Ex n° 1

① $\lim\limits_{x \to -\infty} -4x^5+3x^2+1 = \lim\limits_{x \to -\infty} -4x^5 = +\infty$

$\lim\limits_{X \to +\infty} e^X = +\infty$

Par composée de limites :

$\lim\limits_{x \to -\infty} e^{-4x^5+3x^2+1} = +\infty$

② $\lim\limits_{x \to 2^+} 3x-3 = 3$

$\lim\limits_{x \to 2^+} 2x-4 = 0^+$

Par quotient de limite :

x	$-\infty$		2		$+\infty$
$2x-4$		$-$	0 (0^+)	$+$	

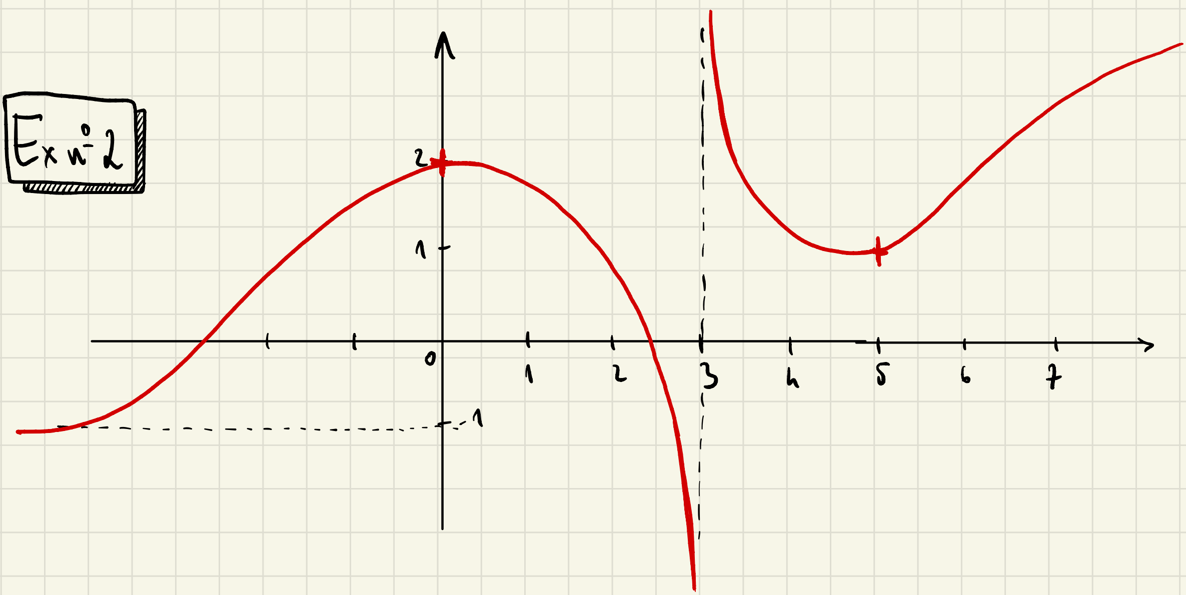
$\lim\limits_{x \to 2^+} \dfrac{3x-3}{2x-4} = +\infty$

$\lim\limits_{X \to +\infty} \ln(X) = +\infty$

Par composée de limite

$\lim\limits_{x \to 2^+} \ln\left(\dfrac{3x-3}{2x-4}\right) = +\infty$

Ex n° 2



Ex n° 3

$$U_{n+1} = \frac{nU_n+1}{2(n+1)} \quad \text{pour } n \in \mathbb{N}^*$$

① $n=1 \longrightarrow U_2 = \frac{1U_1+1}{2(1+1)} = \frac{3+1}{2\times 2} = \frac{4}{4} = 1$ Donc $\boxed{U_2 = 1}$

$n=2 \longrightarrow U_3 = \frac{2U_2+1}{2(2+1)} = \frac{2\times 1+1}{2\times 3} = \frac{3}{6} = \frac{1}{2}$ Donc $\boxed{U_3 = \frac{1}{2}}$

② $\forall n \geqslant 1$ on pose $V_n = nU_n - 1$

a] $V_{n+1} = (n+1)U_{n+1} - 1 = \cancel{(n+1)} \times \frac{nU_n+1}{2\cancel{(n+1)}} - 1$

$= \frac{nU_n+1}{2} - \frac{1^{\times 2}}{1\times 2}$

$= \frac{nU_n+1}{2} - \frac{2}{2} = \frac{nU_n+1-2}{2} = \frac{nU_n-1}{2} = \frac{V_n}{2} = \frac{1}{2}V_n$

Donc (V_n) est une suite géométrique de raison $q = \frac{1}{2}$

et de 1er terme $V_1 = 1\times U_1 - 1 = 1\times 3 - 1 = 2$

Ainsi $\forall n \geqslant 1$ $V_n = V_1 \times q^{n-1} = 2\times\left(\frac{1}{2}\right)^{n-1} = 2\times\frac{1}{2^{n-1}} = \frac{1}{2^{n-2}} = \left(\frac{1}{2}\right)^{n-2}$

b] On a $V_n = nU_n - 1 \iff V_n + 1 = nU_n$

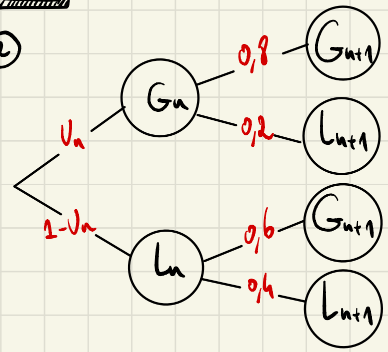
$\iff \frac{1}{n}(1+V_n) = U_n$

$\iff \boxed{U_n = \frac{1}{n}\left(1 + \left(\frac{1}{2}\right)^{n-2}\right)}$ $\forall n \geqslant 1$

Ex n° 4

① $P(L_n) = P(\overline{G_n}) = 1 - p(G_n) = \boxed{1 - U_n}$

②



③ (G_n, L_n) forment un système complet d'évènements. D'après la formule des probabilités totales :

$$P(G_{n+1}) = P(G_n \cap G_{n+1}) + P(L_n \cap G_{n+1})$$

$$= P(G_n) \times P_{G_n}(G_{n+1}) + P(L_n) \times P_{L_n}(G_{n+1})$$

$$= U_n \times 0{,}8 + (1 - U_n) \times 0{,}6$$

Ainsi $U_{n+1} = 0{,}8\,U_n + 0{,}6(1 - U_n) = 0{,}8\,U_n + 0{,}6 - 0{,}6\,U_n$

$$U_{n+1} = 0{,}2\,U_n + 0{,}6 = \boxed{\frac{1}{5}U_n + \frac{3}{5}}$$

④ (U_n) est arithmético-géométrique.

⑤ On résout $x = \frac{1}{5}x + \frac{3}{5} \iff x - \frac{1}{5}x = \frac{3}{5} \iff \frac{5}{5}x - \frac{1}{5}x = \frac{3}{5}$

$$\iff \frac{4}{5}x = \frac{3}{5} \iff x = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4} = \frac{3}{4}$$

⑥ On pose $V_n = U_n - x = U_n - \frac{3}{4}$

Alors pour $n \geqslant 1$ $V_{n+1} = U_{n+1} - \frac{3}{4} = \frac{1}{5}U_n + \frac{3}{5} - \frac{3}{4}$

$$= \frac{1}{5}\left(V_n + \frac{3}{4}\right) + \frac{3^{\times 4}}{5_{\times 4}} - \frac{3^{\times 5}}{4_{\times 5}} = \frac{1}{5}V_n + \frac{3}{20} + \frac{12}{20} - \frac{15}{20}$$

Alors $\boxed{V_{n+1} = \frac{1}{5}V_n}$ La suite (V_n) est donc géométrique de raison $q = \frac{1}{5}$

et de 1^er terme $V_1 = U_1 - \frac{3}{4} = 0{,}1 - \frac{3}{4} = \frac{1^{\times 2}}{10_{\times 2}} - \frac{3^{\times 5}}{4_{\times 5}} = -\frac{13}{20} =$

Donc $\forall n \geqslant 1$ $V_n = V_1 \times q^{n-1} = -\frac{13}{20} \times \left(\frac{1}{5}\right)^{n-1}$

Alors $\forall n \geqslant 1$ $\boxed{U_n = -\frac{13}{20}\left(\frac{1}{5}\right)^{n-1} + \frac{3}{4}}$