Remédiation 2 (racines)

Test de positionnement

1) $\sqrt{32} = 4\sqrt{2}$

2) $\frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

3) $\frac{1}{2-\sqrt{5}} = \frac{2+\sqrt{5}}{4-5} = -2-\sqrt{5}$

4) $16^{1/4} = 2$

5) $\frac{x}{\sqrt{2x}} = \frac{\sqrt{2x}}{2}$

6) $(x-\sqrt{x})\left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) = 2 - \sqrt{x} - \frac{\sqrt{x}}{x}$

7) $\frac{1}{1+\frac{1}{1+\sqrt{3}}} = \sqrt{3}-1$

Exercice 1

1) $\sqrt{72} = 6\sqrt{2}$

2) $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

3) $\frac{9}{\sqrt{3}} = 3\sqrt{3}$

4) $\frac{1}{2-\sqrt{5}} = -2-\sqrt{5}$

5) $\frac{1-2\sqrt{2}}{1+\sqrt{2}} = \frac{(1-2\sqrt{2})(1-\sqrt{2})}{1-2}$

$= -1-4+2\sqrt{2}+\sqrt{2}$

$= 3\sqrt{2}-5$

6) $\frac{\sqrt{8}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{5}-\sqrt{2})(\sqrt{3}+\sqrt{2})}{3-2}$

$= \sqrt{15} - \sqrt{6} + \sqrt{10} - 2$

Exercice 2

1) $\left(\frac{5\sqrt{2}}{1+\sqrt{3}}\right)^2 = \frac{50}{4+2\sqrt{3}} = \frac{50(4-2\sqrt{3})}{16-12} = 25(2-\sqrt{3})$

2) $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}\right)^2 = \frac{1}{2} + \frac{1}{3} - \frac{2}{\sqrt{6}}$

$= \frac{5-2\sqrt{6}}{6}$

Exercice 3 :

1) $\frac{x}{\sqrt{x}} = \sqrt{x}$

2) $\frac{x}{\sqrt{2x}} = \frac{\sqrt{2x}}{2}$

3) $\sqrt{x^4} + \sqrt{y^7} = x^2 + y^3\sqrt{y}$

4) $16^{1/4} = 2$

5) $(x-\sqrt{x})\left(\frac{1}{x} - \frac{1}{\sqrt{x}}\right) = 2 - \sqrt{x} - \frac{\sqrt{x}}{x}$

6) $uv = \sqrt{\sqrt{2}+1} \times \sqrt{\sqrt{2}-1}$

$= \sqrt{\sqrt{2}^2 - 1^2} = 1$

Exercice 4

1) $\frac{\sqrt{45}-\sqrt{5}}{\sqrt{10}} = \frac{2\sqrt{5}}{\sqrt{2}\sqrt{5}} = \sqrt{2}$

2) $\frac{9}{\sqrt{3}} = 3\sqrt{3}$

3) $\frac{1-2\sqrt{2}}{1+\sqrt{2}} = 3\sqrt{2}-5$

4) $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \sqrt{15}-\sqrt{6}+\sqrt{10}-2$

5) $\frac{1}{1+\frac{1}{1+\sqrt{3}}} = \frac{1+\sqrt{3}}{1+\sqrt{3}+1}$

(c'est ça la technique la + rapide, pour ne pas avoir 2 fois des qtés conj.)

$= \frac{(1+\sqrt{3})(2-\sqrt{3})}{2^2-\sqrt{3}^2}$

$= \sqrt{3}-1$

6) $\left(\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{3}}\right)^2 = \frac{5-2\sqrt{6}}{6}$

7) $\left(\frac{5\sqrt{2}}{1+\sqrt{3}}\right)^2 = 25(2-\sqrt{3})$

Exercice 5

1) $\frac{x}{\sqrt{2x}} = \frac{\sqrt{2x}}{2}$

2) $\sqrt{x^4}+\sqrt{y^7} = x^2+y^3\sqrt{y}$

3) $16^{1/4} = 2$

4) $(x-\sqrt{x})\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right) = 2-\sqrt{x}-\frac{\sqrt{x}}{x}$

5) $\frac{\sqrt{3x}+x}{x-\sqrt{x}} = \frac{\sqrt{x}\times(\sqrt{3}+\sqrt{x})}{\sqrt{x}\times(\sqrt{x}-1)}$

$= \frac{(\sqrt{3}+\sqrt{x})(\sqrt{x}+1)}{\sqrt{x}^2-1^2} = \frac{x+\sqrt{3x}+\sqrt{x}+\sqrt{3}}{x-1}$

6) $\frac{\sqrt{2x}}{x+\sqrt{x}} = \frac{\sqrt{2}}{\sqrt{x}+1} = \frac{\sqrt{2x}-\sqrt{2}}{x-1}$

Exercice 6

1) $uv = \sqrt[3]{2+\sqrt{5}} \times \left(-\sqrt[3]{\sqrt{5}-2}\right) = -\sqrt[3]{(2+\sqrt{5})(\sqrt{5}-2)} = -\sqrt[3]{1} = -1$

2) $(u+v)^3 = (u+v)^2 \times (u+v) = (u^2+2uv+v^2)\times(u+v)$

$= u^3+3u^2v+3v^2u+v^3$

$= u^3+v^3+3uv(u+v)$

$= \left(\sqrt[3]{2+\sqrt{5}}\right)^3+\left(-\sqrt[3]{\sqrt{5}-2}\right)^3+3\times(-1)\times(u+v)$

$= 2+\sqrt{5}-(\sqrt{5}-2)-3(u+v)$

$= 4-3(u+v)$.