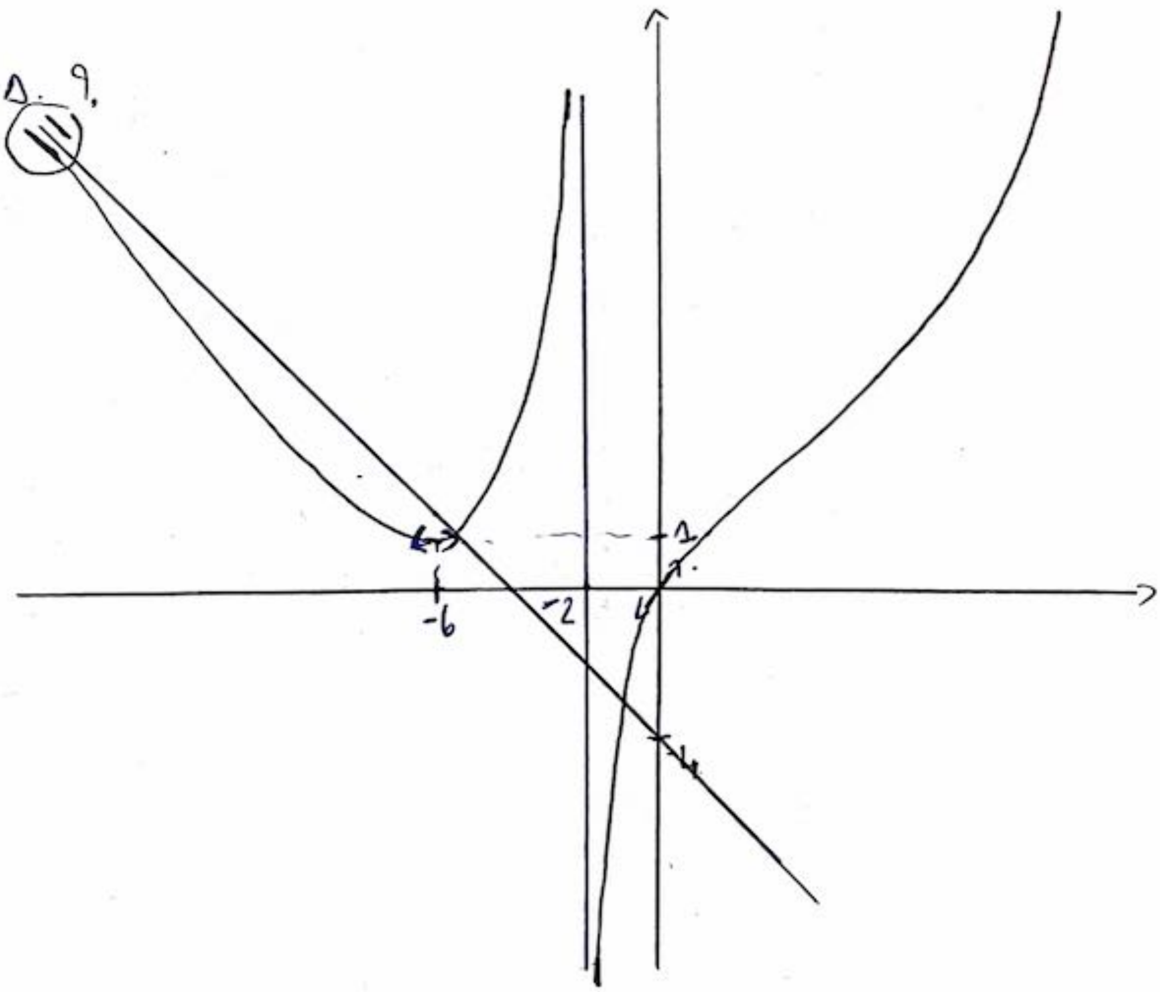
DM6 - Mathématiques -
Corrigé -
15/12/2025

Exercice 1

$$\mathcal{D}_f =]-\infty,-2[\,\cup\,]-2,+\infty[$$

- $\mathcal{C}_f$ admet une tangente horizontale en $(-6,1)$
- $\mathcal{C}_f$ a une asymptote d'équation $y=-x-4$ en $-\infty$
- $\mathcal{C}_f$ a une asymptote verticale $x=2$
- $\mathcal{C}_f$ a une branche parabolique verticale en $+\infty$



Exercice 2.

1.

a.

$$z_1 = \frac{(3+5i)^2\,(1-i\sqrt{3})}{1^2+(\sqrt{3})^2} = \frac{(9+30i-25)\,(1-i\sqrt{3})}{4.} = \frac{(-16+30i)\,(1-i\sqrt{3})}{4}$$

$$= \frac{(-8+15i)\,(1-i\sqrt{3})}{2} = \frac{-8+8i\sqrt{3}+15i-15\sqrt{3}\,i^2}{2}$$

$$= \frac{(-8+15\sqrt{3})+i(8\sqrt{3}+15)}{2}$$

$$\boxed{z_1 = \frac{-8+15\sqrt{3}}{2} + i\;\frac{8\sqrt{3}+15}{2}}$$

b.

- $|1+i\sqrt{3}| = 2\;(>0)$ donc $1+i\sqrt{3} = 2\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right) = 2e^{i\frac{\pi}{3}}$
- $|1-i| = \sqrt{2}\;(>0)$ donc $1-i = \sqrt{2}\left(\frac{1}{2}+i\left(-\frac{1}{\sqrt{2}}\right)\right) = \sqrt{2}\,e^{i\left(-\frac{\pi}{4}\right)}$

donc $$z_2 = \left(\frac{2e^{i\frac{\pi}{3}}}{\sqrt{2}\,e^{i(-\pi/4)}}\right)^{20} = \sqrt{2}^{20}\left(e^{i\left(\frac{\pi}{3}+\frac{\pi}{4}\right)}\right)^{20}$$

$$= 2^{10}\left(e^{i\frac{7\pi}{12}}\right)^{20} = 2^{10}\,e^{i\,\cancel{20}^{5}\times\frac{7\pi}{\cancel{12}_{3}}}$$

$$= 2^{10}\,e^{i\frac{5\times 7\pi}{3}}$$

or $\frac{35\pi}{3} = \frac{36\pi}{3} - \frac{\pi}{3} = 12\pi - \frac{\pi}{3} = 6\times(2\pi) - \frac{\pi}{3}$

donc $\underline{z_2 = 2^{10}\,e^{-i\frac{\pi}{3}}}$

soit $z_2 = 2^{10}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right) = 2^{10} \times \frac{1}{2} + i\left(2^{10} \times \left(-\frac{\sqrt{3}}{2}\right)\right)$

$$\boxed{z_2 = 2^9 + i\left(-2^9\sqrt{3}\right)}$$

(2.)

(a)

. $1+i = \sqrt{2}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\, e^{i\frac{\pi}{4}}$ $\quad (\sqrt{2} > 0)$

. $-i\sqrt{2} = \sqrt{2}\, e^{i\pi}\, e^{i\pi/2} = \sqrt{2}\, e^{i\frac{3\pi}{2}}$

donc $z_3 = \dfrac{\cancel{\sqrt{2}}\, e^{i\frac{3\pi}{2}}}{\cancel{\sqrt{2}}\, e^{i\pi/4}} = e^{i\left(\frac{3\pi}{2} - \frac{\pi}{4}\right)} = e^{i\frac{5\pi}{4}}$

donc $\boxed{z_3 = \cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)}$

(b)

$z_4 = \dfrac{3^3(-1+i)^3}{\sqrt{3}\,(1+\sqrt{3}i)}$ avec $-1+i = \sqrt{2}\left(-\frac{1}{2} + i\frac{1}{\sqrt{2}}\right) = \sqrt{2}\, e^{i\frac{3\pi}{4}}$

et $1+\sqrt{3}i = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2\, e^{i\frac{\pi}{3}}$

donc $z_4 = \dfrac{3^3}{\sqrt{3}} \times \dfrac{(\sqrt{2})^3\left(e^{i\frac{3\pi}{4}}\right)^3}{2\, e^{i\pi/3}} = \dfrac{3^3}{\sqrt{3}} \times \sqrt{2} \times \dfrac{e^{i\left(\frac{9\pi}{4}\right)}}{e^{i\pi/3}}$

$\left(\begin{array}{l} \text{avec } \frac{9\pi}{4} = \frac{\pi}{4} + 2\pi \\ \text{avec } \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12} \end{array}\right)$ donc $z_4 = \dfrac{3^3\sqrt{2}}{\sqrt{3}}\, e^{i\left(\frac{\pi}{4} - \frac{\pi}{3}\right)} = \dfrac{3^3\sqrt{2}}{\sqrt{3}}\, e^{-i\frac{\pi}{12}}$

donc $\boxed{z_4 = \dfrac{3^3\sqrt{2}}{\sqrt{3}}\left(\cos\left(-\frac{\pi}{12}\right) + i\sin\left(-\frac{\pi}{12}\right)\right)}$