Feuille Calculs_7 : La fonction de répartition à partir d'une densité.

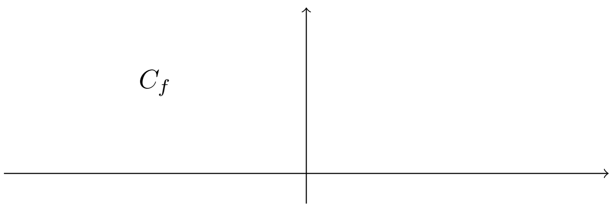
Méthode. *Comment déterminer la fonction de répartition à partir d'une densité ?*

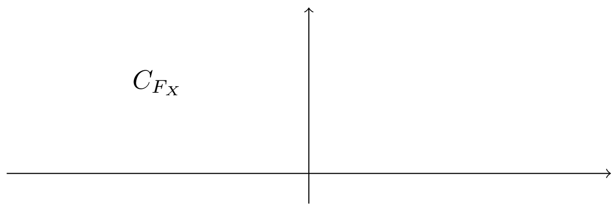
Dans tous les exercices suivants on donne f une densité de probabilité d'une variable aléatoire X,

On vérifiera rapidement avec une représentation graphique que f est bien une densité de probabilité.

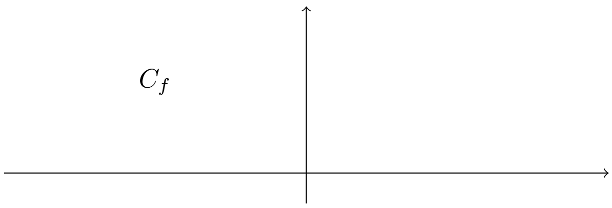
Donner sa fonction de répartition F_X et tracer l'allure des deux représentations graphiques dans deux repères.

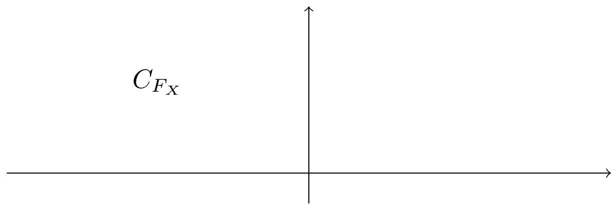
Ex 1 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = \dfrac{1}{2} & \text{si} \quad t \in]-1, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]-1, 1] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]1; +\infty[\end{cases}$



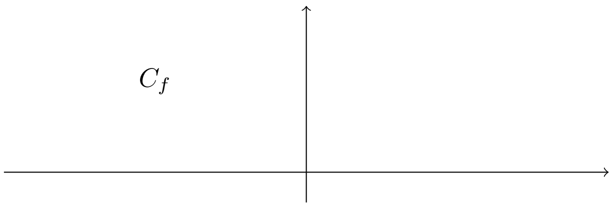


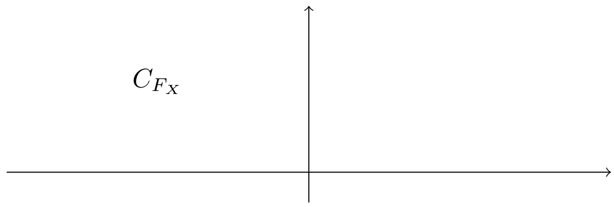
Ex 2 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = t+1 & \text{si} \quad t \in]-1, 0[\\ f(t) = 1-t & \text{si} \quad t \in]\,0\,,\,1\,[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]-1, 0] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]\,0\,,\,1\,] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]1; +\infty[\end{cases}$



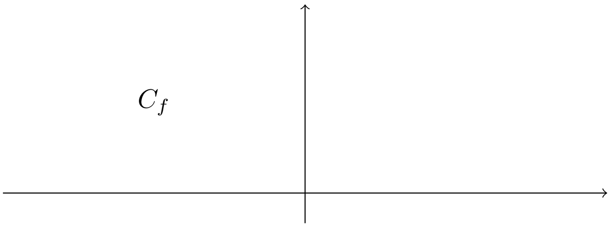


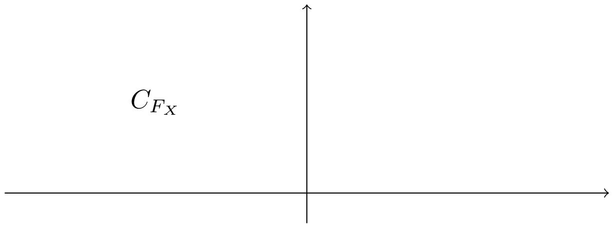
Ex 3 : $\begin{cases} f(t) = e^t & \text{si} \quad t \in]-\infty, 0[\\ f(t) = 0 & \text{si} \quad t \in]0; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \ldots\ldots\ldots & \text{si} \quad t \in]0; +\infty[\end{cases}$



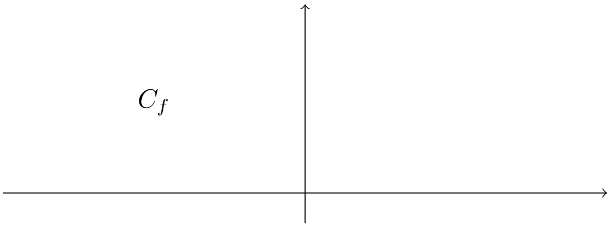


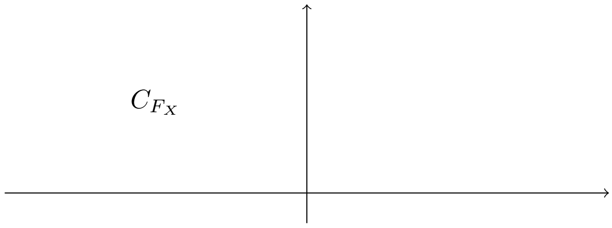
Ex 4 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = 1 & \text{si} \quad t \in]-1, -\frac{1}{2}[\\ f(t) = 0 & \text{si} \quad t \in]-\frac{1}{2}, \frac{1}{2}[\\ f(t) = 1 & \text{si} \quad t \in]\ \frac{1}{2}\ ,\ 1\,[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \text{...............} & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]-1, -\frac{1}{2}] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]-\frac{1}{2}, \frac{1}{2}] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]\ \frac{1}{2}\ ,\ 1\,] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]1; +\infty[\end{cases}$



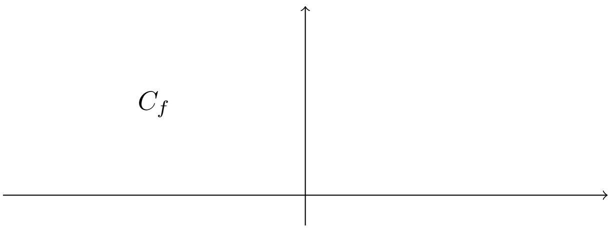


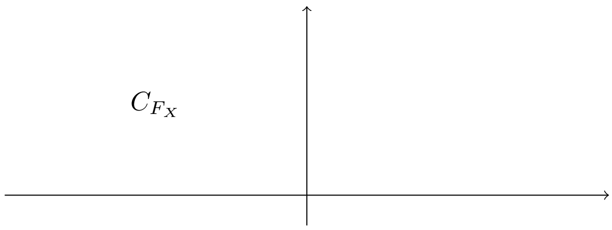
Ex 5 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{2t}{3} & \text{si} \quad t \in]0, \sqrt{3}[\\ f(t) = 0 & \text{si} \quad t \in]\sqrt{3}; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \text{...............} & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]0, \sqrt{3}[\\ F_X(t) = \text{...............} & \text{si} \quad t \in]\sqrt{3}; +\infty[\end{cases}$



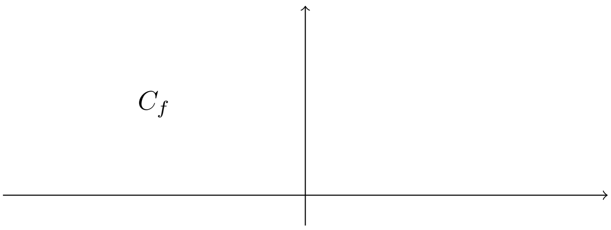


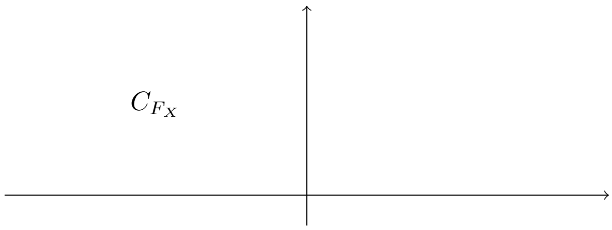
Ex 6 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 1[\\ f(t) = \dfrac{1}{4\sqrt{t}} & \text{si} \quad t \in]1, 9[\\ f(t) = 0 & \text{si} \quad t \in]9; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \text{...............} & \text{si} \quad t \in]-\infty, 1] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]1, 9] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]9; +\infty[\end{cases}$



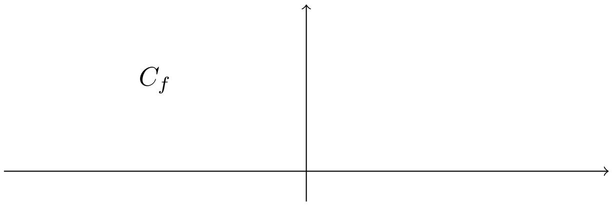


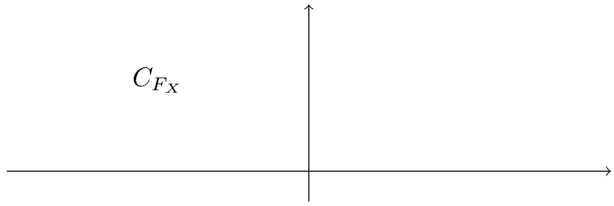
Ex 7 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = \sqrt{t+1} & \text{si} \quad t \in]-1, 0[\\ f(t) = 1 - \sqrt{t} & \text{si} \quad t \in]0, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \text{...............} & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]-1, 0] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]0, 1] \\ F_X(t) = \text{...............} & \text{si} \quad t \in]1; +\infty[\end{cases}$



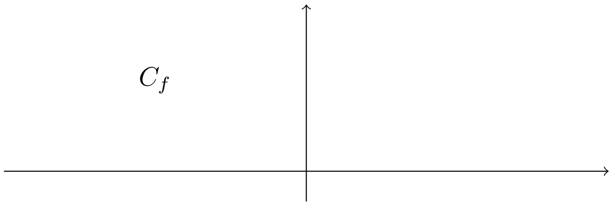


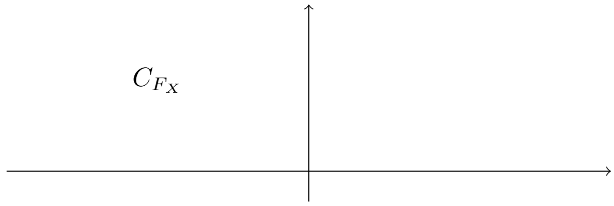
Ex 8 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = t \exp\left(-\dfrac{t^2}{2}\right) & \text{si} \quad t \in]0; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]0; +\infty[\end{cases}$



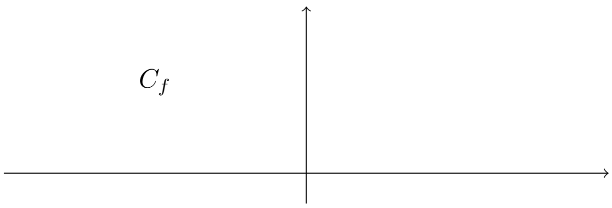


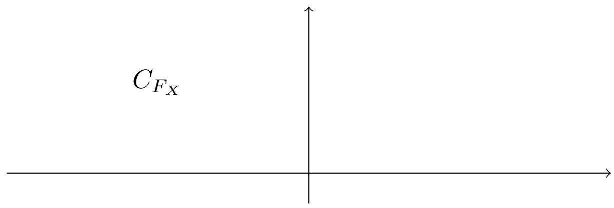
Ex 9 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{1}{2\sqrt{t}} & \text{si} \quad t \in]0, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]0, 1] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]1; +\infty[\end{cases}$





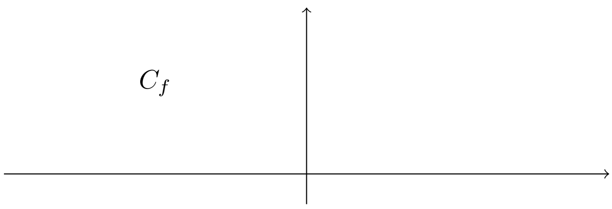
Ex 10 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{1}{t^2} \exp\left(-\dfrac{1}{t}\right) & \text{si} \quad t \in]0; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]0; +\infty[\end{cases}$

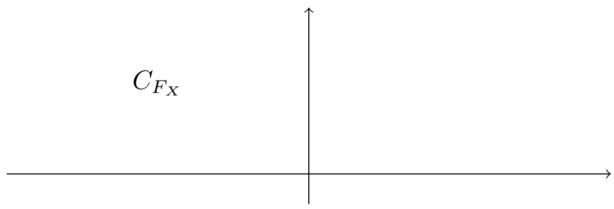




Ex 11 : Soit $\theta \in \mathbb{R}_+^*$,

$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty\,,\ \theta\ [\\ f(t) = e^{\theta - t} & \text{si} \quad t \in]\ \theta\ , +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-\infty\,,\ \theta\ [\\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]\ \theta\ , +\infty[\end{cases}$





Ex 12 : $\begin{cases} f(t) = \dfrac{1}{2t^2} & \text{si} \quad t \in]-\infty, -1[\\ f(t) = 0 & \text{si} \quad t \in]-1, 1[\\ f(t) = \dfrac{1}{2t^2} & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]-1, 1] \\ F_X(t) = \ldots\ldots\ldots\ldots\ldots & \text{si} \quad t \in]1; +\infty[\end{cases}$