

Remédiation 1 2025/26

Test de penthonne :

$$1) \frac{1}{3} + \frac{2}{5} = \frac{5+6}{15} = \frac{11}{15}$$

$$2) \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{3}$$

$$3) \frac{\frac{2}{3}}{4} = \frac{2}{12} = \frac{1}{6}$$

$$4) \frac{1}{240} + \frac{1}{360} = \frac{1}{120} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{5}{720} = \frac{1}{144}$$

$$5) 3 - \frac{x-1}{2-x} = \frac{3(2-x) - (x-1)}{2-x} = \frac{7-4x}{2-x}$$

$$6) \frac{1}{x^3y} + \frac{1}{xy^2} = \frac{y+x^2}{x^3y^2}$$

$$7) \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$$

$$8) (-2x+y)(3y-x) = -7xy + 3y^2 + 2x^2$$

$$9) (x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$10) \frac{\frac{1}{x} - \frac{1}{x-1}}{\frac{1}{x} + \frac{1}{x-1}} = \frac{x-1-x}{x-1+x} = -\frac{1}{2x-1} = \frac{1}{1-2x}$$

Exercice 1

$$1) \frac{3}{12} + \frac{5}{12} = \frac{8}{12} = \frac{2}{3} ; \quad \frac{2}{3} + \frac{5}{6} = \frac{9}{6} = \frac{3}{2} ; \quad \frac{1}{\frac{5}{3}} + \frac{1}{\frac{4}{5}} = \frac{3}{5} + \frac{5}{4} = \frac{2}{10}$$

$$2) \frac{\frac{2}{3}}{\frac{8}{3}} = \frac{1}{12} ; \quad \frac{2}{\frac{4}{3}} = \frac{3}{2} ; \quad \frac{\frac{7}{5}}{\frac{7}{3}} = \frac{3}{5}$$

$$3) \frac{4 \times 6 \times 9}{5 \times 8 \times 3} = \frac{4 \times 2 \times 3 \times 9}{5 \times 4 \times 2 \times 3} = \frac{9}{5}$$

$$4) 5 \times \frac{2+3}{20} = \frac{2+3}{4} = \frac{5}{4}$$

$$5) \frac{3 \times 4 + 3 \times 7}{24} = \frac{3 \times 11}{3 \times 8} = \frac{11}{8}$$

Exercice 2

$$1) \frac{1}{240} + \frac{1}{360} = \frac{1}{120} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{5}{720} = \frac{1}{144}$$

$$2) \frac{1}{2^3 \times 3^2 \times 5^2} + \frac{1}{2^4 \times 3 \times 5^3} = \frac{2 \times 5 + 3}{2^4 \times 3^2 \times 5^3} = \frac{13}{2^4 3^2 5^3}$$

$$3) \frac{1}{x^2 y^2} + \frac{1}{x y^3} = \frac{y + x}{x^2 y^3}$$

$$4) \frac{x+y^3}{x y^4} - \frac{x-y}{x^2 y} = \frac{x^2 + x y^3 - y^3(x-y)}{x^2 y^4} = \frac{x^2 + y^4}{x^2 y^4}$$

Exercice 3

$$1) 2 + \frac{2+x}{3-x} = \frac{2(3-x) + 2+x}{3-x} = \frac{8-x}{3-x}$$

$$2) 3 - \frac{2-x}{1+x} = \frac{3(1+x) - 2+x}{1+x} = \frac{1+4x}{1+x}$$

$$3) \frac{x}{\frac{1}{x} + \frac{1}{y}} = \frac{x^2 y}{y+x}$$

$$4) \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} = \frac{y+x}{y-x}$$

$$5) \frac{\frac{1}{x} - \frac{1}{x-1}}{\frac{1}{x} + \frac{1}{x-1}} = \frac{x-1-x}{x-1+x} = \frac{1}{1-2x}$$

Exercice 4

$$1) \frac{(a^2 b)^3 (a^{-1} b)^{-2}}{a^3 b^{-2}} = a^6 b^3 a^{-2} b^{-2} a^{-3} b^2 = a^1 b^3$$

$$2) \frac{(2ab)^{-2} (a^{-1} b)^2}{(2a)^{-3} b^2} = \frac{1}{4} a^{-2} b^{-2} a^{-2} b^2 8 a^3 b^{-2} = 2 a^{-1} b^{-2} = \frac{2}{ab^2}$$

$$3) \left(\frac{a^3}{b^2}\right)^3 \times \left(\frac{b^3}{a^2}\right)^{-2} = a^9 b^{-6} b^{-6} a^{-4} = \frac{a^5}{b^{12}}$$

$$4) \frac{(a\sqrt{a})^3}{a} \times \frac{1}{a^2 \sqrt{a}} = \frac{a^3 \sqrt{a}^2 \sqrt{a}}{a^3 \sqrt{a}} = a$$

Exercise 5

$$1) (x+2y)^2 = x^2 + 4xy + 4y^2$$

$$2) (2x-y)^2 = 4x^2 - 4xy + y^2$$

$$3) \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$$

$$4) (-2x+3y)(-y+x) = +2xy - 3y^2 - 2x^2 + 3xy = 5xy - 3y^2 - 2x^2$$

$$5) (x+y)\left(x^2 + \frac{1}{xy} + xy\right) = x^3 + \frac{1}{y} + 2x^2y + \frac{1}{x} + xy^2$$

$$6) (x-y-z)(x+y-z) = x^2 + xy - xz - xy - y^2 + yz - xz - yz + z^2 \\ = x^2 - y^2 + z^2 - 2xz$$

$$7) (x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2xz + 2yz$$

Exercice 7:

$$1) \frac{\frac{2}{3}}{8} = \frac{1}{12} ; \quad \frac{2}{\frac{4}{3}} = \frac{3}{2} ; \quad \frac{\frac{7}{5}}{\frac{7}{3}} = \frac{3}{5}$$

$$2) \frac{1}{240} + \frac{1}{360} = \frac{5}{720} \quad (\text{factoriez par 120 d'abord!})$$

$$3) \frac{1}{2^3 \times 3^2 \times 5^2} + \frac{1}{2^4 \times 3 \times 5^3} = \frac{13}{2^4 \times 3^2 \times 5^3}$$

$$4) \frac{x+y^3}{xy^4} - \frac{x-y}{x^2y} = \frac{x^2+y^4}{x^2y^4}$$

$$5) 3 - \frac{2-x}{1+x} = \frac{1+4x}{1+x}$$

$$6) \frac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{x} - \frac{1}{y}} = \frac{y+x}{y-x} \quad (\text{ne mettez pas } \frac{1}{x} + \frac{1}{y} \text{ et } \frac{1}{x} - \frac{1}{y} \text{ sur le même dénominateur, multipliez simplement par } xy!)$$

$$7) \frac{\frac{1}{x} - \frac{1}{x-1}}{\frac{1}{x} + \frac{1}{x-1}} = \frac{1}{1-2x} \quad (\text{multipliez en haut et en bas par } x(x-1)!)$$

$$8) \frac{1}{1 - \frac{1}{2+x}} = \frac{2+x}{1+x}$$

$$9) \frac{1}{\frac{1}{x} - \frac{2+x}{1 + \frac{1}{x}}} = \frac{1}{\frac{1}{x} - \frac{(2+x)x}{x+1}} = \frac{x(x+1)}{x+1 - x^2(2+x)} = \frac{x^2+x}{-x^3 - 2x^2 + x + 1}$$

$$10) \frac{(2ab)^{-2}(a^{-1}b)^2}{(2a)^{-3}b^2} = \frac{2}{ab^2} \quad // \quad 11) \left(\frac{a^3}{b^2}\right)^3 \times \left(\frac{b^3}{a^2}\right)^{-2} = \frac{a^{13}}{b^{12}}$$

$$12) \frac{(a\sqrt{a})^3}{a} \times \frac{1}{a^2\sqrt{a}} = a$$

Exercice 8:

$$1) (x+2y)^2 = x^2 + 4xy + 4y^2 \quad // \quad (2x-y)^2 = 4x^2 - 4xy + y^2 \quad // \quad (x-\frac{1}{x})^2 = x^2 - 2 + \frac{1}{x^2}$$

$$2) (x+y)(x^2 + \frac{1}{xy} + xy) = x^3 + \frac{x}{xy} + x^2y + yx^2 + \frac{y}{xy} + xy^2 = x^3 + \frac{1}{y} + 2x^2y + \frac{1}{x} + xy^2$$

$$3) (\sqrt{x}y + \frac{\sqrt{x}}{2y})(\sqrt{x}y - 2 + \frac{1}{y}) = \sqrt{x}^2y - 2\sqrt{x} + \frac{\sqrt{x}}{y} - \sqrt{x}y^2 + 2y - \frac{y}{y} \\ + \frac{\sqrt{x}^2y}{2y} - \frac{2\sqrt{x}}{2y} + \frac{\sqrt{x}}{2y^2} \\ = xy - 2\sqrt{x} - \sqrt{x}y^2 + 2y - 1 + \frac{x}{2} + \frac{\sqrt{x}}{2y^2}$$

$$4) (2x - \frac{y}{2})^2 = 4x^2 - 2 \times 2x \times \frac{y}{2} + \frac{y^2}{4} = 4x^2 - 2xy + \frac{y^2}{4}$$

$$5) (\frac{1}{2} + \sqrt{x})^2 - (x + \frac{1}{2\sqrt{x}})^2 = \frac{1}{4} + 2 \times \frac{1}{2} \times \sqrt{x} + \sqrt{x}^2 - (x^2 + 2x \frac{1}{2\sqrt{x}} + \frac{1}{4\sqrt{x}^2}) \\ = \frac{1}{4} + \sqrt{x} + x - x^2 - \sqrt{x} - \frac{1}{4x} = \frac{1}{4} + x - x^2 - \frac{1}{4x}$$

$$6) (xy + \frac{1}{xy} + 1)^2 = (xy + \frac{1}{xy} + 1)(xy + \frac{1}{xy} + 1) = x^2y^2 + 2xy + \frac{2}{xy} + 3 + \frac{1}{x^2y^2}$$

$$7) \frac{xy}{\sqrt{\frac{x}{y}} - 1} \times \frac{x-y}{1 + \sqrt{\frac{x}{y}}} = \frac{xy(x-y)}{\frac{x}{y} - 1} = \frac{-xy^2(x-y)}{x-y} = xy^2$$

(identité rigable) (xy en haut et en bas)

Exercice 9:

$$1) (\frac{1}{a} + \frac{1}{b} + \frac{1}{c})abc = \frac{abc}{a} + \frac{abc}{b} + \frac{abc}{c} = bc + ac + ab$$

$$2) a \times \frac{\frac{1}{a} + 1}{\frac{1}{a} + 2} = \frac{\frac{a}{a} + a}{\frac{1}{a} + 2} = \frac{1+a}{\frac{1}{a} + 2} = \frac{a(1+a)}{\frac{1}{a} + 2a} = \frac{a^2 + a}{2a + 1}$$

$$3) (\frac{2}{a} + \frac{a}{2} + 2a)^2 = (\frac{2}{a} + \frac{a}{2} + 2a)(\frac{2}{a} + \frac{a}{2} + 2a) \\ = \frac{4}{a^2} + \frac{2a}{2a} + \frac{4a}{a} + \frac{2a}{2a} + \frac{a^2}{4} + \frac{2a^2}{2} + \frac{4a}{a} + \frac{2a^2}{2} + 4a^2 \\ = \frac{4}{a^2} + 1 + 4 + 1 + 4 + a^2(\frac{1}{4} + 1 + 1 + 4) \\ = \frac{4}{a^2} + 10 + \frac{25}{4}a^2$$

$$4) (\frac{a}{b} + \frac{b}{a}) \times (\frac{1}{a} - \frac{1}{b}) = \frac{a}{b} \times \frac{1}{a} - \frac{a}{b} \times \frac{1}{b} + \frac{b}{a} \times \frac{1}{a} - \frac{b}{a} \times \frac{1}{b} \\ = \frac{1}{b} - \frac{a}{b^2} + \frac{b}{a^2} - \frac{1}{a}$$

Exercice 10:

$$1) (a-b)(a^2+ab+b^2) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = a^3 - b^3$$

$$2) \frac{(a^3-b^3)(a+b)}{a^2-b^2} = \frac{(a-b)(a^2+ab+b^2)(a+b)}{(a-b)(a+b)} = a^2+ab+b^2$$

3) Écrivons de même que :

$$\begin{aligned} & (a-b)(a^4+a^3b+a^2b^2+ab^3+b^4) \\ &= a^5 + a^4b + a^3b^2 + a^2b^3 + ab^4 \\ & \quad - ba^4 - b^2a^3 - b^3a^2 - b^4a - b^5 \\ &= a^5 - b^5 \end{aligned}$$

Et que :

$$a^4 - b^4 = (a^2)^2 - (b^2)^2 = (a^2 - b^2)(a^2 + b^2) = (a-b)(a+b)(a^2 + b^2)$$

Dès lors :

$$\begin{aligned} \frac{(a^5-b^5)(a^2+b^2)(a+b)}{a^4-b^4} &= \frac{(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)(a^2+b^2)(a+b)}{(a-b)(a+b)(a^2+b^2)} \\ &= a^4 + a^3b + a^2b^2 + ab^3 + b^4. \end{aligned}$$