right); z \in \mathbb{R}\right\}$

16.3 d) $\left\{\left(x, \frac{-5}{12} - \frac{3}{2}x, \frac{-25}{24} - \frac{7}{4}x\right); x \in \mathbb{R}\right\}$

16.4 a) $\{(2, -1, 3)\}$

16.4 b) $\{(-1, 4, 2)\}$

16.4 c) \emptyset

16.4 d) $\left\{\left(-\frac{2}{7} - z, \frac{-3}{7}, z\right); z \in \mathbb{R}\right\}$

16.5 a) $\left\{\left(1, \frac{1}{2}, \frac{1}{2}\right)\right\}$

16.5 b) \emptyset

16.5 c) $\{(5z, 1 - 4z, z); z \in \mathbb{R}\}$

16.5 d) $\left\{\left(1, \frac{1}{a+2}, \frac{1}{a+2}\right)\right\}$

16.6 a) $\{(5, 3, -1)\}$

16.6 b) \emptyset

16.6 c) .. $\left\{\left(\frac{a^2+a-1}{a^3-1}c, \frac{a^2-a-1}{a^3-1}c, \frac{-a^2+a+1}{a^3-1}c\right)\right\}$

16.7 a) $\{(0, 0, 0)\}$

16.7 b) $\{(x, y, -x - y); (x, y) \in \mathbb{R}^2\}$

16.7 c) $\{(z, z, z); z \in \mathbb{R}\}$

Corrigés

16.1 a)

$$\begin{cases} x - 2y = 1 \\ 3x + 4y = 13 \end{cases} \xrightarrow{L_2 \leftarrow L_2 - 3L_1} \begin{cases} x - 2y = 1 \\ 10y = 10 \end{cases} \Leftrightarrow \begin{cases} x = 1 + 2 \times 1 \\ y = 1 \end{cases} \Leftrightarrow \begin{cases} y = 1 \\ x = 3 \end{cases}$$

16.1 b)

$$\begin{cases} 2x + y = 16 \\ x - y = 5 \end{cases} \xrightarrow{L_1 \leftarrow L_1 + L_2} \begin{cases} 3x = 21 \\ x - y = 5 \end{cases} \Leftrightarrow \begin{cases} x = 7 \\ y = 7 - 5 = 2 \end{cases}$$

16.1 c)

$$\begin{cases} 3x - 6y = -3 \\ 2x + 2y = 2 \end{cases} \Leftrightarrow \begin{cases} x - 2y = -1 \\ x + y = 1 \end{cases} \xrightarrow{L_2 \leftarrow L_2 - L_1} \begin{cases} x - 2y = -1 \\ 3y = 2 \end{cases} \Leftrightarrow \begin{cases} x = -1 + 2 \times \frac{2}{3} = \frac{1}{3} \\ y = \frac{2}{3} \end{cases}$$

16.1 d)

$$\begin{cases} 3x - 4y = -\sqrt{2} \\ 6x + 2y = 3\sqrt{2} \end{cases} \xrightarrow{L_2 \leftarrow L_2 - 2L_1} \begin{cases} 3x - 4y = -\sqrt{2} \\ 10y = 5\sqrt{2} \end{cases} \Leftrightarrow \begin{cases} 3x = 4 \times \frac{\sqrt{2}}{2} - \sqrt{2} \\ y = \frac{\sqrt{2}}{2} \end{cases} \Leftrightarrow \begin{cases} x = \frac{\sqrt{2}}{3} \\ y = \frac{\sqrt{2}}{2} \end{cases}$$