ex1 :

1.a) $\left[\frac{x^3}{3}-x\right]_1^2 = \frac{8}{3}-2-\frac{1}{3}+1 = \frac{4}{3}$

b) $\left[\frac{x^6}{6}-5\frac{x^4}{4}\right]_{-1}^1 = 0$

c) $\left[-\cos(t)\right]_{\pi/3}^{\pi/2} = \frac{1}{2}$

d) $\left[2\sqrt{x}\right]_1^2 = 2\sqrt{2}-2$

2.a) $\left[\frac{(x+1)^8}{8}\right]_0^2 = \frac{3^8-1}{8}$

b) $\left[\frac{(2x+1)^8}{16}\right]_0^2 = \frac{5^8-1}{16}$

c) $\left[\frac{e^{2x}}{2}-e^{1-x}\right]_0^1 = \frac{e^2}{2}-1-\frac{1}{2}+e = \frac{e^2+2e-3}{2}$

d) $\left[\frac{2}{3}x\sqrt{x}-\frac{2}{5}x^2\sqrt{x}\right]_0^1 = \frac{2}{3}-\frac{2}{5} = \frac{4}{15}$

3.a) $\left[\frac{1}{6}\sin(x^6)\right]_0^1 = \frac{1}{6}\sin(1)$

b) $\left[-\frac{1}{2}e^{1-x^2}\right]_1^2 = -\frac{1}{2}\left(e^{-3}-1\right) = \frac{1-e^{-3}}{2}$

c) $\left[\frac{\ln(x)^6}{6}\right]_1^e = \frac{1}{6}$

d) $\left[-\frac{1}{3}(t^2+1)^{-3}\right]_0^1 = -\frac{1}{3}\left(2^{-3}-1\right) = \frac{7}{24}$

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ex 2

1.a) $\int_0^{\pi/4} t\cos(2t)\,dt = \left[t\frac{\sin(2t)}{2}\right]_0^{\pi/4} - \int_0^{\pi/4}\frac{\sin(2t)}{2}dt$

$= \frac{\pi}{8}\sin\left(\frac{\pi}{2}\right) - \left[\frac{-\cos(2t)}{4}\right]_0^{\pi/4} = \frac{\pi}{8}+\frac{\cos(\pi/2)}{4}-\frac{\cos(0)}{4} = \frac{\pi-2}{8}$

b) $\int_0^1 t^2e^{-t}dt = \left[-t^2e^{-t}\right]_0^1 - \int_0^1 -2te^{-t}dt = -e^{-1}+2\int_0^1 te^{-t}dt$

$= -e^{-1}+2\left(\left[-te^{-t}\right]_0^1 - \int_0^1 -e^{-t}dt\right)$

$= -e^{-1}+2\left(-1e^{-1}+\left[-e^{-t}\right]_0^1\right)$

$= -e^{-1}-2e^{-1}+2\left(-e^{-1}+1\right) = 2-\frac{5}{e}$

c) $\int_1^2\sqrt{t}\ln(t)\,dt = \left[\frac{2}{3}t\sqrt{t}\ln(t)\right]_1^2 - \int_1^2\frac{2}{3}t\sqrt{t}\,\frac{1}{t}dt$

$= \frac{4\sqrt{2}\ln(2)}{3}-\frac{2}{3}\int_1^2\sqrt{t}\,dt = \frac{4\sqrt{2}\ln(2)}{3}-\frac{2}{3}\left[\frac{2}{3}t\sqrt{t}\right]_1^2$

$= \frac{4\sqrt{2}\ln(2)}{3}-\frac{4}{9}\left(2\sqrt{2}-1\right) = \frac{12\sqrt{2}\ln(2)-8\sqrt{2}+4}{9}$

2.a) $\int_0^x e^t\sin(t)\,dt = \left[e^t\sin(t)\right]_0^x - \int_0^x e^t\cos(t)\,dt$

[ou dans l'autre sens]

$= e^x\sin(x) - \left(\left[e^t\cos(t)\right]_0^x - \int_0^x e^t(-\sin(t))\,dt\right)$

$= e^x\sin(x) - \left(e^x\cos(x)-1+\int_0^x e^t\sin(t)\,dt\right)$

donc $\int_0^x e^t\sin(t)\,dt = \frac{e^x\sin(x)-e^x\cos(x)+1}{2}$

b) $\int_1^x \sin(\ln(t))\,dt = \left[t\sin(\ln(t))\right]_1^x - \int_1^x t\times\frac{1}{t}\cos(\ln(t))\,dt$

$= x\sin(\ln(x)) - \int_1^x\cos(\ln(t))\,dt$

$= x\sin(\ln(x)) - \left[t\cos(\ln(t))\right]_1^x + \int_1^x t\times\frac{1}{t}\times(-\sin(\ln(t)))\,dt$

$= x\sin(\ln(x)) - x\cos(\ln(x)) + 1 - \int_1^x\sin(\ln(t))\,dt$

donc $\int_1^x\sin(\ln(t))\,dt = \frac{x\sin(\ln(x))-x\cos(\ln(x))+1}{2}$

ex 2 :

2\. c) $\int_0^x e^{-t}\ln(e^{2t}+1)\,dt = \left[-e^{-t}\ln(e^{2t}+1)\right]_0^x - \int_0^x -e^{-t} \times \frac{2e^{2t}}{(e^{2t}+1)}\,dt$

$= -e^{-x}\ln(e^{2x}+1) + \ln(2) + \int_0^x \frac{2e^t}{(e^t)^2+1}\,dt$

$= -e^{-x}\ln(e^{2x}+1) + 2\left[\arctan(e^t)\right]_0^x + \ln(2)$

$= -e^{-x}\ln(e^{2x}+1) + 2\arctan(e^x) - \frac{\pi}{2} + \ln(2)$

3\. a) $\int_0^x \arctan(t)\,dt = \left[t\arctan(t)\right]_0^x - \int_0^x \frac{t}{1+t^2}\,dt$

$= x\arctan(x) - \left[\frac{1}{2}\ln(1+t^2)\right]_0^x$

$= x\arctan(x) - \frac{1}{2}\ln(1+x^2)$

b) $\int_1^x t^2\ln(t)\,dt = \left[\frac{t^3}{3}\ln(t)\right]_1^x - \int_1^x \frac{t^3}{3} \times \frac{1}{t}\,dt$

$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\int_1^x t^2\,dt$

$= \frac{x^3}{3}\ln(x) - \frac{1}{3}\left[\frac{t^3}{3}\right]_1^x = \frac{x^3}{3}\ln(x) - \frac{x^3}{9} \quad \left(+\frac{1}{9}\right)$

c) $\int_1^x \ln(t)^2\,dt = \left[\ln(t) \times (t\ln(t)-t)\right]_1^x - \int_1^x (t\ln(t)-t) \times \frac{1}{t}\,dt$

$= \ln(x)\,(x\ln(x)-x) - \int_1^x (\ln(t)-1)\,dt$

$= \ln(x)(x\ln(x)-x) - (x\ln(x)-x+1) + x - 1$

$= \ln(x)(x\ln(x)-x) - x\ln(x) + 2x \quad (-2)$

$= x\ln^2(x) - 2x\ln(x) + 2x \quad (-2)$

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exo 3 :

1: a) $\int_{\ln(\pi/4)}^{\ln(\pi)} e^t\cos(e^t)\,dt = \int_{\pi/4}^{\pi}\cos(u)\,du = \left[\sin(u)\right]_{\pi/4}^{\pi} = -\frac{\sqrt{2}}{2}$

b) $\int_1^4 \frac{(\sqrt{t}-1)^4}{\sqrt{t}}\,dt = \int_1^2 (u-1)^4\,2du = \left[2\frac{(u-1)^5}{5}\right]_1^2 = \frac{2}{5}$

c) $\int_0^{\pi/4}\sqrt{1-\sin(t)}\,dt = \int_0^{\sqrt{2}/2}\sqrt{1-u}\,\frac{du}{\sqrt{1-u^2}} = \int_0^{\sqrt{2}/2}\frac{1}{\sqrt{1+u}}\,du = \left[2\sqrt{1+u}\right]_0^{\sqrt{2}/2}$

$= 2\sqrt{1+\frac{\sqrt{2}}{2}} - 2$

$u = \sin(t)$, $du = \cos(t)\,dt$

$= \sqrt{1-\sin^2(t)}\,dt = \sqrt{1-u^2}\,dt$

car $\cos(t) \geq 0$

car $t \in [0, \pi/4]$

$$\int_1^2 \frac{\ln(x+1)-\ln(x)}{x^2}\,dx = \int_1^2 \ln\left(1+\frac{1}{x}\right) \times \frac{1}{x^2}\,dx$$

$$= \int_1^{1/2} \ln(1+u) \times -du = \int_{1/2}^1 \ln(1+u)\,du = \Big[(1+u)\ln(1+u) - (1+u)\Big]_{1/2}^1$$

$$= 2\ln(2) - 2 - \frac{3}{2}\ln\left(\frac{3}{2}\right) + \frac{3}{2} = -\frac{1}{2} + 2\ln(2) - \frac{3}{2}\ln\left(\frac{3}{2}\right) = \frac{7\ln(2)-3\ln(3)-1}{2}$$

b) $$\int_0^1 \frac{1}{2+e^{-t}}\,dt = \int_1^e \frac{1}{1+2u}\,du = \left[\frac{\ln(1+2u)}{2}\right]_1^e = \frac{\ln(1+2e)-\ln(3)}{2}$$

$u = e^t \quad du = e^t dt = u\,dt$

$\frac{1}{2+e^{-t}}dt = \frac{1}{2+\frac{1}{u}} \times \frac{du}{u} = \frac{1}{1+2u}du$

$u = \sqrt{1+x} \quad du = \frac{1}{2\sqrt{1+x}}dx$

$x = u^2 - 1$

c) $$\int_0^3 \frac{x}{\sqrt{1+x}}\,dx = \int_1^2 (u^2-1) \times 2du = 2\left[\frac{u^3}{3} - u\right]_1^2 = 2\left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right) = \frac{8}{3}$$

3.a) $$\int_e^x \frac{\ln(\ln(t))}{t}\,dt = \int_1^{\ln(x)} \ln(u)\,du = \Big[u\ln(u) - u\Big]_1^{\ln(x)}$$

$$= \ln(x)\ln(\ln(x)) - \ln(x) + 1$$

b) $$\int_0^1 \frac{1}{1+\left(\frac{2}{\sqrt{3}}t - \frac{1}{\sqrt{3}}\right)^2}\,dt = \int_{-\frac{1}{\sqrt{3}}}^{1/\sqrt{3}} \frac{1}{1+u^2} \times \frac{\sqrt{3}}{2}\,du = \frac{\sqrt{3}}{2}\Big[\arctan(u)\Big]_{-\sqrt{3}/3}^{\sqrt{3}/3}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\pi}{6} + \frac{\pi}{6}\right) = \frac{\sqrt{3}\pi}{6}$$

$u = \frac{2}{\sqrt{3}}t - \frac{1}{\sqrt{3}}$

$du = \frac{2}{\sqrt{3}}dt$

c) $$\int_1^x e^{\sqrt{t}}\,dt = \int_1^{\sqrt{x}} e^u\, 2u\,du = 2\Big[ue^u\Big]_1^{\sqrt{x}} - 2\int_1^{\sqrt{x}} e^u\,du$$

$$= 2\sqrt{x}e^{\sqrt{x}} - 2e - 2e^{\sqrt{x}} + 2e$$

$$= 2(\sqrt{x}-1)e^{\sqrt{x}}$$

$u = \sqrt{t} \;/\; du = \frac{1}{2\sqrt{t}}dt = \frac{1}{2u}dt$