

Exercice 1

$$\begin{aligned}
 1. \quad & 25 - (10x+3)^2 \\
 &= (5 - (10x+3)) (5 + (10x+3)) \\
 &= (2 - 10x) (8 + 10x) \\
 &= 2(1-5x)2(4+5x) \\
 &= \boxed{4(1-5x)(4+5x)}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & 3x^2 - 3x + \frac{3}{4} \\
 &= 3\left(x^2 - x + \frac{1}{4}\right) \\
 &= \boxed{3\left(x - \frac{1}{2}\right)^2}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & (-9x-8)(8x+8) + 64x^2 - 64 \\
 &= -(9x+8)8(x+1) + 64(x^2-1) \\
 &= -8(9x+8)(x+1) + 64(x-1)(x+1) \\
 &= 8(x+1)[-9x-8 + 8(x-1)] \\
 &= 8(x+1)[-x-16] \\
 &= \boxed{-8(x+1)(x+16)}
 \end{aligned}$$

Exercice 2

$$\begin{aligned}
 1. \quad & (1-\sqrt{2})^2 (2+\sqrt{2}) \\
 &= (1-2\sqrt{2}+2)(2+\sqrt{2}) \\
 &= (3-2\sqrt{2})(2+\sqrt{2}) \\
 &= 6+3\sqrt{2}-4\sqrt{2}-4 \\
 &= \boxed{2-\sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1) \\
 &= x^4 - \cancel{\sqrt{2}x^3} + x^2 + \cancel{\sqrt{2}x^3} - 2x^2 + \cancel{\sqrt{2}x} + x^2 - \cancel{\sqrt{2}x} + 1 \\
 &= \boxed{x^4+1}
 \end{aligned}$$

Exercise 3

$$1. \frac{\frac{4}{9}}{\frac{1}{6}} = \frac{4}{9} \times \frac{6}{1} = \frac{2}{3} = \boxed{\frac{2}{27}}$$

$$2. \frac{10}{\frac{15}{6}} = 10 \times \frac{6}{15} = \frac{2 \times 3 \times 2 \times 3}{3 \times 3} = \boxed{4}$$

$$\begin{aligned} 3. \quad & \frac{1}{(n+1)^2} - \frac{2}{n+1} + n \\ &= \frac{1 - 2(n+1) + n(n+1)^2}{(n+1)^2} \\ &= \frac{1 - 2n - 2 + n(n^2 + 2n + 1)}{(n+1)^2} \\ &= \frac{-1 - 2n + n^3 + 2n^2 + n}{(n+1)^2} \\ &= \boxed{\frac{n^3 + 2n^2 - n - 1}{(n+1)^2}} \end{aligned}$$

$$\begin{aligned} 4. \quad & \frac{a^3 - b^3}{(a-b)^2} - \frac{(a+b)^2}{a-b} \\ &= \frac{(a^3 - b^3) - (a+b)^2(a-b)}{(a-b)^2} \\ &= \frac{a^3 - b^3 - (a^2 + 2ab + b^2)(a-b)}{(a-b)^2} \\ &= \frac{a^3 - b^3 - (a^3 + 2a^2b + ab^2 - a^2b - 2ab^2 - b^3)}{(a-b)^2} \\ &= \frac{\cancel{a^3} - \cancel{b^3} - \cancel{a^3} - a^2b + a^2b + \cancel{ab^2} + \cancel{b^3}}{(a-b)^2} \\ &= \frac{ab(-a+b)}{(a-b)^2} \\ &= \boxed{\frac{ab}{b-a}} \end{aligned}$$

$$\begin{aligned} 5. \quad & \frac{x}{1 - \frac{1}{2 + \frac{3n+1}{n-1}}} = \frac{x}{1 - \frac{1}{\frac{2(n-1) + 3n+1}{n-1}}} = \frac{x}{1 - \frac{n-1}{5n-1}} \\ &= \frac{x}{\frac{5n-1 - (n-1)}{5n-1}} = \frac{x(5n-1)}{4x} = \boxed{\frac{5n-1}{4}} \end{aligned}$$

Exercice 4

$$\begin{aligned} 1. \quad \sqrt{72} &= \sqrt{8 \times 9} = \sqrt{9} \times \sqrt{8} \\ &= 3 \times 2\sqrt{2} \\ &= \boxed{6\sqrt{2}} \end{aligned}$$

$$\begin{aligned} 2. \quad \sqrt{2}-3 < 0 \quad \text{donc } \sqrt{(\sqrt{2}-3)^2} &= \boxed{3-\sqrt{2}} \\ 3. \quad \text{si } a \leq 3 : \quad 3-a &\geq 0 \\ &\text{donc } \sqrt{(3-a)^2} = 3-a \end{aligned}$$

$$\begin{aligned} \text{si } a > 3 : \quad 3-a &< 0 \\ &\text{donc } \sqrt{(3-a)^2} = a-3. \end{aligned}$$

$$\begin{aligned} 4. \quad \left(\frac{5-\sqrt{3}}{\sqrt{2}} \right)^2 &= \frac{(5-\sqrt{3})^2}{(\sqrt{2})^2} = \frac{25 - 2 \times 5\sqrt{3} + 3}{2} = \frac{28}{2} - \frac{2 \times 5\sqrt{3}}{2} \\ &= \boxed{14 - 5\sqrt{3}} \end{aligned}$$