TD 4

exo 17 :

1) $\displaystyle\sum_{k=0}^{n-1} \sin((2k+1)x) = \sum_{k=0}^{n-1} \operatorname{Im}\left(e^{i(2k+1)x}\right)$

$\displaystyle = \operatorname{Im}\left(\sum_{k=0}^{n-1} e^{i(2k+1)x}\right)$

et $\displaystyle\sum_{k=0}^{n-1} e^{i(2k+1)x} = e^{ix} \sum_{k=0}^{n-1} \left(e^{2ix}\right)^k$

$\displaystyle = e^{ix} \times \frac{1-\left(e^{2ix}\right)^n}{1-e^{2ix}}$ correct car $e^{2ix} \neq 1$ car $2x \in]0, 2\pi[$

$\displaystyle = e^{ix} \times \frac{e^{2inx}-1}{e^{2ix}-1}$

$\displaystyle = e^{ix} \times \frac{e^{inx}\left(e^{inx}-e^{-inx}\right)}{e^{ix}\left(e^{ix}-e^{-ix}\right)}$

$\displaystyle = e^{inx} \times \frac{2i\sin(nx)}{2i\sin(x)}$

$\displaystyle = \frac{\sin(nx)}{\sin(x)}\left(\cos(nx)+i\sin(nx)\right)$

donc $\displaystyle\sum_{0}^{n-1} \sin((2k+1)x) = \frac{\sin^2(nx)}{\sin(x)}$

2) Avec $n = 4$ on a $\displaystyle\sum_{k=0}^{3} \sin((2k+1)x) = \sin(x)+\sin(3x)+\sin(5x)+\sin(7x)$

$\displaystyle = \frac{\sin^2(4x)}{\sin(x)}$

Dès lors,

$\sin(x)+\sin(3x)-\sin(4x)+\sin(5x)+\sin(7x)=0$

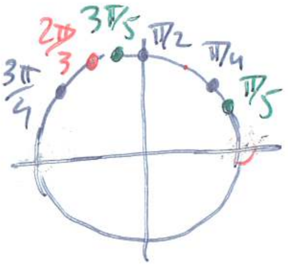
$\displaystyle\Leftrightarrow \frac{\sin^2(4x)}{\sin(x)} - \sin(4x) = 0$

$\displaystyle\Leftrightarrow \sin(4x)=0$ ou $\displaystyle\frac{\sin(4x)}{\sin(x)} - 1 = 0$

$\Leftrightarrow \sin(4x)=\sin(0)$ ou $\sin(4x)=\sin(x)$

$\Leftrightarrow \exists k \in \mathbb{Z} : 4x = k\pi$ ou $4x = x+2k\pi$ ou $4x = \pi - x + 2k\pi$

$\Leftrightarrow \exists k \in \mathbb{Z} : x = k\frac{\pi}{4}$ ou $x = \frac{2k\pi}{3}$ ou $x = \frac{(2k+1)\pi}{5}$



Comme $x \in]0,\pi[$ on trouve finalement comme solu° $S = \left\{ \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{2\pi}{3}, \frac{\pi}{5}, \frac{3\pi}{5} \right\}$