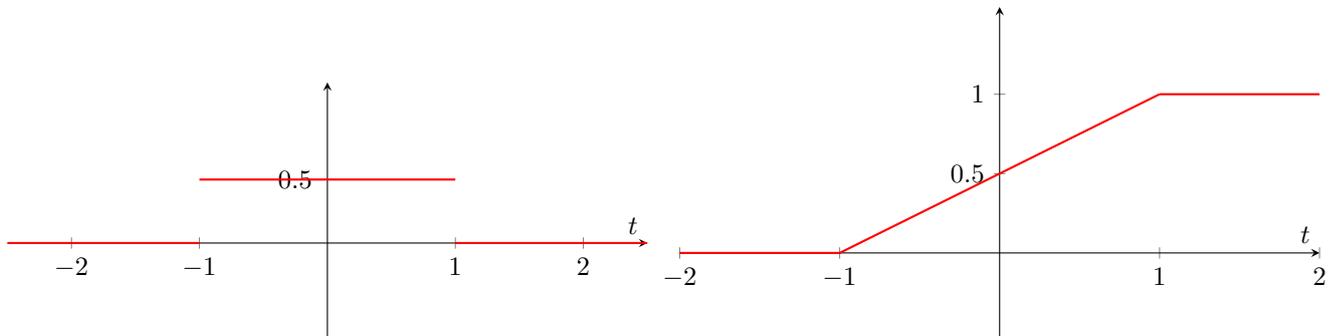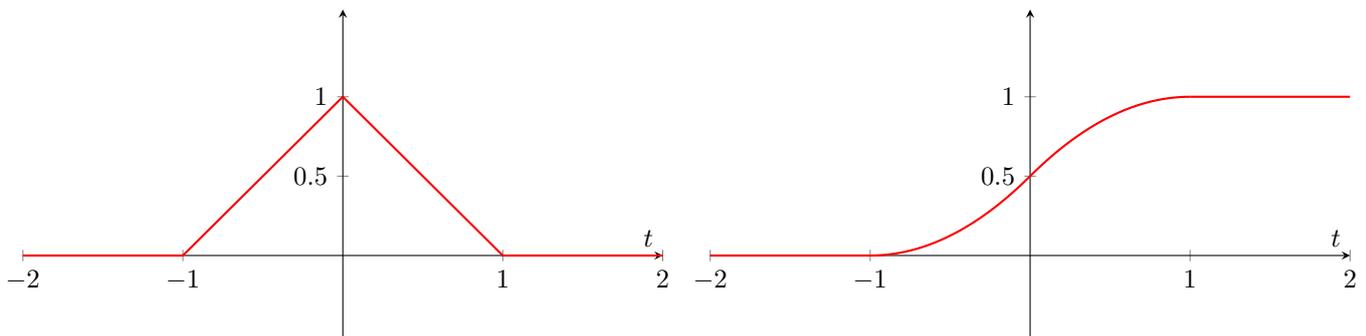Correction de la feuille Calculs_7 : Des fonctions de répartition et des densités.
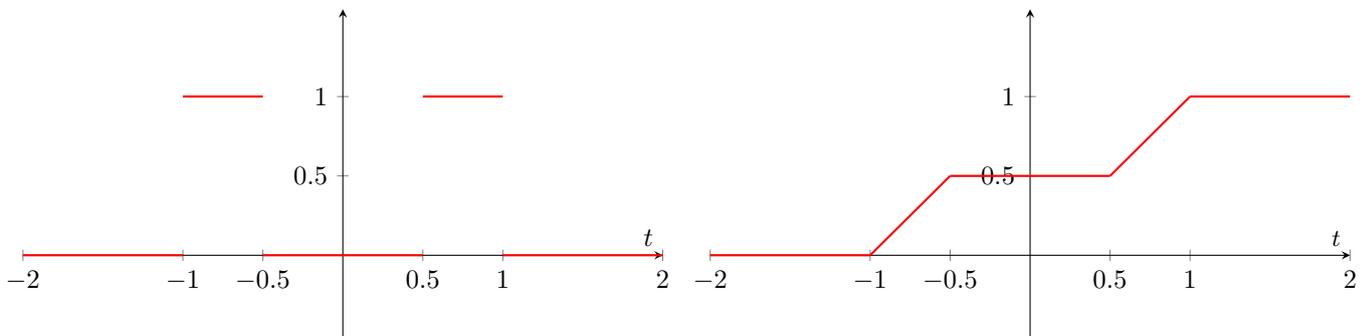
Ex 1 :
$$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = \dfrac{1}{2} & \text{si} \quad t \in]-1, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
donc
$$\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \frac{t}{2} + \frac{1}{2} & \text{si} \quad t \in]-1, 1] \\ F_X(t) = 1 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
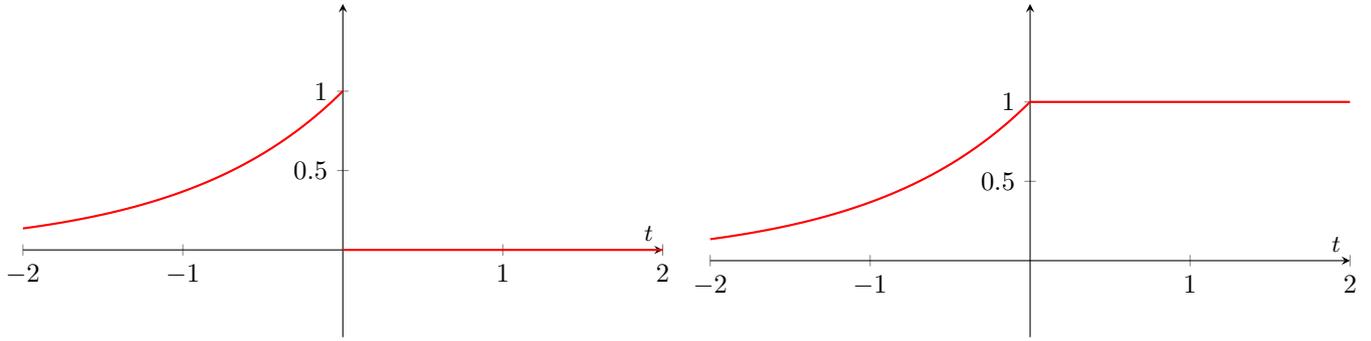
Ex 2 :
$$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = t + 1 & \text{si} \quad t \in]-1, 0[\\ f(t) = 1 - t & \text{si} \quad t \in]0, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
donc
$$\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \frac{t^2}{2} + t + \frac{1}{2} & \text{si} \quad t \in]-1, 0] \\ F_X(t) = t - \frac{t^2}{2} + \frac{1}{2} & \text{si} \quad t \in]0, 1] \\ F_X(t) = 1 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
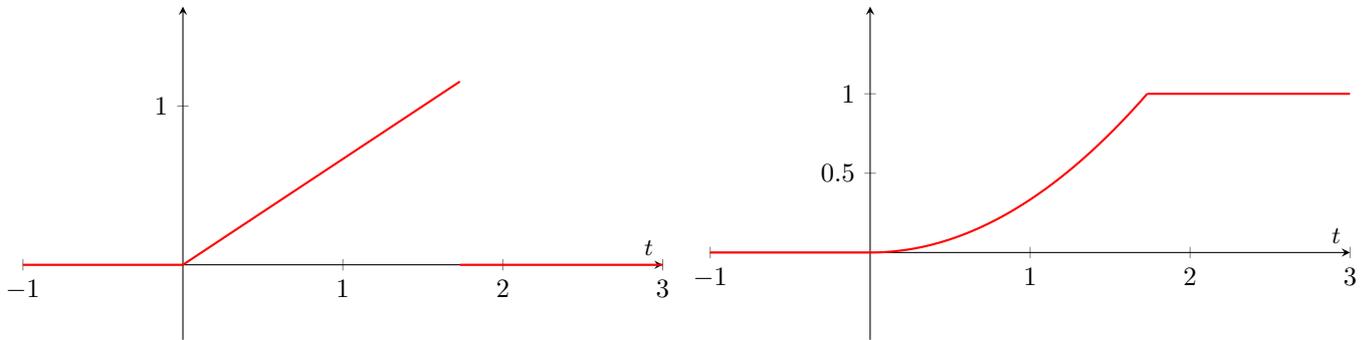
Ex 3 :
$$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = 1 & \text{si} \quad t \in]-1, -\frac{1}{2}[\\ f(t) = 0 & \text{si} \quad t \in]-\frac{1}{2}, \frac{1}{2}[\\ f(t) = 1 & \text{si} \quad t \in]\frac{1}{2}, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
donc
$$\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = t + 1 & \text{si} \quad t \in]-1, -\frac{1}{2}] \\ F_X(t) = \frac{1}{2} & \text{si} \quad t \in]-\frac{1}{2}, \frac{1}{2}] \\ F_X(t) = t & \text{si} \quad t \in]\frac{1}{2}, 1] \\ F_X(t) = 1 & \text{si} \quad t \in]1; +\infty[\end{cases}$$
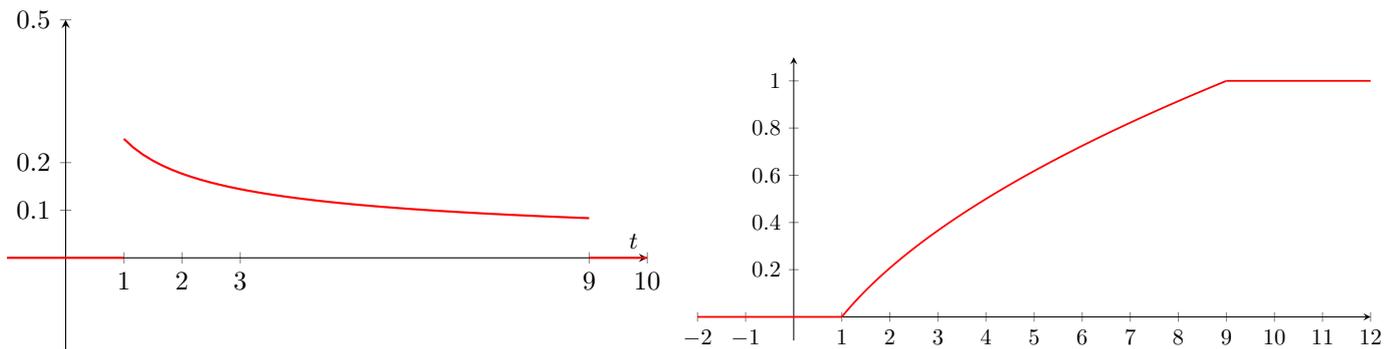
Ex 4 : $\begin{cases} f(t) = e^t & \text{si} \quad t \in]-\infty, 0[\\ f(t) = 0 & \text{si} \quad t \in]0; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = e^t & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = 1 & \text{si} \quad t \in]0; +\infty[\end{cases}$
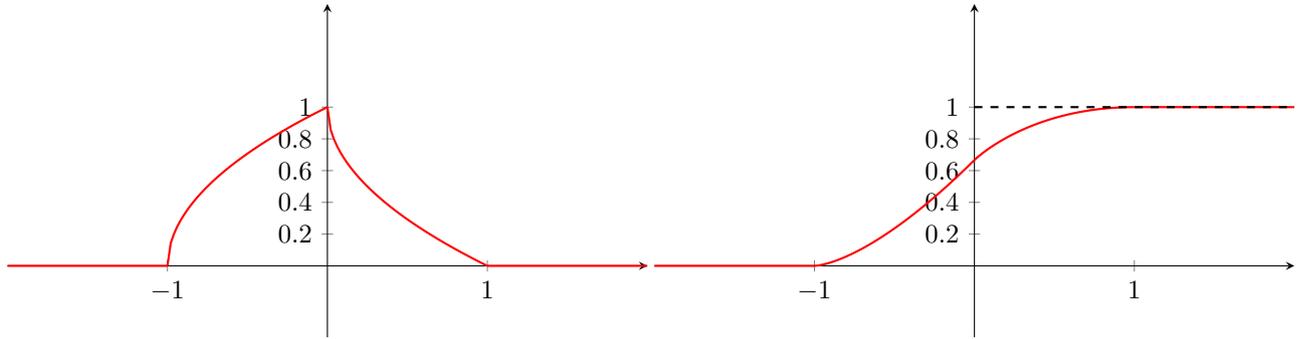
Ex 5 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{2t}{3} & \text{si} \quad t \in]0, \sqrt{3}[\\ f(t) = 0 & \text{si} \quad t \in]\sqrt{3}; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \frac{t^2}{3} & \text{si} \quad t \in]0, \sqrt{3}[\\ F_X(t) = 1 & \text{si} \quad t \in]\sqrt{3}; +\infty[\end{cases}$
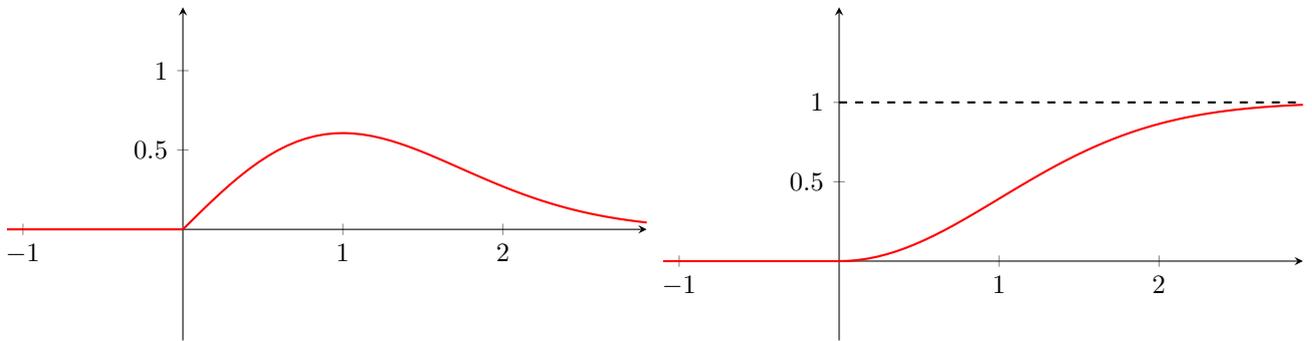
Ex 6 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 1[\\ f(t) = \dfrac{1}{4\sqrt{t}} & \text{si} \quad t \in]1, 9[\\ f(t) = 0 & \text{si} \quad t \in]9; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, 1] \\ F_X(t) = \frac{\sqrt{t}}{2} - \frac{1}{2} & \text{si} \quad t \in]1, 9] \\ F_X(t) = 1 & \text{si} \quad t \in]9; +\infty[\end{cases}$
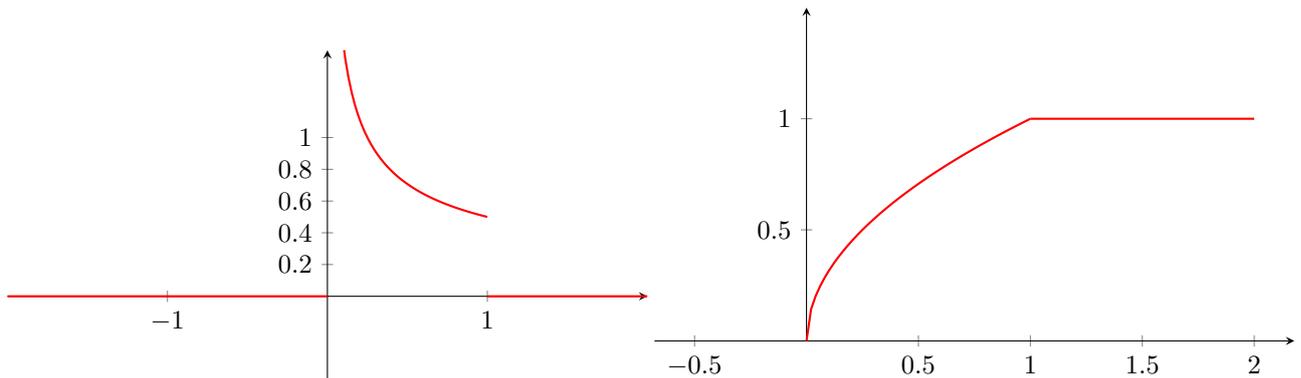
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Ex 7 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, -1[\\ f(t) = \sqrt{t+1} & \text{si} \quad t \in]-1, 0[\\ f(t) = 1 - \sqrt{t} & \text{si} \quad t \in]0, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \frac{2}{3}(t+1)^{\frac{3}{2}} & \text{si} \quad t \in]-1, 0] \\ F_X(t) = t - \frac{2}{3}t^{\frac{3}{2}} + \frac{2}{3} & \text{si} \quad t \in]0, 1] \\ F_X(t) = 1 & \text{si} \quad t \in]1; +\infty[\end{cases}$
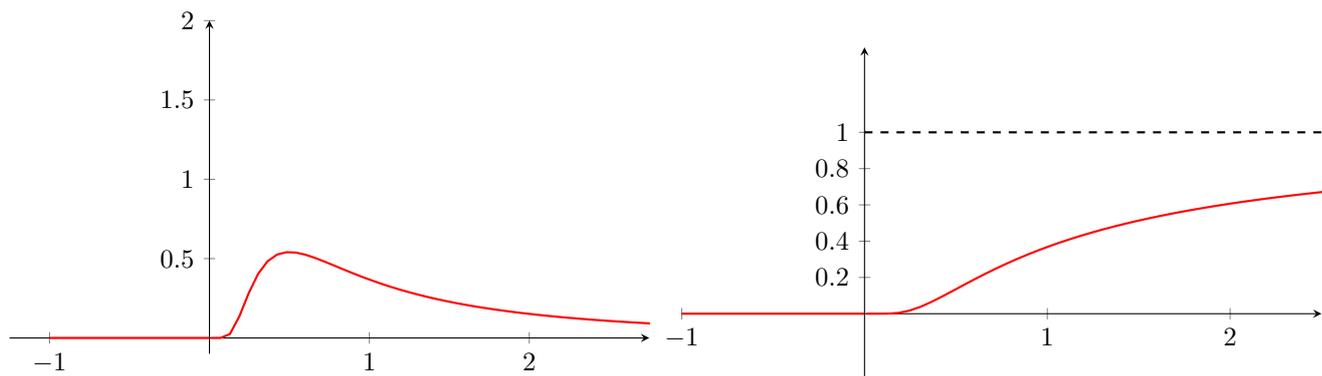

Ex 8 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = t\exp\left(-\dfrac{t^2}{2}\right) & \text{si} \quad t \in]0; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = 1 - e^{-\frac{t^2}{2}} & \text{si} \quad t \in]0; +\infty[\end{cases}$

Ex 9 : $\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{1}{2\sqrt{t}} & \text{si} \quad t \in]0, 1[\\ f(t) = 0 & \text{si} \quad t \in]1; +\infty[\end{cases}$ donc $\begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = \sqrt{t} & \text{si} \quad t \in]0, 1] \\ F_X(t) = 1 & \text{si} \quad t \in]1; +\infty[\end{cases}$

Ex 10 :
$$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, 0[\\ f(t) = \dfrac{1}{t^2} \exp\left(-\dfrac{1}{t}\right) & \text{si} \quad t \in]0; +\infty[\end{cases} \quad \text{donc} \quad \begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, 0] \\ F_X(t) = e^{-\frac{1}{t}} & \text{si} \quad t \in]0; +\infty[\end{cases}$$

Ex 11 : Pour $\theta \in \mathbb{R}_+^*$,
$$\begin{cases} f(t) = 0 & \text{si} \quad t \in]-\infty, \theta[\\ f(t) = e^{\theta - t} & \text{si} \quad t \in]\theta, +\infty[\end{cases} \quad \text{donc} \quad \begin{cases} F_X(t) = 0 & \text{si} \quad t \in]-\infty, \theta[\\ F_X(t) = 1 - e^{\theta - t} & \text{si} \quad t \in]\theta, +\infty[\end{cases}$$

Ex 12 :
$$\begin{cases} f(t) = \dfrac{1}{2t^2} & \text{si} \quad t \in]-\infty, -1[\\ f(t) = 0 & \text{si} \quad t \in]-1, 1[\\ f(t) = \dfrac{1}{2t^2} & \text{si} \quad t \in]1; +\infty[\end{cases} \quad \text{donc} \quad \begin{cases} F_X(t) = -\dfrac{1}{2t} & \text{si} \quad t \in]-\infty, -1] \\ F_X(t) = \dfrac{1}{2} & \text{si} \quad t \in]-1, 1] \\ F_X(t) = 1 - \dfrac{1}{2t} & \text{si} \quad t \in]1; +\infty[\end{cases}$$