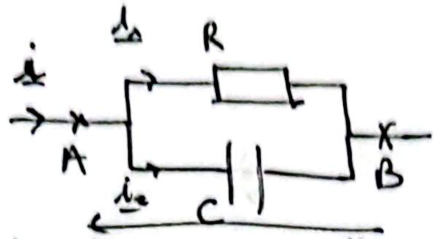
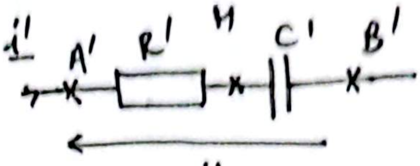
TD Phys 1

Exercice n° 1 :





1) $u_{AB} = u_{A'B'} = u(t) = U_M \cos(\omega t)$. on introduit $\underline{u}$: $\underline{u} = U_M e^{j\omega t}$

- **Dipôle AB :**

$\underline{i} = \underline{i}_1 + \underline{i}_2$ avec $\underline{i}_1 = \dfrac{\underline{u}}{R}$ et $\underline{i}_2 = C\dfrac{d\underline{u}}{dt}$

donc $\underline{i} = \dfrac{\underline{u}}{R} + \dfrac{C d\underline{u}}{dt} = \underline{u}\left(\dfrac{1}{R} + j\omega C\right) \Rightarrow \boxed{\underline{u} = \dfrac{R}{1+j\omega RC}\,\underline{i}}$

- **Dipôle A'B' :**

$\underline{u} = \underline{u}_{A'M} + \underline{u}_{MB'}$ avec $\underline{u}_{A'M} = R'\underline{i}'$ et $\underline{i}' = C'\dfrac{d\underline{u}_{MB'}}{dt}$

on a donc $\underline{i}' = j\omega C'\underline{u}_{MB'}$

donc $\boxed{\underline{u} = R'\underline{i}' + \dfrac{\underline{i}'}{j\omega C'} = R'\left(1 - \dfrac{j}{R'C'\omega}\right)\underline{i}'}$

2) On cherche ω tel que $\dfrac{R}{1+j\omega RC} = R'\left(1 - \dfrac{j}{R'C'\omega}\right)$

$\Leftrightarrow R = R'\left(1 + j\omega RC - \dfrac{j}{R'C'\omega} + \dfrac{\omega RC}{\omega R'C'}\right) = \left[\left(1 + \dfrac{RC}{R'C'}\right) + j\left(\omega RC - \dfrac{1}{\omega R'C'}\right)\right]R'$

Par identification : $R = R'\left(1 + \dfrac{RC}{R'C'}\right)$ et $\omega RC - \dfrac{1}{\omega R'C'} = 0$.

$\Leftrightarrow \omega^2 RCR'C' = 1$

$\Rightarrow R'C' = \dfrac{1}{\omega^2 RC} \Rightarrow \boxed{R' = \dfrac{R}{1+\omega^2R^2C^2}}$ et $\boxed{C' = \dfrac{1+\omega^2R^2C^2}{R^2C\omega^2}}$

3) $\underline{i} = \left(\dfrac{1}{R} + j\omega C\right)\underline{u} = U_m\left(\dfrac{1}{R} + j\omega C\right)e^{j\omega t} = I_M e^{j(\omega t + \varphi)}$

$\Rightarrow |\underline{i}| = |U_M|\left|\left(\dfrac{1}{R} + j\omega C\right)\right|\left|e^{j\omega t}\right| = \boxed{U_M\sqrt{\left(\dfrac{1}{R}\right)^2 + \omega^2C^2} = I_M}$

$\varphi = \arg\left(\dfrac{\underline{i}}{\underline{u}}\right) = \arg\left(\dfrac{1}{R} + j\omega C\right) = \underline{\arctan(RC\omega)} > 0$ i en avance sur u.