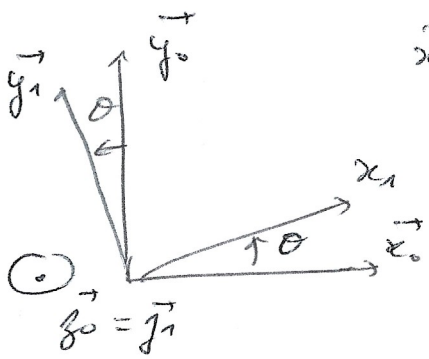
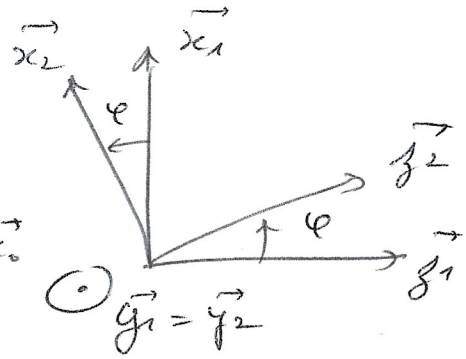
Correction DS de SI, MPSI 1, mars 26

Exo 1 Centrifugeuse

$\overrightarrow{OB} = a\vec{x}_1 + b\vec{x}_2$





$$\left(\frac{d\vec{x}_2}{dt}\right)_0 = \left(\frac{d\vec{x}_2}{dt}\right)_1 + \vec{\Omega}_{1/0} \wedge \vec{x}_2 = -\dot{\varphi}\vec{z}_2 + \dot{\theta}\vec{z}_1 \wedge (\cos\varphi \vec{x}_1 - \sin\varphi \vec{z}_1)$$
$$= -\dot{\varphi}\vec{z}_2 + \dot{\theta}\cos\varphi \vec{y}_1 \quad (\times b)$$

$$\vec{v}(B \in 2/0) = (a + b\cos\varphi)\dot{\theta}\vec{y}_1 - b\dot{\varphi}\vec{z}_2$$

Exo 2 Variateur

(Q1) $\vec{v}(A \in 3/2) = \vec{0}$; $\vec{v}(B \in 4/3) = \vec{0}$

(Q2) $\vec{v}(A \in 3/1) + \vec{v}(A \in 1/2) = \vec{0}$

$\vec{v}(A \in 3/1) = \vec{v}(C \in 3/1) + \vec{\Omega}_{3/1} \wedge \overrightarrow{CA} = \vec{0} + \omega_{31}\vec{x}_3 \wedge R_1\vec{y}_3 = \omega_{31}R_1\vec{z}_3$

$\vec{v}(A \in 2/1) = \vec{v}(O \in 2/1) + \vec{\Omega}_{2/1} \wedge \overrightarrow{OA} = \vec{0} + \omega_{21}\vec{x}_1 \wedge \delta\vec{y}_1 = \omega_{21}\delta\vec{z}_1$

$$\Rightarrow \omega_{31} = \frac{\delta}{R_1}\omega_{21}$$

(Q3) $\vec{v}(B \in 4/1) + \vec{v}(B \in 1/3) = \vec{0}$

$\vec{v}(B \in 4/1) = \vec{v}(E \in 4/1) + \vec{\Omega}_{4/1} \wedge \overrightarrow{EB} = \vec{0} + \omega_{41}\vec{x}_1 \wedge -(a-\delta)\vec{y}_1 = -\omega_{41}(a-\delta)\vec{z}_1$

$\vec{v}(B \in 3/1) = \vec{v}(D \in 3/1) + \vec{\Omega}_{3/1} \wedge \overrightarrow{DB} = \vec{0} + \omega_{31}\vec{x}_3 \wedge -R_2\vec{y}_3 = -\omega_{31}R_2\vec{z}_3$

$$\Rightarrow \omega_{31} = \omega_{41}\cdot\frac{a-\delta}{R_2}$$

(Q4) $\frac{\delta}{R_1}\omega_{21} = \omega_{41}\frac{a-\delta}{R_2} \Rightarrow \frac{\omega_{21}}{\omega_{41}} = \frac{(a-\delta)R_1}{\delta R_2}$

(2)

EX03 Inverseur

Q1 $\lambda \vec{x_0} + L_3 \vec{x_3} + L_4 \vec{y_4} = L \vec{x_0} + H \vec{y_0}$

$\vec{x_3} = \cos\theta_{b} \vec{x_0} + \sin\theta_{b} \vec{y_0}$ $(\times L_3)$

$\vec{y_4} = -\sin\theta_{40} \vec{x_0} + \cos\theta_{40} \vec{y_0}$ $(\times L_4)$

$$\begin{cases} \lambda + L_3 \cos\theta_{b} - L_4 \sin\theta_{40} = L \\ L_3 \sin\theta_{b} + L_4 \cos\theta_{40} = H \end{cases}$$

$$\begin{cases} L_4 \sin\theta_{40} = \lambda - L + L_3 \cos\theta_{b} \\ L_4 \cos\theta_{40} = H - L_3 \sin\theta_{b} \end{cases}$$

$$L_4^2 = (\lambda - L + L_3 \cos\theta_{b})^2 + (H - L_3 \sin\theta_{b})^2$$

$$\lambda = L - L_3 \cos\theta_{b} + \sqrt{L_4^2 - (H - L_3 \sin\theta_{b})^2}$$

Q2 Courbe $\Rightarrow \Delta\lambda = 800 - 250 = 550\,mm > 500\,mm$

Q3

$K = \dfrac{\Delta\theta}{\Delta\lambda} = \dfrac{50}{300} = \dfrac{1}{6}$ °/mm

Q4 $N = 2000$ tr/min

$V = 2000 \times 8$ mm/min $= \dfrac{2000 \times 8}{60} = \dfrac{800}{3}$ mm/s

$\omega_b = \dfrac{1}{6} \times \dfrac{800}{3}$ °/s $= \dfrac{400}{3}$ °/s

$\omega_b = \dfrac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \dfrac{\Delta\theta}{\omega_b} = 80 \times \dfrac{3}{400} = \dfrac{72}{40} = 1{,}8$ s

Q bonus