TD 5 exo 4 :

3) $$\frac{n(n+1)\times(n+2)\times\dots\times(3n)}{(2n+1)(2n+3)\times\dots\times(4n+1)} = \frac{\prod_{k=n}^{3n} k}{\prod_{k=n}^{2n}(2k+1)} = \frac{P}{Q}$$

où $$P = \prod_{k=n}^{3n} k = \frac{\prod_{k=1}^{3n} k}{\prod_{k=1}^{n-1} k} = \frac{(3n)!}{(n-1)!}$$ et

$$Q = \prod_{k=n}^{2n}(2k+1) = \frac{(2n+1)\times(2n+2)\times(2n+3)\times(2n+4)\times\dots\times(4n)\times(4n+1)}{(2n+2)(2n+4)\times\dots\times(4n)}$$

$$= \frac{\prod_{k=2n+1}^{4n+1} k}{\prod_{k=n+1}^{2n} 2k} = \frac{(4n+1)!}{(2n)!} = \frac{(4n+1)!}{(2n)!}$$

$$\prod_{k=n+1}^{2n} 2 \times \prod_{k=n+1}^{2n} k \qquad 2^n \times \frac{(2n)!}{n!}$$

$$= \frac{(4n+1)!\, n!}{2^n (2n)!^2}$$

Donc le produit souhaité vaut :

$$\frac{P}{Q} = \frac{(3n)!}{(n-1)!} \times \frac{2^n (2n)!^2}{(4n+1)!\, n!}$$