Lycée Turgot

Corrigé I_2

ECT – 1

2023 / 2024

Sujet A

Exercice I

1°/ (E_1) : $2x^2 - 5x + 12 = 0$

On pose $\Delta_1 = 25 - 4 \times 2 \times 12 = 25 - 96 < 0$

donc $\Delta_1 = -71 < 0$ soit $S_{E_1} = \emptyset$ (dans $\mathbb{R}$)

2°/ (E_2) : $3x - 5 = 3x^2 - 6x$

$\iff 0 = 3x^2 - 9x + 5$

On calcule $\Delta_2 = 81 - 4 \times 3 \times 5 = 81 - 60 = 21 \geq 0$

On peut déterminer $\begin{matrix} x_1 \\ x_2 \end{matrix} = \dfrac{9 \pm \sqrt{21}}{6} = \dfrac{3}{2} \pm \dfrac{\sqrt{21}}{6}$

Finalement $S_{E_2} = \left\{ \dfrac{3}{2} - \dfrac{\sqrt{21}}{6} ; \dfrac{3}{2} + \dfrac{\sqrt{21}}{6} \right\}$

3°/ (E_3) : $2x^2 + 5x - 1 = x^2 - 5x + 10$

$\iff x^2 + 10x - 11 = 0$

On calcule $\Delta_3 = 100 - 4 \times (-11) = 144 \geq 0$

d'où $\begin{matrix} x_1 \\ x_2 \end{matrix} = \dfrac{-10 \pm \sqrt{144}}{2} = -5 \pm \dfrac{12}{2} = \begin{matrix} 1 \\ -11 \end{matrix}$

Soit encore $S_{E_3} = \{-11 ; 1\}$

Exercice II

1°/ D : $1-x^2 \neq 0 \iff x^2 \neq 1 \iff x \neq 1$ et $x \neq -1$

Soit $D = \mathbb{R} \setminus \{-1; 1\}$

2°/ On calcule : $f(2) = \dfrac{2 \times 2}{1-2^2} = \dfrac{4}{-3} = -\dfrac{4}{3}$

$f(\sqrt{3}) = \dfrac{2\sqrt{3}}{1-3} = -\sqrt{3}$

$f\left(\frac{1}{2}\right) = \dfrac{2 \times \frac{1}{2}}{1-\frac{1}{4}} = \dfrac{1}{3/4} = 4/3$

3°/ On calcule, pour $x \in D$:

$f(-x) = \dfrac{2 \times (-x)}{1-(-x)^2} = \dfrac{-2x}{1-x^2} = -\dfrac{2x}{1-x^2} = -f(x)$

4°/ par ce qui précède on trouve :

$f(-2) = -f(2) = -\left(-\frac{4}{3}\right) = \frac{4}{3}$

$f(-\sqrt{3}) = -f(\sqrt{3}) = -(-\sqrt{3}) = \sqrt{3}$

$f\left(-\frac{1}{2}\right) = -f(1/2) = -\frac{4}{3}$

Sujet B

Exercice I

1°/ (E_1) : $4x^2 - 8x + 6 = 0$

On calcule $\Delta_1 = 64 - 4 \times 4 \times 6 = 64 - 144 < 0$

d'où $S_{E_1} = \emptyset$ (dans $\mathbb{R}$)

2°/ (E_2) : $2x - 5 = 7 + x^2$

$\iff x^2 - 2x + 12 = 0$

On calcule $\Delta_2 = 4 - 48 = -44 < 0$

Donc $S_{E_2} = \emptyset$ (dans $\mathbb{R}$)

3°/ (E_3) : $6x^2 - 5x + 2 = 7x^2 - 4x + 12$

$\iff x^2 + x + 10 = 0$

On calcule $\Delta_3 = 1 - 40 = -39 < 0$

D'où $S_{E_3} = \emptyset$ (dans $\mathbb{R}$)

Exercice II

1°/ On a D : $\begin{cases} x - 1 \neq 0 \\ x + 1 \neq 0 \end{cases} \iff \begin{cases} x \neq 1 \\ x \neq -1 \end{cases}$

Soit finalement $D = \mathbb{R} \setminus \{-1; 1\}$

2/ On calcule :

$$f(3) = \frac{1}{3-1} - \frac{1}{3+1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$f(\sqrt{2}) = \frac{1}{\sqrt{2}-1} + \frac{-1}{\sqrt{2}+1}$$

$$= \frac{(\sqrt{2}+1) - (\sqrt{2}-1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{2}{2-1} = 2$$

$$f\left(\frac{1}{3}\right) = \frac{1}{\frac{1}{3}-1} - \frac{1}{\frac{1}{3}+1} = -\frac{1}{2/3} - \frac{1}{4/3}$$

$$= -\frac{3}{2} - \frac{3}{4} = -\frac{6+3}{4} = -\frac{9}{4}$$

3/ pour $x \in D$ on calcule :

$$f(-x) = \frac{1}{-x-1} + \frac{-1}{-x+1} = -\left[\frac{1}{x+1} + \frac{1}{1-x}\right]$$

$$= -\frac{1-x+1+x}{(1+x)(1-x)} = -\frac{2}{1-x^2}$$

$$\Rightarrow f(-x) = \frac{2}{x^2-1}$$

4/ On a alors, par ce qui précède :

$$f(-3) = \frac{2}{9-1} = \frac{2}{8} = \frac{1}{4}$$

$$f(-\sqrt{2}) = \frac{2}{2-1} = 2$$

$$f\left(-\frac{1}{3}\right) = \frac{2}{\frac{1}{9}-1} = -\frac{2}{8/9} = -\frac{18}{8} = -\frac{9}{4}$$