Remédiation 10

Exercice 1

1) $\int_1^2 t^2-3\,dt = \left[\frac{t^3}{3}\right]_1^2 - \left[3t\right]_1^2 = \frac{7}{3} - 3 = -\frac{2}{3}$

2) $\int_{-1}^1 e^{2t}\,dt = \left[\frac{1}{2}e^{2t}\right]_{-1}^1 = \frac{e^2 - e^{-2}}{2}$

3) $\int_0^1 \sin(2\pi x)\,dx = \left[-\frac{\cos(2\pi x)}{2\pi}\right]_0^1 = \frac{-1+1}{2\pi} = 0$

4) $\int_{-2}^2 e^{-t/2}\,dt = \left[-2e^{-t/2}\right]_{-2}^2 = -2e^{-1} + 2e^1 = 2\left(e - \frac{1}{e}\right)$

5) $\int_0^2 \frac{t}{2t^2+1}\,dt = \left[\frac{1}{4}\ln(2t^2+1)\right]_0^2 = \frac{\ln(9)}{4} = \frac{\ln(3)}{2}$

6) $\int_0^2 \frac{t}{\sqrt{2t^2+1}}\,dt = \left[\frac{1}{2}\sqrt{2t^2+1}\right]_0^2 = \frac{\sqrt{9}-\sqrt{1}}{2} = 1$

Exercice 2

1) $\int_1^2 t^2 x\,dt = x\left[\frac{t^3}{3}\right]_1^2 = \frac{7x}{3}$

2) $\int_1^2 t^2 x\,dx = t^2\left[\frac{x^2}{2}\right]_1^2 = \frac{3t^2}{2}$

3) $\int_x^{2x} \frac{1}{t^2}\,dt = \left[-\frac{1}{t}\right]_x^{2x} = -\frac{1}{2x} + \frac{1}{x} = \frac{1}{2x}$

4) $\int_{-T}^T e^{-t/T}\,dt = \left[-Te^{-t/T}\right]_{-T}^T = -Te^{-1} + Te^1 = T\left(e - \frac{1}{e}\right)$

5) $\int_0^\pi \cos(\omega t + \varphi)\,dt = \left[\frac{1}{\omega}\sin(\omega t + \varphi)\right]_0^\pi = \frac{\sin(\varphi + \pi\omega) - \sin(\varphi)}{\omega}$

6) $\int_0^\pi \sin(\omega x + \varphi)\,d\varphi = \left[-\cos(\omega x + \varphi)\right]_0^\pi = \cos(\omega x) - \cos(\omega x + \pi)$

$= \cos(\omega x) + \cos(\omega x)$

$= 2\cos(\omega x)$

Exercice 3 :

1) $\int_1^2 \frac{1}{t^3} dt = \left[-\frac{1}{2t^2}\right]_1^2 = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

2) $\int_0^1 t^n dt = \left[\frac{t^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$

3) $\int_0^\pi \cos^2(x)\sin(x)\,dx = \left[-\frac{\cos^3(x)}{3}\right]_0^\pi = \frac{\cos^3(0) - \cos^3(\pi)}{3} = \frac{2}{3}$

4) $\int_0^{\pi/4} \tan(x)\,dx = \left[-\ln(|\cos(x)|)\right]_0^{\pi/4} = -\ln\left(\cos\left(\frac{\pi}{4}\right)\right) = \frac{1}{2}\ln(2)$

5) $\int_0^x \sqrt{t}\,dt = \left[\frac{2t\sqrt{t}}{3}\right]_0^x = \frac{2x\sqrt{x}}{3}$

6) $\int_{\ln(2)}^{\ln(3)} e^{-2s+1} ds = \left[-\frac{1}{2}e^{-2s+1}\right]_{\ln(2)}^{\ln(3)} = \frac{e^{-2\ln(2)+1} - e^{-2\ln(3)+1}}{2}$

$= \frac{e}{2}\left(e^{\ln(\frac{1}{4})} - e^{\ln(\frac{1}{9})}\right) = \frac{e}{2}\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5e}{72}$

7) $\int_0^\lambda \frac{t}{\lambda} e^{-\lambda t^2} dt = \left[-\frac{1}{2\lambda^2}e^{-\lambda t^2}\right]_0^\lambda = \frac{1 - e^{-\lambda^3}}{2\lambda^2}$

8) $\int_{-x}^{x} \frac{t^3}{1+t^4} dt = \left[\frac{1}{4}\ln(1+t^4)\right]_{-x}^{x} = \frac{\ln(1+x^4) - \ln(1+(-x)^4)}{4} = 0$

9) $\int_{-\theta}^{\theta} \sin(t)\,dt = \left[-\cos(t)\right]_{-\theta}^{\theta} = -\cos(\theta) + \cos(-\theta) = 0$

10) $\int_{-\theta}^{0} \cos(t)\,dt - \int_0^\theta \cos(t)\,dt = \left[\sin(t)\right]_{-\theta}^{0} - \left[\sin(t)\right]_0^\theta$

$= \sin(0) - \sin(-\theta) - \sin(\theta) + \sin(0) = 0$

11) $\int_1^y \ln(t)\,dt = \left[t\ln(t) - t\right]_1^y = y\ln(y) - y + 1$

12) $\int_1^y t\ln(t^2)\,dt = \left[\frac{1}{2}\left(t^2\ln(t^2) - t^2\right)\right]_1^y = \frac{y^2\ln(y^2) - y^2 + 1}{2}$