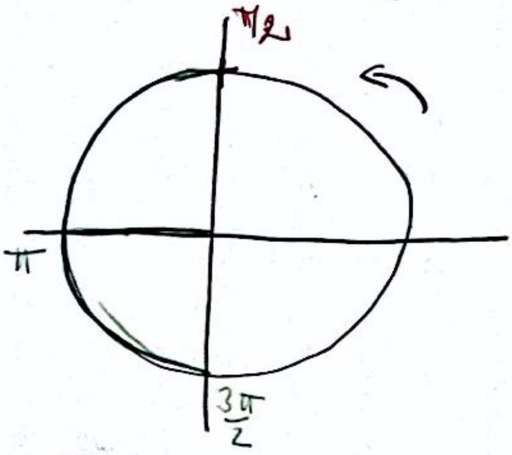
DM2 - 21/10/2022
Corrigé

Exercice 1.

$0 \leq x < \frac{2\pi}{3}$ donc $0 \leq 3x < 2\pi$

$$\frac{\pi}{2} \leq 3x + \frac{\pi}{2} < 2\pi + \frac{\pi}{2}$$



un tour complet "en partant" de $\frac{\pi}{2}$.

$$\cos\left(3x+\frac{\pi}{2}\right) < 0 \iff \frac{\pi}{2} < 3x + \frac{\pi}{2} < \frac{3\pi}{2}$$
$$\iff 0 < 3x < \pi$$
$$\iff 0 < x < \frac{\pi}{3}.$$

Donc :

x	0		$\frac{\pi}{3}$		$\frac{2\pi}{3}$
$\cos\left(3x+\frac{\pi}{2}\right)$		$-$	0	$+$	

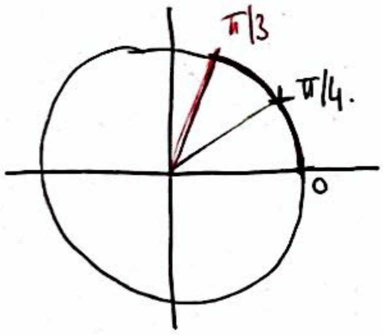
* Si $x \in \left[0, \frac{\pi}{3}\right]$:

$$(*) \iff -\cos\left(3x+\frac{\pi}{2}\right) = \sin x$$
$$\iff \cos\left(3x+\frac{\pi}{2}\right) = -\sin x$$
$$\iff \sin\left(\frac{\pi}{2} - \left(3x+\frac{\pi}{2}\right)\right) = -\sin x \qquad \left(\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta\right)$$
$$\iff \sin(-3x) = -\sin x$$
$$\iff \cancel{-}\sin(3x) = \cancel{-}\sin x$$

$$\iff \begin{cases} 3x = x + 2k\pi,\ k \in \mathbb{Z} \\ \text{ou} \\ 3x = \pi - x + 2l\pi,\ l \in \mathbb{Z} \end{cases}$$

$$\iff \begin{cases} x = k\pi,\ k \in \mathbb{Z} \\ \text{ou} \\ x = \frac{\pi}{4} + \frac{l\pi}{2},\ l \in \mathbb{Z} \end{cases}$$

or $x \in \left[0, \frac{\pi}{3}\right]$ donc $\underline{(*) \iff x = 0 \text{ ou } x = \frac{\pi}{4}}$



<u>* si $x \in \left]\frac{\pi}{3}, \frac{2\pi}{3}\right[$:</u>

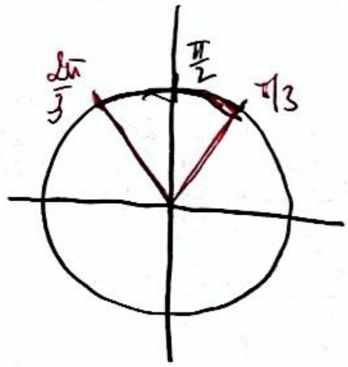
$$(*) \iff \cos\left(3x + \frac{\pi}{2}\right) = \sin x$$

$$\iff \sin\left(\frac{\pi}{2} - \left(3x + \frac{\pi}{2}\right)\right) = \sin x$$

$$\iff \sin(-3x) = \sin x$$

$$\iff \begin{cases} -3x = x + 2k\pi,\ k \in \mathbb{Z} \\ \text{ou} \\ -3x = \pi - x + 2l\pi,\ l \in \mathbb{Z} \end{cases}$$

$$\iff \begin{cases} x = \frac{k\pi}{2},\ k \in \mathbb{Z} \\ \text{ou} \\ x = -\frac{\pi}{2} + l\pi,\ l \in \mathbb{Z} \end{cases}$$



or $x \in \left]\frac{\pi}{3}, \frac{2\pi}{3}\right[$ donc : $\underline{(*) \iff x = \frac{\pi}{2}}$

<u>Conclusion:</u> $\boxed{S = \left\{0, \frac{\pi}{4}, \frac{\pi}{2}\right\}}$

Exercice 2.

$$\forall n \in \mathbb{N}^*, \quad S_n = \sum_{k=1}^{n} \frac{1}{k}\binom{n}{k-1}.$$

Formule du chef : $\binom{n+1}{k} = \frac{n+1}{k}\binom{n}{k-1}$ donc :

$$S_n = \sum_{k=1}^{n} \frac{1}{n+1}\binom{n+1}{k} = \frac{1}{n+1}\sum_{k=1}^{n}\binom{n+1}{k}$$

$$= \frac{1}{n+1}\left(\underbrace{\sum_{k=0}^{n+1}\binom{n+1}{k}}_{2^{n+1}} - \underbrace{\binom{n+1}{0}}_{1} - \underbrace{\binom{n+1}{n+1}}_{1}\right)$$

Conclusion : $\boxed{S_n = \frac{2^{n+1}-2}{n+1}}$