

Cours du Test : 09/10.

1. Résolvons les systèmes :

$$\textcircled{a} \begin{cases} x+y = 1 \\ y-z = 2 \\ x+z = 3 \end{cases}$$

$$\Leftrightarrow \begin{matrix} \Leftrightarrow \\ L_3 - L_1 \end{matrix} \begin{cases} x+y = 1 \\ y-z = 2 \\ -y+z = 2 \end{cases} \text{ incompatible.}$$
$$S = \emptyset$$

$$\textcircled{c} \begin{cases} x+y+z = 1 \\ 2x+2y+4z = 2 \end{cases}$$

$$L_2 - 2L_1 \Leftrightarrow \begin{cases} x+y+z = 1 \\ 0 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} z = -x-y \\ x \in \mathbb{R} \\ y \in \mathbb{R} \end{cases}$$

$$S = \{(x, y, -x-y) \text{ ou } (x, y) \in \mathbb{R}^2\}$$

$$\textcircled{b} \begin{cases} 2x+y-z = 0 \\ x+2y = 2 \\ -x+y+2z = 0 \end{cases}$$

$$\Leftrightarrow \begin{matrix} \Leftrightarrow \\ L_3 + 2L_1 \end{matrix} \begin{cases} 2x+y-z = 0 \\ x+2y = 2 \\ 3x+3y = 0 \end{cases}$$

$$\Leftrightarrow \begin{matrix} \Leftrightarrow \\ L_3 - 3L_2 \end{matrix} \begin{cases} 2x+y-z = 0 \\ x+2y = 2 \\ -3y = -6 \end{cases}$$

$$\Leftrightarrow \begin{cases} z = 2x+y = -2 \\ x = -2 \\ y = 2 \end{cases}$$

$$S = \{(-2, 2, -2)\}$$

$$\textcircled{d} \begin{cases} x-2y = 0 \\ -x+2y = 0 \end{cases}$$

$$\Leftrightarrow \begin{matrix} \Leftrightarrow \\ L_2 - L_1 \end{matrix} \begin{cases} x-2y = 0 \\ 0 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2y \\ y \in \mathbb{R} \end{cases}$$

$$S = \{(2y, y) \text{ ou } y \in \mathbb{R}\}$$

$$(e) \begin{cases} x+y & -t = 1 \\ 2x+y+z & = 0 \\ -x+y & +3t = 2 \end{cases}$$

$$\Leftrightarrow \begin{array}{l} L_2 - 2L_1 \\ L_3 + L_1 \end{array} \left\{ \begin{array}{l} x+y & -t = 1 \\ -y+z+2t & = -2 \\ 2y & +2t = 3 \end{array} \right.$$

$$\Leftrightarrow \begin{cases} x = 1 - y + t = \frac{1}{2} \\ z = -2 + y - 2t = -\frac{1-4t}{2} = -\frac{1}{2} - 2t \\ y = \frac{3-2t}{2} \\ t \in \mathbb{R} \end{cases}$$

$$S = \left\{ \left(1, \frac{3-2t}{2}, -\frac{1}{2} - 2t, t \right) \mid t \in \mathbb{R} \right\}.$$

$$(f) \quad x+y+z=0$$

$$\Leftrightarrow \begin{cases} x = -y-z \\ y \in \mathbb{R}, z \in \mathbb{R} \end{cases}$$

$$S = \left\{ (-y-z, y, z) \mid (y, z) \in \mathbb{R}^2 \right\}.$$

$$(g) \begin{cases} 2x+y+z+t=1 \\ x+y+z+2t=0 \\ x+y+2z+t=0 \\ x+2y+z+t=0 \end{cases}$$

$$S = \left\{ \left(\frac{4}{5}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5} \right) \right\}$$