Exercice 1

1) $x \mapsto \dfrac{1}{3} + \lambda \, e^{-3x}$

2) $x \mapsto \dfrac{1}{2}\cos x + \dfrac{1}{2}\sin x - e^{-2x} + \dfrac{3}{2} e^{-x}$

3) $x \mapsto (\lambda + e^x) \, e^{-x^2}$

4) $x \mapsto \dfrac{2}{3} x^2 \ln x - \dfrac{4x^2}{9} + \lambda \sqrt{x}$

5) $x \mapsto \operatorname{Arctan} x - \dfrac{\ln(1+x^2)}{2x} + \dfrac{\lambda}{x}$

6) $x \mapsto -2\cos^2 x + 4\cos x$.

Exercice 2 2) $x \mapsto -\dfrac{1}{2x} + \lambda x$

Exercice 3

1) Sur $\mathbb{R}_+^*$, $x \mapsto \dfrac{x^2}{3} + \dfrac{\lambda_1}{x}$. Sur $\mathbb{R}_-^*$, $x \mapsto -x^2 + \lambda_2 x$.

2) $x \mapsto \begin{cases} \dfrac{x^2}{3} & \text{si } x \geqslant 0 \\ -x^2 & \text{si } x < 0 \end{cases}$

Exercice 4

1) $x \mapsto \lambda \, e^{2x} + \mu \, e^x + 1 - 3x \, e^x$

2) $x \mapsto \lambda + \mu \, e^{-4x} + \dfrac{3x^2}{8} - \dfrac{3x}{16} - \dfrac{1}{20}\sin(2x) - \dfrac{1}{10}\cos(2x)$

3) $x \mapsto (\lambda x + \mu) \, e^{2x} - \dfrac{1}{16}\sin(2x) + \dfrac{1}{8}$

4) $x \mapsto e^{-x}(\lambda \cos x + \mu \sin x) + \dfrac{1}{10} e^x + \dfrac{1}{2} e^{-x}$

Exercice 5 $x \mapsto \lambda \, e^{2x} + \mu \, e^{-\,i\,x}$ où $\lambda \in \mathbb{C}$ et $\mu \in \mathbb{C}$

Exercice 6 $x \mapsto \dfrac{9}{8} e^{-x}(\cos x + \sin x) + \dfrac{1}{8} e^x(\sin x - \cos x)$

Exercice 7

1) $x \mapsto \begin{cases} \lambda \cos x + \mu \sin x + x + 1 & \text{si } x \geqslant 0 \\ \lambda \cos x + (2 + \mu)\sin x - x + 1 & \text{si } x < 0 \end{cases}$

2) $x \mapsto \begin{cases} \lambda \, e^{2x} + \mu \, e^{-2x} - \frac{1}{3} e^{2x} & \text{si } x \geqslant 0 \\ (\lambda - \frac{1}{6}) e^{2x} + (\mu + \frac{1}{6}) e^{-2x} - \frac{1}{3} e^{-x} & \text{si } x < 0 \end{cases}$

Exercice 8 Si $m \neq \pm 2$, $x \mapsto \lambda \cos(2x) + \mu \sin(2x) + \dfrac{\cos(mx)}{4 - m^2}$

Si $m = \pm 2$, $x \mapsto \lambda \cos(2x) + \mu \sin(2x) + \dfrac{1}{8}\cos(2x) + \dfrac{1}{4}x \sin(2x)$

Exercice 9 1) (F) $z'' + 2z' + z = 0$ 3) $x \mapsto \dfrac{\lambda \ln x + \mu}{x}$.

Exercice 10 $x \mapsto \lambda \dfrac{1}{\sqrt{1+x^2}} + \mu \dfrac{x}{\sqrt{1+x^2}}$.

Exercice 11 $x \mapsto \lambda \cos x + \mu \sin x + \nu \, e^{-x}$ où $(\lambda, \mu, \nu) \in \mathbb{R}^3$

Exercice 12 $x \mapsto \lambda x + \mu \ln(x)$

Exercice 13 $x(t) = \lambda \, e^{-t} + \mu \, e^{2t} + \dfrac{1}{2} e^t$ et $y(t) = 2\lambda \, e^{-t} - \mu \, e^{2t}$

Exercice 14 $x \mapsto \lambda(\dfrac{e^{-1} - 1}{2} + e^{-x})$