

TD 5 :

exo 12

Via $k' = n - k$ c'est-à-dire $k = n - k'$, on a, puisque :

$$0 \leq k \leq n \Leftrightarrow 0 \leq n - k' \leq n \Leftrightarrow -n \leq -k' \leq 0 \\ \Leftrightarrow n \geq k' \geq 0$$

$$S = \sum_{k=0}^n \cos^2\left(\frac{k\pi}{2n}\right) = \sum_{k'=0}^n \cos^2\left(\frac{(n-k')\pi}{2n}\right) \\ = \sum_{k'=0}^n \cos^2\left(\frac{\pi}{2} - \frac{k'\pi}{2n}\right)$$

Or $\forall x \in \mathbb{R}, \cos\left(\frac{\pi}{2} - x\right) = \sin(x)$, donc :

$$S = \sum_{k'=0}^n \sin^2\left(\frac{k'\pi}{2n}\right) = \sum_{k'=0}^n \left(1 - \cos^2\left(\frac{k'\pi}{2n}\right)\right) \\ = \sum_{k'=0}^n 1 - \sum_{k'=0}^n \cos^2\left(\frac{k'\pi}{2n}\right) = (n+1) - S$$

$$\text{dmc } 2S = n+1 \quad \text{dmc } S = \frac{n+1}{2}.$$