DM5 - Corrigé
25/11/2024.

Exercice 1

$$\forall n \in \mathbb{N}^*, \quad S_n = \frac{1}{2} \sum_{k=1}^{2n} \binom{2n}{k} (-1)^k 2^k$$

$$= \frac{1}{2} \sum_{k=1}^{2n} \binom{2n}{k} (-2)^k$$

$$= \frac{1}{2} \left(\sum_{k=0}^{2n} \binom{2n}{k} (-2)^k - \binom{2n}{0} (-2)^0 \right)$$

$$= \frac{1}{2} \left((-2+1)^{2n} - 1 \right)$$

$$= \frac{1}{2} \left((-1)^{2n} - 1 \right) = \frac{1}{2} (1 - 1) = \underline{0}$$

Exercice 2

On pose $X = \cos(x) \in [-1, 1]$.

$(E) \Leftrightarrow 2X^2 - 9X + 4 \geq 0.$

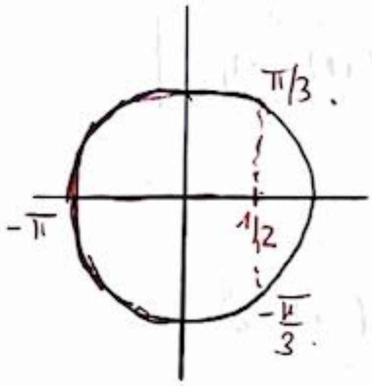
$\Delta = 81 - 4 \times 4 \times 2 = 49 = 7^2 > 0$

donc 2 racines réelles : $X = \frac{9-7}{4} = \frac{2}{4} = \frac{1}{2} \in [-1, 1]$

ou $X = \frac{9+7}{4} = \frac{16}{4} = 4 \notin [-1, 1]$

X	-1		$1/2$		1
$2X^2 - 9X + 4$		$+$	0	$-$	

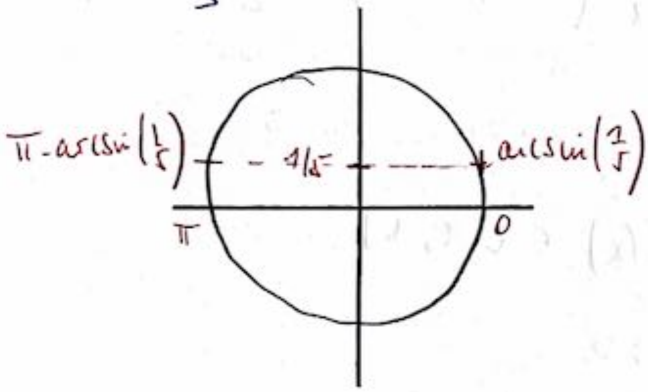
Donc $(E) \Leftrightarrow -1 \leq X \leq \frac{1}{2} \Leftrightarrow -1 \leq \cos(x) \leq \frac{1}{2}$



$$\boxed{\mathcal{S} = \left[-\pi, -\frac{\pi}{3}\right] \cup \left[\frac{\pi}{3}, \pi\right[}$$

Exercice 3

$0 \le x < \frac{2\pi}{3}$ donc $0 \le 3x < 2\pi$ (un tour)



donc $(E) \iff 3x = \arcsin\left(\frac{1}{5}\right)$ ou $3x = \pi - \arcsin\left(\frac{1}{5}\right)$

$\iff x = \frac{1}{3}\arcsin\left(\frac{1}{5}\right)$ ou $x = \frac{\pi}{3} - \frac{1}{3}\arcsin\left(\frac{1}{5}\right)$

avec :

- $\arcsin\left(\frac{1}{5}\right) \in \left[0, \frac{\pi}{2}\right]$ donc $\frac{1}{3}\arcsin\left(\frac{1}{5}\right) \in \left[0, \frac{2\pi}{3}\right[$ ✓
- $0 \le \arcsin\left(\frac{1}{5}\right) \le \frac{\pi}{2}$ donc $-\frac{\pi}{6} \le -\frac{1}{3}\arcsin\left(\frac{1}{5}\right) \le 0$

$$0 \le \frac{\pi}{6} \le \frac{\pi}{3} - \frac{1}{3}\arcsin\left(\frac{1}{5}\right) \le \frac{\pi}{3} < \frac{2\pi}{3} \checkmark$$

Conclusion : $\boxed{\mathcal{S} = \left\{ \frac{1}{3}\arcsin\left(\frac{1}{5}\right) \; ; \; \frac{\pi}{3} - \frac{1}{3}\arcsin\left(\frac{1}{5}\right) \right\}}$