Correction Exercice 1 Feuille 1 (compléments n°2)

7. $u_n = \sqrt{n^2-1} - \cancel{1}\, n$

$$= \sqrt{n^2-1} - n$$

$$= \frac{\left(\sqrt{n^2-1} - n\right)\left(\sqrt{n^2-1} + n\right)}{\sqrt{n^2-1} + n}$$

$$= \frac{n^2-1-n^2}{\sqrt{n^2-1} + n}$$

$$= \frac{-1}{\sqrt{n^2-1} + n}$$

Or $\lim\limits_{n \to +\infty} \sqrt{n^2-1} + n = +\infty$

D'où $\lim\limits_{n \to +\infty} u_n = 0$

8. $u_n = \dfrac{(n+2)!}{(n^2+1)\, n!}$

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 1$$

$$(n+2)! = (n+2) \times (n+1) \times \underbrace{n \times (n-1) \times (n-2) \times \cdots \times 1}_{n!}$$

$$= (n+2)(n+1) \times n!$$

$$\frac{(n+2)!}{n!} = \frac{(n+2)(n+1) \cancel{\times n!}}{\cancel{n!}} = (n+2)(n+1)$$

$$u_n = \frac{1}{n^2+1} \times \frac{(n+2)!}{n!} = \frac{(n+2)(n+1)}{n^2+1}$$

$$= \frac{n^2+3n+1}{n^2+1}$$

$$\lim_{n \to +\infty} u_n = \lim_{n \to +\infty} \frac{n^2}{n^2} = \lim_{n \to +\infty} 1 = 1$$

9. $u_n = e^{-\frac{1}{n}} + \ln\left(\frac{n}{n+2}\right)$

$\lim\limits_{n \to +\infty} -\frac{1}{n} = 0$ donc $\lim\limits_{n \to +\infty} e^{-\frac{1}{n}} = e^0 = 1$

$\lim\limits_{n \to +\infty} \frac{n}{n+2} = \lim\limits_{n \to +\infty} \frac{n}{n} = \lim\limits_{n \to +\infty} 1 = 1$

Donc $\lim\limits_{n \to +\infty} \ln\left(\frac{n}{n+2}\right) = \ln(1) = 0$

D'où $\lim\limits_{n \to +\infty} u_n = \boxed{1}$

Correction Exercice 2 Feuille 1 (compléments n°2)

1. $\lim\limits_{n \to +\infty} \frac{n+2}{n+5} = \lim\limits_{n \to +\infty} \frac{n}{n} = \lim\limits_{n \to +\infty} 1 = 1.$

2. $\lim\limits_{n \to +\infty} \frac{e^n}{2^n - 3^n} = \lim\limits_{n \to +\infty} \frac{e^n}{3^n\left(\frac{2^n}{3^n} - 1\right)}$

$= \lim\limits_{n \to +\infty} \left(\frac{e}{3}\right)^n \times \frac{1}{\left(\frac{2}{3}\right)^n - 1}$

Or $\lim\limits_{n \to +\infty} \left(\frac{e}{3}\right)^n = 0$ car $\left|\frac{e}{3}\right| < 1$ (car $e < 3$)

et $\lim\limits_{n \to +\infty} \left(\frac{2}{3}\right)^n = 0$ car $\left|\frac{2}{3}\right| < 1$

d'où $\lim\limits_{n \to +\infty} \left(\frac{2}{3}\right)^n - 1 = -1$

d'où $\lim\limits_{n \to +\infty} u_n = 0 \times (-1) = 0$

3. $\lim\limits_{n \to +\infty} \frac{e^n}{n^2} = +\infty$ par croissance comparée.

4. $\lim\limits_{n \to +\infty} \frac{e^{n^2}}{n} \underset{\substack{X = n^2 \\ n = \sqrt{X}}}{=} \lim\limits_{X \to +\infty} \frac{e^X}{\sqrt{X}}$

$= +\infty$ par croissance comparée.

5. $\lim\limits_{n \to +\infty} \frac{n+1}{n-1} = \lim\limits_{n \to +\infty} \frac{n}{n} = \lim\limits_{n \to +\infty} 1 = 1$

Donc $\lim\limits_{n \to +\infty} e^{\frac{n+1}{n-1}} = e^1 = e$

6. $\lim\limits_{n \to +\infty} \frac{2^n - 3^n}{2^n + 3^n} = \lim\limits_{n \to +\infty} \frac{\cancel{3^n}\left(\frac{2^n}{3^n} - 1\right)}{\cancel{3^n}\left(\frac{2^n}{3^n} + 1\right)} = \lim\limits_{n \to +\infty} \frac{\left(\frac{2}{3}\right)^n - 1}{\left(\frac{2}{3}\right)^n + 1}$

Or $\lim\limits_{n \to +\infty} \left(\frac{2}{3}\right)^n = 0$ car $\left|\frac{2}{3}\right| < 1$

Donc $\lim\limits_{n \to +\infty} u_n = \frac{-1}{1} = -1.$

7. $\lim\limits_{n \to +\infty} (n^2 + 4n - 3^n)(-4n^2 + 2n - 1)$

$= \lim\limits_{n \to +\infty} 3^n\left(\frac{n^2}{3^n} + 4\frac{n}{3^n} - 1\right) \times n^2\left(-4 + \frac{2}{n} - \frac{1}{n^2}\right)$

$= \lim\limits_{n \to +\infty} 3^n \times n^2 \left(-1 + \frac{n^2}{3^n} + 4 \times \frac{n}{3^n}\right)\left(-4 + \frac{2}{n} - \frac{1}{n^2}\right)$

Or $\lim\limits_{n \to +\infty} 3^n \times n^2 = +\infty$ (par simple produit)

$\lim\limits_{n \to +\infty} \frac{n^2}{3^n} = 0$ (par croissance comparée)

$\lim\limits_{n \to +\infty} \frac{n}{3^n} = 0$ (" " ")

Donc $\lim\limits_{n \to +\infty} u_n = +\infty.$