4. MISE EN ÉQUATIONS DU RUGOSIMÈTRE

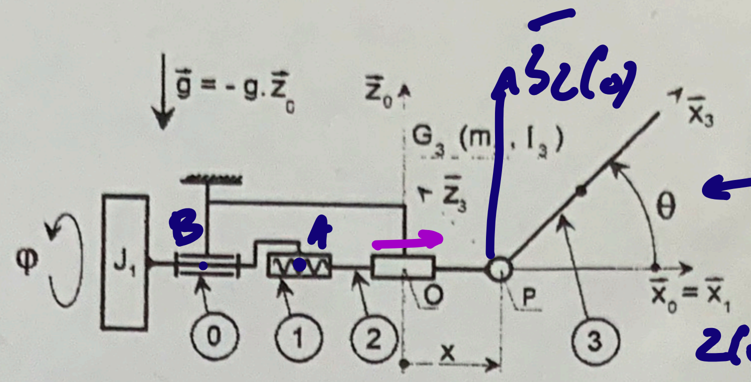


Figure 9 : Schéma paramétré du prototype de rugosimètre 2D

Les calculs prévisionnels du projet de rugosimètre s'adressent maintenant au dimensionnement des moteurs et à l'étude de la commande asservie de la tête optique. Pour cela, une mise en équations dynamiques est un préalable nécessaire. La Figure 9 reprend le schéma de la Figure 3 avec le paramétrage géométrique : $\overrightarrow{OP} = x.\vec{x}_0$ (x variable) ; $\overrightarrow{PG_3} = r.\vec{x}_3$, (r constant) ; $\varphi = (\widehat{\vec{z}_0, \vec{z}_1})$, $\theta = (\widehat{\vec{x}_0, \vec{x}_3})$ variables ; hélicoïdale de pas p « à droite » (p>0, en mm/tour).

Les données dynamiques suivantes :

- **Solide 0** : considéré comme galiléen.
- **Solide 1** : Moment d'inertie par rapport à l'axe $(O, \vec{x}_0)$: J_1. Son centre d'inertie G_1 est sur $(O, \vec{x}_0)$.
- **Solide 2** : Masse m_2.
- **Solide 3** : Masse m_3 ; matrice d'inertie en son centre d'inertie G_3, $[I_3(G_3)]_{\mathcal{B}_3} = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_{\mathcal{B}_3}$ dans la base $\mathcal{B}_3(\vec{x}_3, \vec{y}_3, \vec{z}_3)$:

Le pivot 0-1 présente un frottement visqueux de coefficient f_1 , créant un moment $\vec{\mathcal{M}}_O(0 \to 1) = -f_1.\dot{\varphi}\vec{x}_0$

La glissière 0-2 présente un frottement visqueux de coefficient f_2, créant une force $\vec{\mathcal{F}}(0 \to 2) = -f_2.\dot{x}.\vec{x}_0$

Le pivot 2-3 présente un frottement visqueux de coefficient f_3, créant un moment $\vec{\mathcal{M}}_P(2 \to 3) = -f_3.\dot{\theta}.\vec{y}_0$

L'hélicoïdale 1-2 présente un frottement visqueux de coefficient f_4 , créant un moment $\vec{\mathcal{M}}_O(1 \to 2) = -f_4.\dot{\varphi}.\vec{x}_0$

Les inter-efforts provenant d'autres composants que les liaisons mécaniques sont :

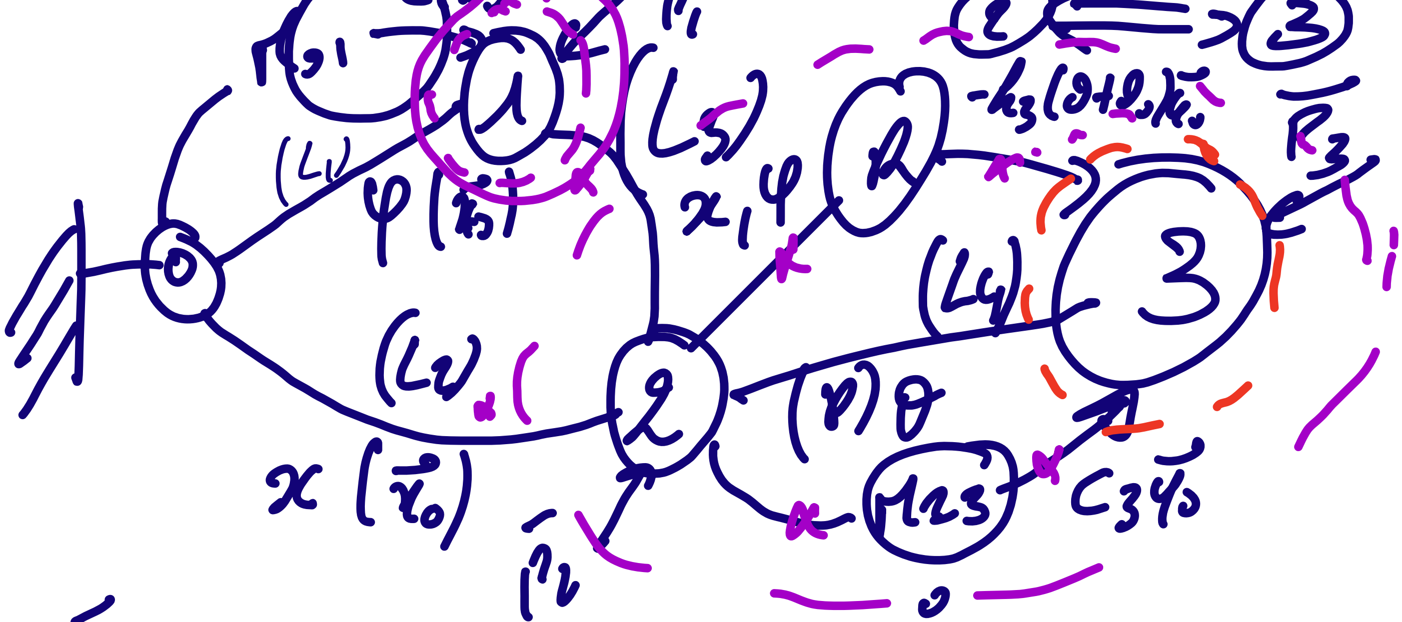
- **La motorisation de 0 sur 1** : le moteur de l'UT exerce un moment variable $\vec{\mathcal{M}}_O(0 \to 1) = C_1.\vec{x}_0$
- **La motorisation de 2 sur 3** : le moteur de l'UR exerce un moment variable $\vec{\mathcal{M}}_P(2 \to 3) = C_3.\vec{y}_0$
- **Un ressort de rappel entre 2 et 3** exerce un moment $\vec{\mathcal{M}}_P(2 \to 3) = -k_3.(\theta + \theta_0).\vec{y}_0$, (k_3 est une raideur constante, et le moment $-k_3.\theta_0.\vec{y}_0$ compense le moment du poids de 3 pour $\theta = 0$)

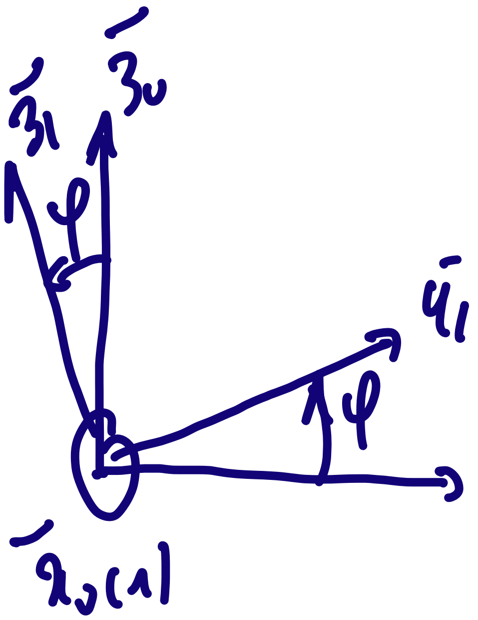
Le système étudié possède deux mobilités : on choisit, pour caractériser celles-ci, x et θ comme paramètres géométriques indépendants. Il est donc nécessaire de déterminer deux équations différentielles en x et θ pour mettre en équations le système dynamique formé des solides (0,1,2,3), en vue de sa commande asservie.

Question 4 (quarante minutes) : Déterminer deux équations différentielles sans autre paramétrage géométrique que (x,θ), par l'application de principes, théorèmes et méthodes laissés à l'initiative du candidat. Constater que ces équations sont non-linéaires, et couplées (c'est à dire qu'elles ne sont pas réductibles à deux équations à une seule variable chacune).

Dans la rédaction de la réponse, avant toute mise en équation, les bilans d'efforts extérieurs ou les bilans énergétiques seront obligatoirement présentés d'abord sous forme graphique. Toutes les écritures matricielles (matrices 3x3 ou 3x1) seront présentées entre crochets, avec en indice la base de projection.

On notera $\frac{d_k \vec{u}}{dt}$ la dérivée par rapport à la base $\mathcal{B}_k$ de la fonction vectorielle $\vec{u}$.





$$\vec\Omega(1/0) = \dot\varphi\, \vec x_{0/1}$$

$$\mathcal{V}(1/0) = \left\{ \dot\varphi\, \vec x_{0/1} \ , \ \vec 0 \right\}_{B,A,O,P}$$

$$T(1/0) = \frac{1}{2} J_1 \dot\varphi^2$$

f_1 en $\dfrac{Nm}{rad.s^{-1}}$

$$\mathcal{T}(0\to 1) = \left\{\begin{matrix} X_{01} & -f_1\dot\varphi \\ Y_{01} & M_{01} \\ Z_{01} & N_{01} \end{matrix}\right\}_{B,(O),(P),(A)}$$

$(\vec x_{0/1}, -, -)$

$$\vec\Omega(2/0) = \vec 0 \Rightarrow (b_2) = (b_0)$$

$$\mathcal{V}(2/0) = \left\{ \vec 0 \ , \ \dot x\, \vec x_0 \right\}_{P,\forall M}$$

$$\vec V(P, 2/0) = \left.\frac{d\vec{OP}}{dt}\right|_{(0)} = \left.\frac{d\, x\vec x_0}{dt}\right|_{(0)} = \dot x\, \vec x_0$$

$$\mathcal{F}(0\to2) = \begin{vmatrix} -f_2\ddot{x} & L_{02} \\ Y_{02} & M_{02} \\ Z_{02} & N_{02} \end{vmatrix}_{O} \quad f_2 \text{ en } \frac{N}{m.s^{-1}}$$

$(\vec{x}_0, -, -)$
(2)
(1)

$$T(2/0) = \frac{1}{2} m_2 \dot{x}^2$$

Hélicoïdale d'axe $(A, \vec{x_0})$

$$\mathcal{V}(2/1) = \begin{Bmatrix} \omega_{x21}\ \vec{x}_0 \ , \ V_{x21}\ \vec{x}_0 \end{Bmatrix}_{A_1B_1O,P}$$

(d) m/tr — rad/s

m/s $V_{x21} = +\frac{P}{2\pi}\ \omega_{x21}$ $\quad N_c = 1$

(g) → (−) $\quad N_s = 5$

$$\mathcal{F}(1\to2) = \begin{vmatrix} X_{12} & L_{12} \\ Y_{12} & M_{12} \\ Z_{12} & N_{12} \end{vmatrix}_{O} \quad b_0(2)$$

$$L_{12} \overset{(d)}{=} - \frac{p}{2\pi} X_{12} - f_4 \varphi^0$$

$(f) \rightarrow (g)$

Fermeture de chaîne cinématique

$$\mathcal{V}(0/2) + \mathcal{V}(2/1) + \mathcal{V}(1/0) = \{\vec{0}\ \vec{0}\}$$

$$\mathcal{V}(2/0) = \mathcal{V}(2/1) + \mathcal{V}(1/0)$$

$$\vec{\Omega}(2/0) = \vec{\Omega}(2/1) + \vec{\Omega}(1/0)$$

$$\vec{0} = \omega_{x21}\, \vec{x}_0 + \varphi^0\, \vec{x}_0$$

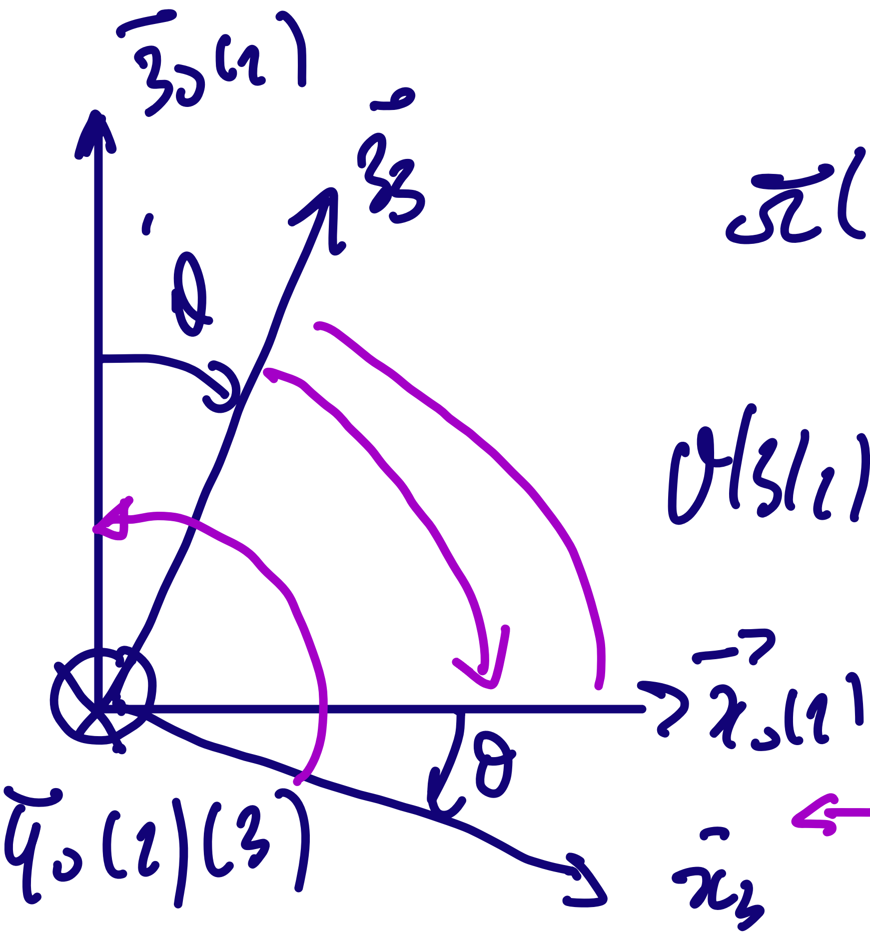
$$\Rightarrow \omega_{x21} = -\varphi^0$$

$$\vec{v}(O,2/0) = \vec{v}(O,2/1) + \vec{v}(O,1/0)$$

$$\dot{x}\,\vec{x}_0 = V_{x21}\,\vec{x}_0 + \vec{0}$$

$$\Rightarrow \boxed{V_{x21} = \dot{x}}$$

$$V_{x21} = +\frac{p}{2\pi}\,\omega_{x21} \Rightarrow \underbrace{\dot{x}}_{2/0} = -\frac{p}{2\pi}\underbrace{\dot{\varphi}}_{1/0}$$



$$\vec{\Omega}(3/2) = \dot{\theta}\,\vec{y}_{2(3)(0)}$$

$$\mathcal{V}(3/2) = \left\{\dot{\theta}\,\vec{y}_{2(3)},\ \vec{0}\right\}_{P,P} \quad \forall P \in (P, \vec{y}_{2(3)})$$

$$(X_{23}\quad L_{23})$$

$$\mathcal{F}(2\to3)=\left|\begin{matrix} Y_{23} & -l_3\dot\theta \\ Z_{23} & N_{23} \end{matrix}\right|_P$$

$(-|\vec{\psi}_2(3)|-)$

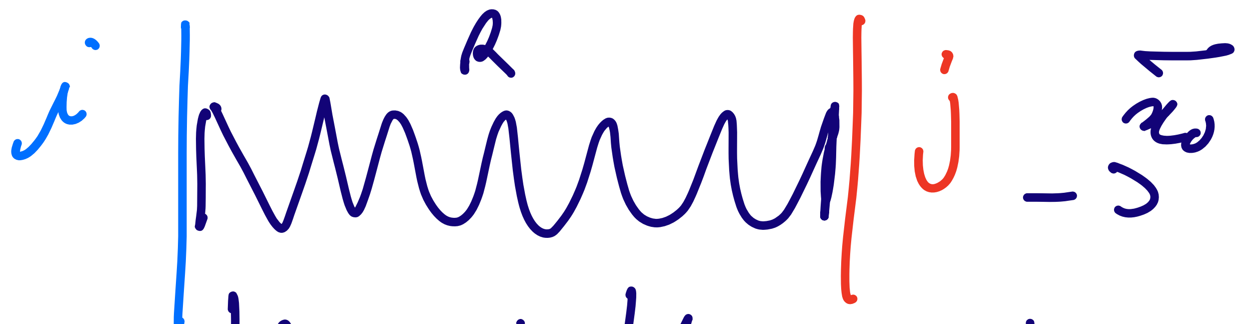
$$\mathcal{F}(\Pi_{01}\to1)=\left\{\vec{0}\ ,\ C_1\vec{x}_0\right\}_{\forall M}$$

$$\mathcal{F}(\Pi_{23}\to3)=\left\{\vec{0}\ ,\ C_3\vec{y}_0\right\}_{\forall M}$$

$$\mathcal{F}(pes\to i)=\left\{\vec{P}_i\ ,\ \vec{0}\right\}_{G_i}$$

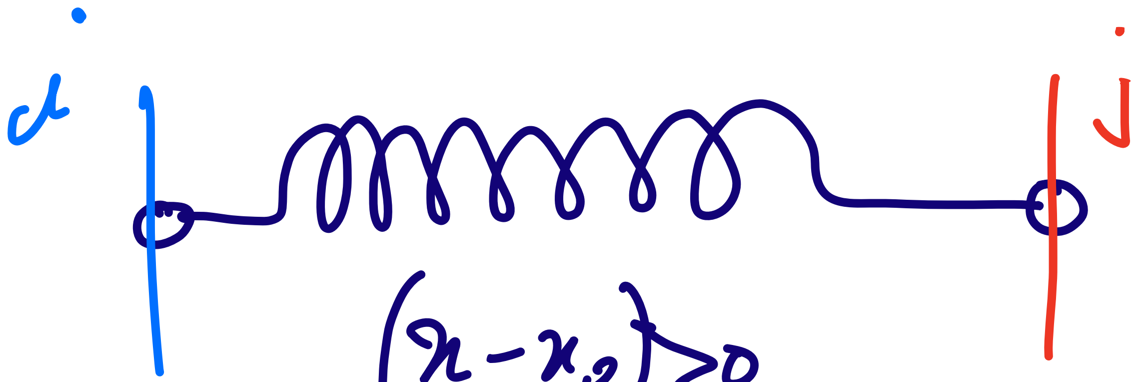
$$\vec{P}_i=-m_i g\,\vec{z}_0$$

$$\mathcal{F}(\text{Ressort}\to3)=\left\{\vec{0}\ ,\ -k_3(\theta+\theta_0)\vec{y}_0\right\}_{\forall P}$$



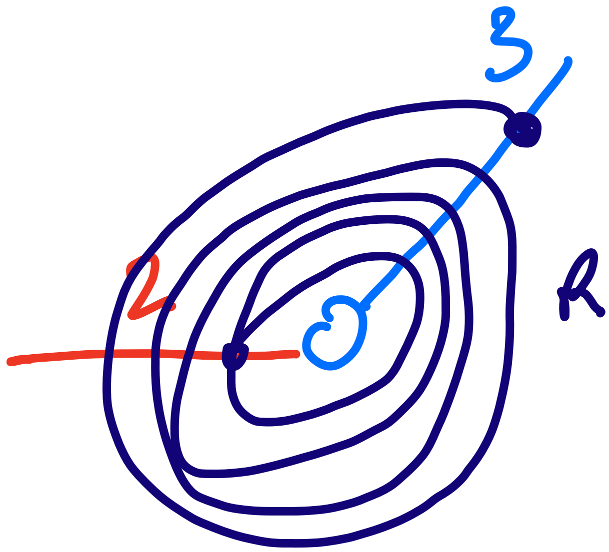
$$F_R=|k\,\Delta x|=|k(x-x_0)|$$

$$\vec{F}(R \to i) = -\overset{<0}{F_R}\, \vec{x}_0 \qquad k\,(N/mm)$$



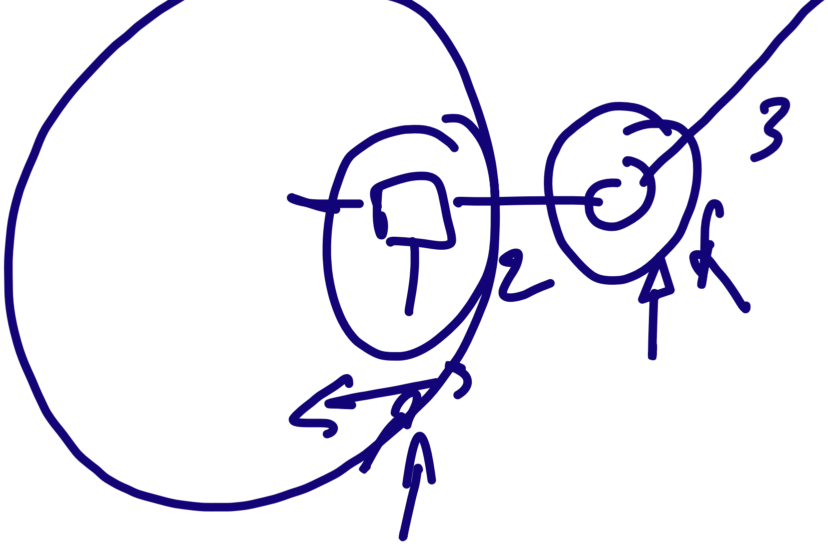
$$(x - x_0) > 0$$

$$\vec{F}(R \to i) = F_R\, \vec{x}_0$$



$$C_R = \left| k(\theta - \theta_0) \right|$$

Nm/rad



$\boxed{3}$: $\mathcal{F}(\bar{3}\to 3) = \mathcal{D}(3/0)$

$\mathcal{F}(pes\to 3) + \mathcal{F}(R_{23}\to 3) + \mathcal{F}(R\to 3) + \mathcal{F}(2/3)$
$= \mathcal{D}(3/0)$

M.D. à $\boxed{3}$ en P $/\vec{y}_0$

$$\boxed{\underline{(\overrightarrow{PG_3}\wedge\vec{P_3})\cdot\vec{y}_0} + C_3 - k_3(\theta+\theta_0) - f_3\dot{\theta} = \underline{\vec{\delta}(P,3/0)\cdot\vec{y}_0}}$$

$$\left\{\begin{array}{l} \vec{\delta}(P, 3/0) = \left.\dfrac{d\vec{\sigma}(P, 3/0)}{dt}\right|_{(0)} + m_3 \left.\dfrac{d\vec{OP}}{dt}\right|_{(0)} \wedge \vec{V}(G_3, 3/0) \\ \\ \vec{\sigma}(P, 3/0) = \bar{\bar{J}}(P, 3)\vec{\Omega}(3/0) + m_3 \vec{PG_3} \wedge \vec{V}(P, 3/0) \end{array}\right.$$

$$\bar{\bar{J}}(G_3, 3) \to \vec{\sigma}(G_3, 3/0) = \bar{\bar{J}}(G_3, 3)\vec{\Omega}(3/0)$$

$$\vec{\delta}(G_3, 3/0) = \left.\dfrac{d\vec{\sigma}(G_3, 3/0)}{dt}\right|_{(0)}$$

$$\vec{\delta}(P, 3/0) \cdot \vec{y}_0 = \left(\vec{\delta}(G_3, 3/0) + \vec{PG_3} \wedge m_3 \vec{A}(G_3, 3/0)\right) \cdot \vec{y}_0$$

$$\vec{\delta}(P, 3/0) \cdot \vec{y}_0 = \underbrace{\left.\dfrac{d\vec{\sigma}(G_3, 3/0)}{dt}\right|_{(0)} \cdot \vec{y}_0}_{} + \underbrace{\left(\vec{PG_3} \wedge m_3 \vec{A}(G_3, 3/0)\right) \cdot \vec{y}_0}_{}$$

$$\vec{\delta}(P, 3/0)\cdot\vec{y}_0 = \frac{d\,\vec{\sigma}(G_3, 3/0)\cdot\vec{y}_0}{dt} - \cancel{\vec{\sigma}\cdot\left.\frac{d\vec{y}_0}{dt}\right|_0}$$

$$+ \left(\vec{PG_3}\wedge M_3\vec{A}(G_3, 3/0)\right)\cdot\vec{y}_0$$

$$\vec{\sigma}(G_3, 3/0) = J(G_3, 3)\,\vec{\Omega}(3/0)$$

$$\vec{\Omega}(3/0) = \vec{\Omega}(3/2) + \underbrace{\cancel{\vec{\Omega}(2/0)}}_{\vec{0}} = \dot{\theta}\,\vec{y}_0(3)$$

$$J(G_3, 3) = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_{R_3}$$

$$\vec{\sigma}(G_3, 3/0) = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_{R_3} \begin{pmatrix} 0 \\ \dot{\theta} \\ 0 \end{pmatrix}_{R_5} = \begin{pmatrix} -F\dot{\theta} \\ B\dot{\theta} \\ -D\dot{\theta} \end{pmatrix}_{R_3}$$

$$\vec{\Gamma}(G_3, 3/_0) = -F\dot{\theta}\,\vec{x_3} + B\dot{\theta}\,\vec{y_3}_{(0)} - D\dot{\theta}\,\vec{z_3}$$

$$\vec{\Gamma}(G_3, 3/_0) \cdot \vec{y_0} = B\dot{\theta} \qquad \text{" } J\omega \text{ "}$$

$$\frac{d\vec{\Gamma}(G_3, 3/_0) \cdot \vec{y_0}}{dt} = B\ddot{\theta} \qquad \text{" } J\frac{d\omega}{dt} \text{ "}$$

$$\vec{PG_3} \wedge m_3 \vec{A}(G_3, 3/_0) = m_3 r\,\vec{x_3} \wedge \vec{A}(G_3, 3/_0)$$

$$\vec{V}(G_3, 3/_0) = \left(\frac{d\vec{OG_3}}{dt}\right)_{(0)}$$

$$\vec{OG_3} = \vec{OP} + \vec{PG_3} = \underline{x\,\vec{x_0} + r\,\vec{x_3}}$$

$$\vec{V}(G_3, 3/_0) = \dot{x}\,\vec{x_0} + r\left(\frac{d\vec{x_3}}{dt}\right)_{(0)}$$

$$\rightarrow \left(\frac{d\vec{x_3}}{dt}\right)\Bigg|_{(0)} = \vec{\Omega}(3/0) \wedge \vec{x_3} = \dot{\theta}\,\vec{y_3}^{(0)} \wedge \vec{x_3}$$

$$= -\dot{\theta}\,\vec{z_3}$$

$$\vec{V}(G_3, 3/0) = \dot{x}\,\vec{x_0} - r\dot{\theta}\,\vec{z_3}$$

$$\vec{V}(G_3, 3/0) = \vec{V}(G_3, 3/2) + \vec{V}(G_3, 2/0)$$

$$= \vec{G_3P} \wedge \vec{\Omega}(3/2) \quad + \quad \dot{x}\,\vec{x_0}$$

$$= -r\,\vec{x_3} \wedge \dot{\theta}\,\vec{y_3} \quad + \dot{x}\,\vec{x_0}$$

$$= -r\dot{\theta}\,\vec{z_3} + \underline{\dot{x}\,\vec{x_0}} \quad \longleftarrow$$

$$\vec{A}(G_3, 3/0) = \frac{d\vec{V}(G_3, 3/0)}{dt}\Bigg|_{(0)}$$

$$= \ddot{x}\,\vec{x_0} - r\ddot{\theta}\,\vec{z_3} - r\dot{\theta}\,\frac{d\vec{z_3}}{dt}\Bigg|_{(0)}$$

$$\left(\frac{d\vec{z}_3}{dt}\right)_{/(0)} = \vec{\Omega}(3/0) \wedge \vec{z}_3 = \dot{\theta}\,\vec{y}_3 \wedge \vec{z}_3$$

$$= \dot{\theta}\,\vec{x}_3$$

$$\vec{A}(G_3, 3/0) = \ddot{x}\,\vec{x}_0 \underbrace{- \ell\ddot{\theta}\,\vec{z}_3}_{\vec{A}_T} \underbrace{- \ell\dot{\theta}^2\,\vec{x}_3}_{\vec{A}_N}$$

$$\left(\vec{PG}_3 \wedge m_3 \vec{A}(G_3, 3/0)\right) \cdot \vec{y}_0$$

$$\vec{PG}_3 \wedge m_3 \vec{A}(G_3, 3/0) = m_3 \ell\, \vec{x}_3 \wedge \vec{A}(G_3, 3/0)$$

$$= m_3 \ell \ddot{x} \underbrace{(\vec{x}_3 \wedge \vec{x}_0)}_{-\sin\theta\,\vec{y}_0} - m_3 \ell \ddot{\theta} \underbrace{\vec{x}_3 \wedge \vec{z}_3}_{-\vec{y}_3}$$

$$\left(\vec{PG}_3 \wedge m_3 \vec{A}(G_3, 3/0)\right) \cdot \vec{y}_0 = -\ell m_3 \ddot{x} \sin\theta + m_3 \ell \ddot{\theta}\, \underbrace{\cancel{\vec{y}_3 \cdot \vec{y}_0}}_{1}$$

$$\underline{\left(\vec{PG_3} \wedge \vec{P_3}\right) \cdot \vec{y_0}} = \left(\ell\, \vec{x_3} \wedge -m_3 g\, \vec{z_0}\right) \cdot \vec{y_0}$$

$$= -\ell m_3 g \left(\vec{x_3} \wedge \vec{z_0}\right) \cdot \vec{y_0}$$

$$= -\ell m_3 g \left(-\sin\left(\frac{\pi}{2}+\theta\right) \underbrace{\cancel{\vec{y_0} \cdot \vec{y_0}}}_{1}\right)$$

$$= \ell m_3 g \cos\theta$$

$$\boxed{2+3} + \Gamma_{23} + R = E$$

$$\mathcal{F}\left(\bar{E} \to E\right) = \mathcal{D}\left(2+3/0\right)$$

$$\mathcal{F}\left(pes \to 2\right) + \mathcal{F}\left(pes \to 3\right) + \mathcal{F}\left(0 \to 2\right) + \mathcal{F}\left(1 \to 2\right)$$

$$= \mathcal{D}\left(2/0\right) + \mathcal{D}\left(3/0\right)$$

$$\text{R.D.} / \vec{x_0} : \quad \underbrace{\cancel{\vec{P_2} \cdot \vec{x_0}} + \cancel{\vec{P_3} \cdot \vec{x_0}}}_{0} + \ell_{12}x^0 + X_{12} =$$

$$\vec{P_i} = -m_i g\, \vec{z_0} \qquad \vec{P} \perp \vec{v}$$

$$\underbrace{m_2 \vec{A}(G_2, 2/0) \cdot \vec{x}_0}_{} + m_3 \vec{A}(G_3, 3/0) \cdot \vec{x}_0$$

$$\vec{A}(G_2, 2/0) \cdot \vec{x}_0 = \ddot{x}\, \vec{x}_0 \cdot \vec{x}_0 = \ddot{x}$$

$$\vec{V}(G_2, 2/0) = \dot{x}\, \vec{x}_0$$

$$m_3 \vec{A}(G_3, 3/0) \cdot \vec{x}_0 = m_3 \left[\ddot{x}\, \vec{x}_0 - r\ddot{\theta}\, \vec{z}_3 - r\dot{\theta}^2 \vec{x}_3 \right] \cdot \vec{x}_0$$

$$= m_3 \ddot{x} - r m_3 \ddot{\theta}\, \underbrace{\vec{x}_0 \cdot \vec{z}_3}_{\sin\theta} - r m_3 \dot{\theta}^2\, \underbrace{\vec{x}_3 \cdot \vec{x}_0}_{\cos\theta}$$

$$-f_2 \dot{x} + X_{12} = (m_2 + m_3)\ddot{x} - r m_3 \left(\ddot{\theta} \sin\theta + \dot{\theta}^2 \cos\theta \right)$$

$$L_{12} = -\frac{p}{2\pi} X_{12} - f_4 \dot{\varphi}$$

$\boxed{1}$: $\mathcal{F}(\bar{1} \to 1) = \mathcal{D}(1/0)$

$\mathcal{F}(0 \to 1) + \mathcal{F}(2 \to 1) + \mathcal{F}(\text{M}_{0,t} \to 1) + \mathcal{F}(p_{1,3} \to 1)$

$= \mathcal{D}(1/0)$

MD en O $/ \vec{x}_0$ $\qquad \mathcal{F}(p_{1,3} \to 1) = \{ \vec{P}_1, \vec{0} \}_{G_1}$

$-f_1 \dot{\varphi} + L_{21} + C_1 + \left(\overrightarrow{OG_1} \wedge \vec{P} \right) \cdot \vec{x}_0$

$\cancel{\overrightarrow{OG_1} \cdot \vec{x}_0 \quad -P_{13_0}}$

$= \vec{\delta}(0, 1/0) \cdot \vec{x}_0 = J_1 \ddot{\varphi}$

$\vec{\sigma}(0, 1/0) = J\omega \, \vec{x}_0 = J_1 \dot{\varphi} \, \vec{x}_0$

$($

On isole $\boxed{3}$ $\qquad \mathcal{F}(\overline{3}/3) = \mathcal{D}(3/0)$

$$\mathcal{F}(2\to 3) + \mathcal{F}(F_{23}\to 3) + \mathcal{F}(pes\to 3) + \mathcal{F}(Res_2 t\to 3) = \mathcal{D}(3/0)$$

M.D. à 3 en P $/\ \vec{y}_0$

$$-I_3\ddot{\theta} + C_3 + [\overrightarrow{PG_3}\wedge\vec{P}_3]\cdot\vec{y}_0 - k(\theta+\theta_0) = \delta(P,3/0)\cdot\vec{y}_0 \qquad (I)$$

On isole $\boxed{2+3}$: $\quad \mathcal{F}(\overline{2+3}\to 2+3) = \mathcal{D}(2+3/0)$

$$\mathcal{F}(\overline{2+3}\to 2+3) = \mathcal{D}(2/0) + \mathcal{D}(3/0)$$

$$\mathcal{F}(1\to 2) + \mathcal{F}(0\to 2) + \mathcal{F}(pes\to 2) + \mathcal{F}(pes\to 3)$$

RDà $\boxed{2+3}$ / $\vec{x}_0$

$$X_{12} - \int_2 \ddot{x} + \cancel{\vec{P}_2 \cdot \vec{x}_0} + \cancel{\vec{P}_3 \cdot \vec{x}_0}$$

$$0$$

$$= \underbrace{m_2 \vec{A}(G_2, 2/_0) \cdot \vec{x}_0} + \underbrace{m_3 \vec{A}(G_3, 3/_0) \cdot \vec{x}_0}$$

$$X_{12} = \frac{-2\pi}{P} L_{12}$$

L_{12} ?

On isole ① TDenO / $\vec{x}_0$ $\rightarrow L_{12} = f(C_1)$

Σ : Tout le système sauf bâti

Σ(3, 2, 1, ressort, 2 Rotors)

T.E.C. à Σ

$$\frac{dT(\Sigma/_0)}{dt} = P_{ext}(\Sigma/_0) + P_{int}(\Sigma)$$

$$T(\Sigma/_0) = T(1/_0) + T(2/_0) + T(3/_0) + T(\cancel{R_{01}}/_0) + T(\cancel{R_{23}}/_0)$$

$$+\ T(\cancel{\text{1er son}}\,F/0)$$

$$T(1/0) = \text{"}\frac{1}{2} J\omega^2\text{"} = \frac{1}{2} I_1 \dot{\varphi}^2$$

$$T(2/0) = \text{"}\frac{1}{2} m v^2\text{"} = \frac{1}{2} m_2 \dot{x}^2$$

$$\dot{x} = -\frac{p}{2\pi}\dot{\varphi}$$

$$T(3/0) = \frac{1}{2}\, \mathcal{C}(3/0) \otimes \mathcal{V}(3/0)$$

$$\mathcal{C}(3/0) = \left\{ m_3 \vec{V}(G_3, 3/0) \;,\; \vec{\sigma}(G_3, 3/0) \right\}_{G_3}$$

$$\mathcal{V}(3/0) = \left\{ \vec{\Omega}(3/0) \;,\; \vec{V}(G_3, 3/0) \right\}_{G_3}$$

$$T(3/0) = \underbrace{\frac{1}{2} m_3 \vec{V}^2(G_3, 3/0)}_{\text{"}\frac{1}{2} m v^2\text{"}} + \underbrace{\frac{1}{2} \vec{\sigma}(G_3, 3/0) \cdot \vec{\Omega}(3/0)}_{\text{"}\frac{1}{2} J\omega^2\text{"}}$$

$\vec{\theta}_3$

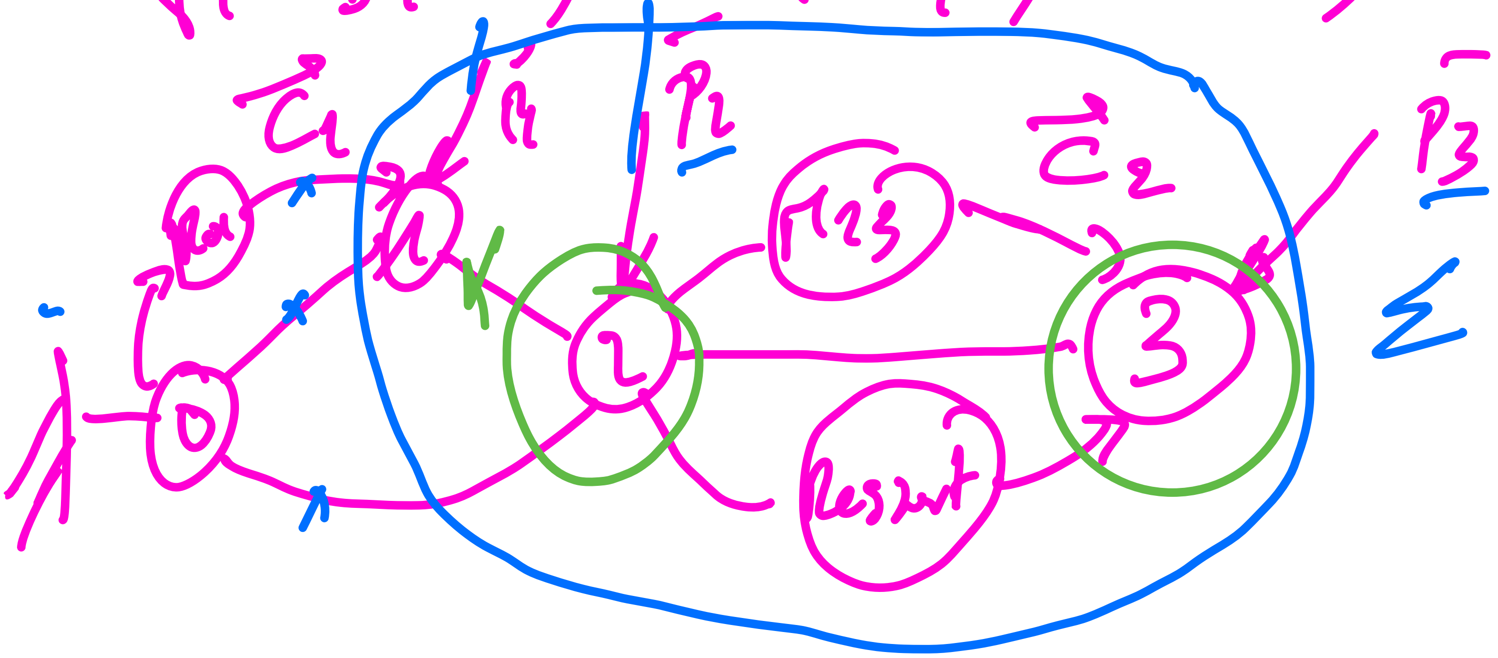
$$\vec{V}(G_3, 3/0) = \left(\frac{d\,\overrightarrow{OG_3}}{dt}\right)_{(0)} = \left(\frac{d\,\overrightarrow{OP}}{dt}\right)_{(0)} + \left(\frac{d\,\overrightarrow{PG_3}}{dt}\right)_{(0)}$$

$x^\circ \bar{x}_0$

$$\vec{v}(G_3, 3/0) = \vec{v}(G_3, 3/2) + \underbrace{\vec{v}(G_3, 2/0)}_{x^\circ \bar{x}_0}$$

$$= \vec{v}(P, \cancel{3}/2) + \overrightarrow{G_3 P_1} \wedge \vec{\Omega}(3/2) +$$

$$\vec{v}(G_3, 3/0) = \dot{J}(G_3, 3)\,\vec{\Omega}(3/0)$$



$$P_{ext}(\Sigma/0) = P(\bar{\Sigma} \to \Sigma/0)$$

$$= P(R_{01} \to 1/0) + \underbrace{P(0 \to 1/0)}_{P(0 \leftrightarrow 1)}$$

$C_1 \omega_1$

$C_1 \dot{\varphi}$

Nulle si L.P

$-f_1 \dot{\varphi}^2$

$+P(0 \to 2/_0)$ $\quad +P(pes \to 1/_0)$

$P(0 \leftrightarrow 2)$ $\quad \vec{P}_1 \cdot \cancel{\vec{v}(G_1, 1/_0)}$

Nulle si L.P. $\quad \vec{0}$ car $G_1 \in \Delta$

$-f_2 \dot{\alpha}^2$

$+ P(pes \to 2/_0)$

$\vec{P}_2 \cdot \cancel{\vec{v}(G_2, 2/_0)}$

$\vec{0}$ car $\vec{P}_2 \perp \vec{v}(G_2, 2/_0)$

$+ P(pes \to 3/_0)$

$\vec{P}_3 \cdot \vec{v}(G_3, 3/_0) \neq \vec{0}$

$P_{int}(\Sigma) = P(M_{23}) + P(ressort)$

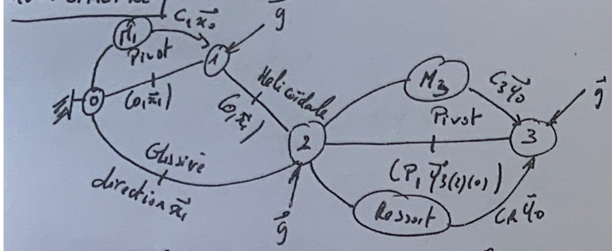
$C_2 \dot{\theta}$ $\quad$ " C_ω "

$+ P(2 \leftrightarrow 3) + P(1 \leftrightarrow 2)$ $\quad -k(\theta+\theta_0)\dot{\theta}$

Nulle si L.P $\quad$ Nulle si L.

$-f_3 \dot{\theta}^2$ $\quad -f_4 \dot{\varphi}^2$

RUGO SIMETRE



$\mathcal{V}(1/0) = \{\dot\varphi \vec{x}_{1(0)}, \vec{0}\}_{O,P}$; $\mathcal{V}(2/1) = \{-\dot\varphi\vec{x}_{1(0)}, \dot x \vec{x}_{1(0)}\}_{O,P}$

$\mathcal{V}(2/0) = \{\vec{0}, \dot x \vec{x}_{1(0)}\}_{\forall M}$ $\qquad \boxed{\dot x = -\frac{p}{2\pi}\dot\varphi}$

$\mathcal{F}(M_1/1) = \{\vec 0, C_1\vec{x}_0\}_{\forall M}$; $\mathcal{F}(M_3/3) = \{\vec 0, C_3\vec{y}_0\}_{\forall M}$

$\mathcal{F}(\text{ressort}/3) = \{\vec 0, C_R\vec{y}_0\}_{\forall M}$ $\quad C_R = -k_3(\theta+\theta_0)\vec{y}_0$

$\mathcal{F}(2/3) = \begin{vmatrix} X_{23} & L_{23} \\ Y_{23} & -f_3\dot\theta \\ Z_{23} & N_{23} \end{vmatrix}_P$ $(-,\vec{y}_{0(3)},-)$; $\mathcal{F}(1/2) = \begin{vmatrix} X_{12} & L_{12}+f_1\dot\varphi \\ Y_{12} & M_{12} \\ Z_{12} & N_{12} \end{vmatrix}_O$ $(\vec{x}_{1(0)},-,-)$ $\quad \boxed{L_{12} = \frac{-p}{2\pi}X_{12}}$

$\mathcal{F}(0/1) = \begin{vmatrix} X_{01} & -f_1\dot\varphi \\ Y_{01} & M_{01} \\ Z_{01} & N_{01} \end{vmatrix}_O$ $(\vec{x}_{1(0)},-,-)$ $\qquad \mathcal{F}(0/2) = \begin{vmatrix} -f_2\dot x & L_{02} \\ Y_{02} & M_{02} \\ Z_{02} & N_{02} \end{vmatrix}_O$ $(\vec{x}_{1(0)},-,-)$

$\mathcal{F}(\text{pes}\to 3) = \{-m_3 g\vec{z}_0, \vec 0\}_{G_3}$

$\mathcal{F}(\text{pes}\to 2) = \{-m_2 g\vec{z}_0, \vec 0\}_{G_2}$

$\mathcal{F}(\text{pes}\to 1) = \{-m_1 g\vec{z}_0, \vec 0\}_{G_1\in(O,\vec{x}_0)}$

En Utilisant le P.F.D

On isole [3] et M.D. en P / $\vec{y}_{0(3)}$: $\vec{\mathcal{M}}(P,\bar{3}/3)\cdot\vec{y}_{0(3)} = \vec{\delta}(P,3/0)\cdot\vec{y}_0$

$\underbrace{[\vec{PG_3}\wedge\vec{P_3}]\cdot\vec{y}_{0(3)}}_{m_3 g r\cos\theta} + C_R + C_3 - f_3\dot\theta = \underbrace{\vec\delta(P,3/0)\cdot\vec{y}_{0(3)}}_{B\ddot\theta - m_3 r\ddot x\sin\theta + m_3 r^2\ddot\theta}$

Soit $(B+m_3r^2)\ddot\theta - m_3 r\ddot x\sin\theta + f_3\dot\theta + k_3(\theta+\theta_0) = C_3 + m_3 g r\cos\theta$

Pour $\theta=0$ et $C_3=0$: $k_3\theta_0 = m_3 g r$

$\Rightarrow \boxed{[B+m_3r^2]\ddot\theta - m_3 r\ddot x\sin\theta + f_3\dot\theta + k_3\theta = C_3 + m_3 g r(\cos\theta - 1)}$ (1)

On isole [2+3] et RD / $\vec{x}_0$

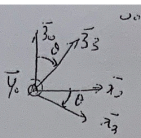
$\rightarrow \boxed{X_{12} = f_2\dot x + (m_2+m_3)\ddot x - m_3 r\ddot\theta\sin\theta - m_3 r\dot\theta^2\cos\theta}$

On isole [1] et M.D en O / $\vec{x}_{1(0)}$

$\Rightarrow \boxed{C_1 = -\frac{p}{2\pi}X_{12} - (f_1+f_1)\frac{2\pi}{p}\dot x - J_1\frac{2\pi}{p}\ddot x}$

Soit $\boxed{C_1 = -(f_1+f_1)\frac{2\pi}{p}\dot x - f_2\frac{p}{2\pi}\dot x - \left[J_1\left(\frac{2\pi}{p}\right) + (m_2+m_3)\frac{p}{2\pi}\right]\ddot x + r m_3\frac{p}{2\pi}[\ddot\theta\sin\theta + \dot\theta^2\cos\theta]}$

T.E.C à $[1+2+3] = \Sigma$ $\qquad \frac{dT(\Sigma/0)}{dt} = P_{ext}(\Sigma/0) + P_{int}(\Sigma)$



$T(\Sigma/0) = T(1/0) + T(2/0) + T(3/0)$

$\boxed{T(1/0) = \frac{1}{2}J_1\dot\varphi^2 = \frac{1}{2}J_1\left(\frac{2\pi}{p}\right)^2\dot x^2}$; $\boxed{T(2/0) = \frac{1}{2}m_2\dot x^2}$

$T(3/0) = \frac{1}{2}m_3\vec{V}^2(G_3,3/0) + \frac{1}{2}\vec{\Omega}(3/0)\cdot\vec{\sigma}(G_3,3/0)$

$\vec V(G_3,3/0) = \dot x\vec{x}_0 - r\dot\theta\vec{z}_3$; $\vec\Omega(3/0) = \dot\theta\vec{y}_0$; $\vec\sigma(G_3,3/0) = -F\dot\theta\vec{x}_3 + B\dot\theta\vec{y}_0 - D\dot\theta\vec{z}_3$

$T(3/0) = \frac{1}{2}m_3(\dot x\vec{x}_0 - r\dot\theta\vec{z}_3)\cdot(\dot x\vec{x}_0 - r\dot\theta\vec{z}_3) + \frac{1}{2}(-F\dot\theta\vec{x}_3 + B\dot\theta\vec{y}_0 - D\dot\theta\vec{z}_3)\cdot\dot\theta\vec{y}_0$

$T(3/0) = \frac{1}{2}m_3\dot x^2 - \frac{1}{2}m_3\dot x r\dot\theta\sin\theta - \frac{1}{2}m_3 r\dot\theta\dot x\sin\theta + \frac{1}{2}m_3r^2\dot\theta^2 + \frac{1}{2}B\dot\theta^2$

$\boxed{T(3/0) = \frac{1}{2}m_3\dot x^2 + \frac{1}{2}(B+m_3r^2)\dot\theta^2 - r m_3\dot x\dot\theta\sin\theta}$

$\frac{dT(\Sigma/0)}{dt} = J_1\left(\frac{2\pi}{p}\right)^2\dot x\ddot x + m_2\dot x\ddot x + m_3\dot x\ddot x + (B+m_3r^2)\dot\theta\ddot\theta - r m_3[\ddot x\dot\theta\sin\theta + \dot x\ddot\theta\sin\theta + \dot x\dot\theta^2\cos\theta]$

$\boxed{\frac{dT(\Sigma/0)}{dt} = \left(J_1\left(\frac{2\pi}{p}\right)^2 + m_2 + m_3\right)\dot x\ddot x + [B+m_3r^2]\dot\theta\ddot\theta - r m_3[\ddot x\dot\theta\sin\theta + \dot x\ddot\theta\sin\theta + \dot x\dot\theta^2\cos\theta]}$

$P_{ext}(\Sigma/0) = P(\text{pes}\to 3/0) + P(\text{pes}\to 2/0) + P(\text{pes}\to 1/0) + P(M_1\to 1/0) + P(0\leftrightarrow 1) + P(0\leftrightarrow 2)$

$\qquad -m_3 g\vec{z}_0\cdot(\dot x\vec{x}_0 - r\dot\theta\vec{z}_3)$ $\qquad \vec 0$ $\qquad + (C_1\dot\varphi)$ $\quad + (-f_1\dot\varphi^2)$ $\quad + (-f_2\dot x^2)$

$\qquad r m_3 g\dot\theta\cos\theta$ $\qquad\qquad\qquad\qquad\qquad \left(-f_1\left(\frac{2\pi}{p}\right)^2\dot x^2\right)$

$\Rightarrow \boxed{P_{ext}(\Sigma/0) = r m_3 g\dot\theta\cos\theta + C_1\dot\varphi - f_1\left(\frac{2\pi}{p}\right)^2\dot x^2 - f_2\dot x^2}$ $\qquad C_1\dot\varphi = C_1\left(-\frac{2\pi}{p}\dot x\right)$

$P_{int}(\Sigma) = P(1\leftrightarrow 2) + P(2\leftrightarrow 3) + P(2(M_3)\to 3) + P(2(\text{ressort})\to 3)$

$\qquad = -f_1\dot\varphi^2 - f_3\dot\theta^2 + C_3\dot\theta + C_R\dot\theta$

$\boxed{P_{int}(\Sigma) = -f_1\left(\frac{2\pi}{p}\right)^2\dot x^2 - f_3\dot\theta^2 + C_3\dot\theta + C_R\dot\theta}$ $\qquad C_R = -k_3(\theta_0+\theta)$

T.E.C à $\Sigma = (1+2+3)$: $\frac{dT(\Sigma/0)}{dt} = P_{ext}(\Sigma/0) + P_{int}(\Sigma)$

$\left[J_1\left(\frac{2\pi}{p}\right)^2 + m_2 + m_3\right]\dot x\ddot x + \cancel{[B+m_3r^2]\dot\theta\ddot\theta} - r m_3(\cancel{\dot x\ddot\theta\sin\theta} + \ddot x\dot\theta\sin\theta + \dot x\dot\theta^2\cos\theta)$

$= \cancel{r m_3 g\dot\theta\cos\theta} + C_1\left(-\frac{2\pi}{p}\right)\dot x - f_1\left(\frac{2\pi}{p}\right)^2\dot x^2 - f_2\dot x^2 - f_1\left(\frac{2\pi}{p}\right)^2\dot x^2 \cancel{- f_3\dot\theta^2} \cancel{+ C_3\dot\theta} \cancel{- k_3(\theta+\theta_0)}$ $C_R\dot\theta$

1ère equation différentielle !

$\rightarrow \boxed{C_1 = -\left[J_1\frac{2\pi}{p} + (m_2+m_3)\frac{p}{2\pi}\right]\ddot x - (f_1+f_1)\frac{2\pi}{p}\dot x - f_2\frac{2\pi}{p}\dot x + \frac{p}{2\pi}r m_3(\ddot\theta\sin\theta + \dot\theta^2\cos\theta)}$

En Utilisant le T.E.C.

3.

$\boxed{1}$ $\dfrac{dT(1/0)}{dt} = P_{ext}(1/0)$

$T(1/0) = \frac{1}{2} J_1 \dot{\varphi}^2 \Rightarrow \dfrac{dT(1/0)}{dt} = J_1 \dot{\varphi} \ddot{\varphi}$

avec $\dot{x} = -\frac{p}{2\pi} \dot{\varphi} \Rightarrow \dot{\varphi} = -\frac{2\pi}{p} \dot{x} \Rightarrow \ddot{\varphi} = \frac{-2\pi}{p} \ddot{x}$

$P_{ext}(1/0) = \mathcal{T}(\bar{1}/1) \otimes \mathcal{V}(1/0)$, $\mathcal{V}(1/0) = \{ \dot{\varphi} \vec{x}_{10}, \vec{0} \}_O$

$\mathcal{T}(\bar{1}/1) = \mathcal{T}(pes \to 1) + \mathcal{T}(0 \to 1) + \mathcal{T}(Mot_1 \to 1) + \mathcal{T}(2/1)$

$\mathcal{T}(pes \to 1) \otimes \mathcal{V}(1/0) = 0$; $\mathcal{T}(0/1) \otimes \mathcal{V}(1/0) = -f_1 \dot{\varphi}^2$

$\mathcal{T}(Mot_1 \to 1) \otimes \mathcal{V}(1/0) = C_1 \dot{\varphi}$, $\mathcal{T}(2/1) \otimes \mathcal{V}(1/0) = (L_{21} - f_4 \dot{\varphi}) \dot{\varphi}$

$= \frac{p}{2\pi} X_{12} \dot{\varphi} - f_4 \dot{\varphi}^2$

T.E.C. $J_1 \dot{\varphi} \ddot{\varphi} = -f_1 \dot{\varphi}^2 + C_1 \dot{\varphi} + \frac{p}{2\pi} X_{12} - f_4 \dot{\varphi}^2$

$\Rightarrow C_1 = -\frac{p}{2\pi} X_{12} + (f_1 + f_4) \dot{\varphi} + J_1 \ddot{\varphi}$

$$\boxed{C_1 = -\frac{p}{2\pi} X_{12} - \frac{2\pi}{p} (f_1 + f_4) \dot{x} - \frac{2\pi}{p} J_1 \ddot{x}}$$

R.D. à $\boxed{2+3}$ / $\vec{x}_0$: $X_{12} = f_2 \dot{x} + (m_2 + m_3) \ddot{x} - r m_3 [\ddot{\theta} \sin\theta + \dot{\theta}^2 \cos\theta]$

$$\Rightarrow \boxed{C_1 = -\frac{2\pi}{p} (f_1 + f_4) \dot{x} - f_2 \frac{p}{2\pi} \dot{x} - \left[J_1 \frac{2\pi}{p} + (m_2 + m_3) \frac{p}{2\pi} \right] \ddot{x} + r m_3 \frac{p}{2\pi} [\ddot{\theta} \sin\theta + \dot{\theta}^2 \cos\theta]}$$

4.

T.E.C à $\boxed{1+2}$: $\dfrac{dT(1+2/0)}{dt} = P_{ext}(1+2/0) + P_{int}(1+2)$

$T(1+2/0) = T(1/0) + T(2/0) = \frac{1}{2} J_1 \dot{\varphi}^2 + \frac{1}{2} m_2 \dot{x}^2 = \boxed{\frac{1}{2} \left[J_1 \left(\frac{2\pi}{p}\right)^2 + m_2 \right] \dot{x}^2}$

$\dfrac{dT(1+2/0)}{dt} = \dot{x} \ddot{x} \left[J_1 \left(\frac{2\pi}{p}\right)^2 + m_2 \right]$

$P_{int}(1+2) = P(1 \leftrightarrow 2) = \mathcal{V}(2/1) \otimes \mathcal{T}(1/2) = -f_4 \dot{\varphi}^2 = \boxed{-f_4 \left(\frac{2\pi}{p}\right)^2 \dot{x}^2 = P_{int}(1+2)}$

$O(a, b, c)(O, \vec{x}_1)$ ... $p_2 \perp$ ... $C_1 \dot{\varphi}$

$P_{ext}(1+2/0) = \cancel{\mathcal{T}(\overline{1+2}/1+2)} \otimes P(pes \to 1/0) + P(pes \to 2/0) + P(Mot \to 1/0)$

$+ P(0 \to 1/0) + P(0 \to 2/0) + P(3 \to 2/0) + P(Mot \to 2/0)$

$+ P(ressort \to 2/0)$

$P(Mot \to 1/0) = C_1 \dot{\varphi}$; $P(0 \to 1/0) = P(0 \leftrightarrow 1) = -f_1 \dot{\varphi}^2 = -f_1 \left(\frac{2\pi}{p}\right)^2 \dot{x}^2$

$P(0 \to 2/0) = P(0 \leftrightarrow 2) = \mathcal{T}(0/2) \otimes \mathcal{V}(2/0) = -f_2 \dot{x}^2$

$P(3 \to 2/0) = X_{32} \dot{x}$

$P(Mot_3 \to 2/0) = -C_3 \vec{y}_0 \cdot \vec{0} = 0$

$P(ressort \to 2/0) = 0$

$(-C_p \vec{y}_0 \cdot \vec{0})$

$\dot{\varphi}$

T.E.C. $\left[J_1 \left(\frac{2\pi}{p}\right)^2 + m_2 \right] \dot{x} \ddot{x} = -f_4 \left(\frac{2\pi}{p}\right)^2 \dot{x}^2 + C_1 \left(-\frac{2\pi}{p}\right) \dot{x} - f_1 \left(\frac{2\pi}{p}\right)^2 \dot{x}^2 - f_2 \dot{x}^2 + X_{32}$

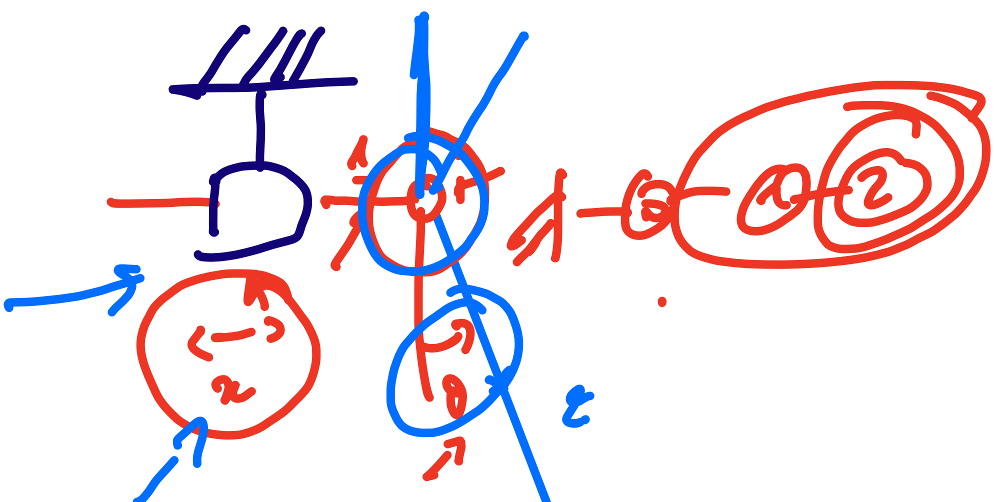
$\Rightarrow C_1 = \left(-\frac{p}{2\pi}\right) \left[J_1 \left(\frac{2\pi}{p}\right)^2 + m_2 \right] \ddot{x} + f_4 \dot{x} \left(\frac{-2\pi}{p}\right) + f_1 \dot{x} \left(\frac{-2\pi}{p}\right) + f_2 \dot{x} \left(\frac{-p}{2\pi}\right) - X_{32}$

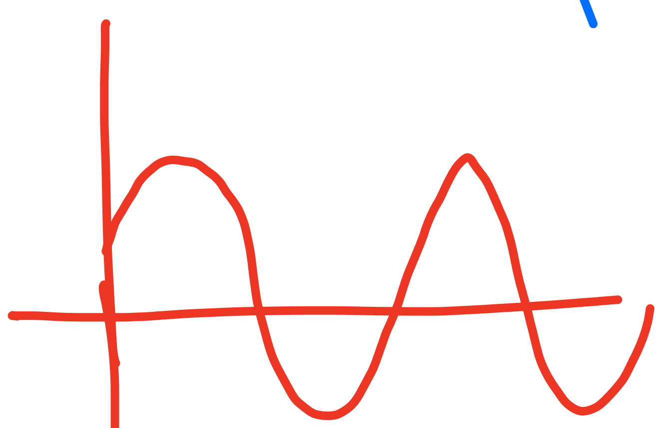
$C_1 = -\left[J_1 \frac{2\pi}{p} + m_2 \frac{p}{2\pi} \right] \ddot{x} - (f_1 + f_4) \frac{2\pi}{p} \dot{x} - f_2 \frac{p}{2\pi} \dot{x} + \frac{p}{2\pi} X_{32}$

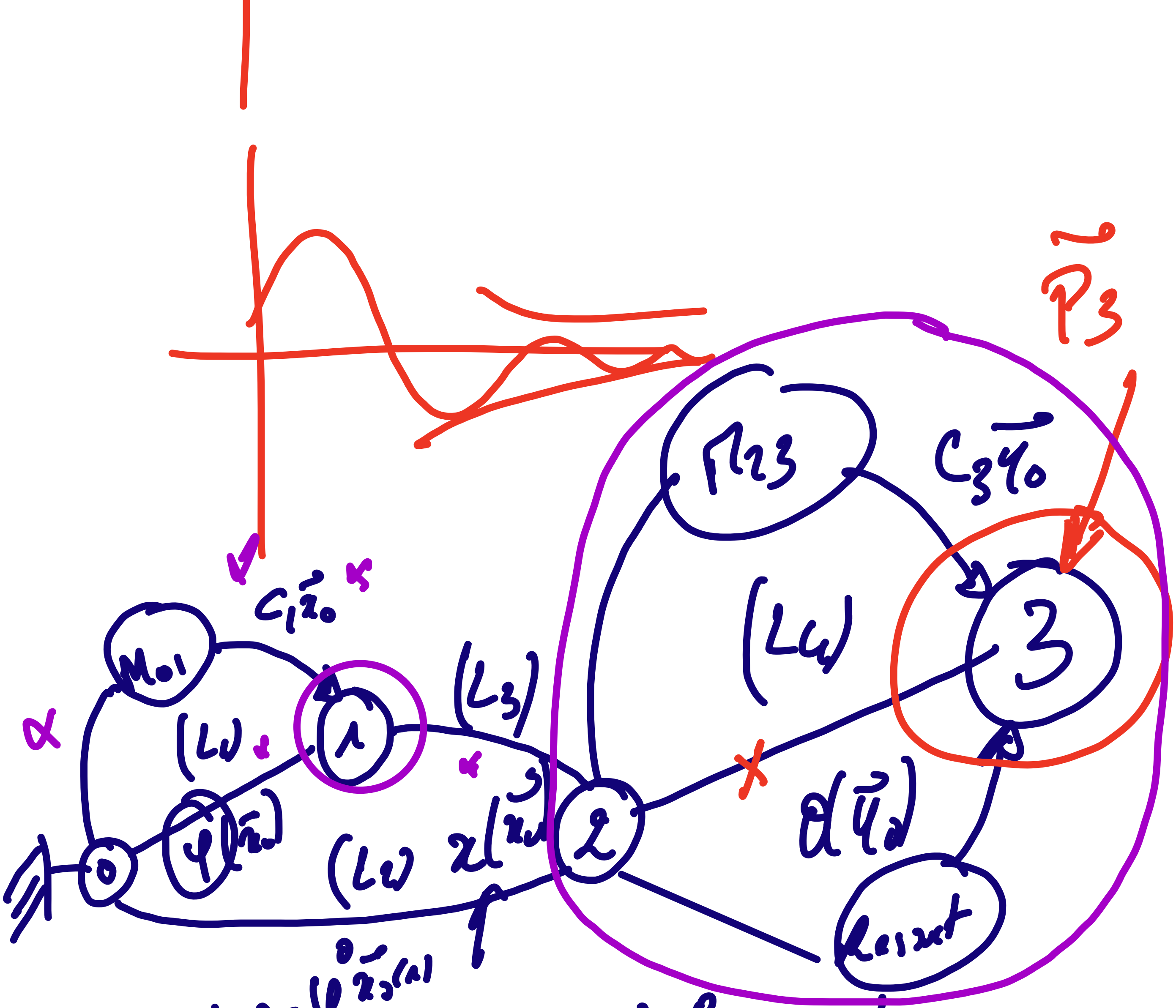
X_{32} ? R.D à $\boxed{3}$ / $\vec{x}_0$ $X_{23} = m_3 \vec{A}(G_3, 3/0) \cdot \vec{x}_0 = m_3 [\ddot{x} \vec{x}_0 - r \ddot{\theta} \vec{y}_3 - r \dot{\theta}^2 \vec{x}_3] \cdot \vec{x}_0$

$\Rightarrow X_{23} = m_3 \ddot{x} - r m_3 \ddot{\theta} \sin\theta - r m_3 \dot{\theta}^2 \cos\theta$

[illegible] $\frac{p}{2\pi} r m_3 [\ddot{\theta} \sin\theta + \dot{\theta}^2 \cos\theta]$

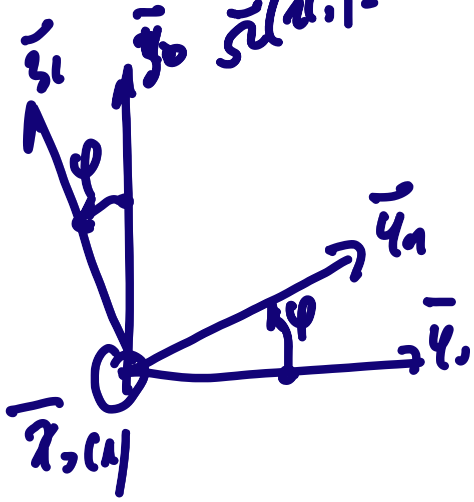






$\vec{\Omega}(1/0) = \dot{\varphi}\, \vec{z}_0(1)$

$\mathcal{V}(1/0) = \left\{ \dot{\varphi}\, \vec{z}_0(1),\ \vec{0} \right\}_B \quad \forall P \in (B, \vec{z}_{01})$



B, A, O, P

$\mathcal{F}(0 \to 1) = \begin{vmatrix} X_{01} & L_{01} \\ Y_{01} & M_{01} \\ Z_{01} & N_{01} \end{vmatrix}_{B, (0)}$ φ, en $\frac{Nm}{rad.s^{-1}}$

$(\vec{x}_{01}, -, -)$

$\mathcal{F}(M_{01} \to 1) = \left\{ \vec{0},\ C_1 \vec{x}_0 \right\}_{\forall P}$

$\vec{\Omega}(1/0) = \vec{0} \Rightarrow (b_1) = (b_0)$

$$\vartheta(2/0) = \left\{ \vec{0} \,,\; x^{\circ}\, \vec{x}_{0(1)(2)} \right\}_{P,\, \forall M}$$

$$\mathcal{F}(0\to 2) = \begin{vmatrix} f_2 x^{\circ} & L_{02} \\ Y_{02} & M_{02} \\ Z_{02} & N_{02} \end{vmatrix}_0 \qquad f_2 \;\; N/m.s^{-1}$$

$(\vec{x}_{0(1)}, -, -)$

$$\vartheta(2/1) = \left\{ \omega_{x21}\, \vec{x}_{0(1)} \,,\; V_{x21}\, \vec{x}_{0(1)} \right\}_{A,B,O,P}$$

$N_C = 1$

$$V_{x21} = +\frac{P}{2\pi}\, \omega_{x21} \qquad \pm \frac{d}{g}$$

$m.s^{-1}$ $\quad$ rad/s $\quad P \to mm/tr$

$$\mathcal{F}(1\to 2) = \begin{vmatrix} X_{12} & L_{12} \\ Y_{12} & M_{12} \\ Z_{12} & N_{12} \end{vmatrix}_0$$

$N_S = 5 \qquad h_{1(7)}$

$$L_{12} = -\frac{P}{2\pi} X_{12} \qquad \mp \frac{d}{g}$$

$P \to m/tr$

$Nm \qquad N$

$$\vartheta(0/1) + \vartheta(1/2) + \vartheta(2/0) = \left\{ \vec{0} \,,\, \vec{0} \right\}$$

$\alpha \quad \vartheta(2/0) = \vartheta(2/1) + \vartheta(1/0)$

1) $\vec{\Omega}(2/0) = \vec{\Omega}(2/1) + \vec{\Omega}(1/0)$

$$\vec{0} = \omega_{x21}\,\vec{x}_0 + \dot{\varphi}\,\vec{x}_0$$

$$/\vec{x}_0 \Rightarrow \boxed{\omega_{x21} = -\dot{\varphi}}$$

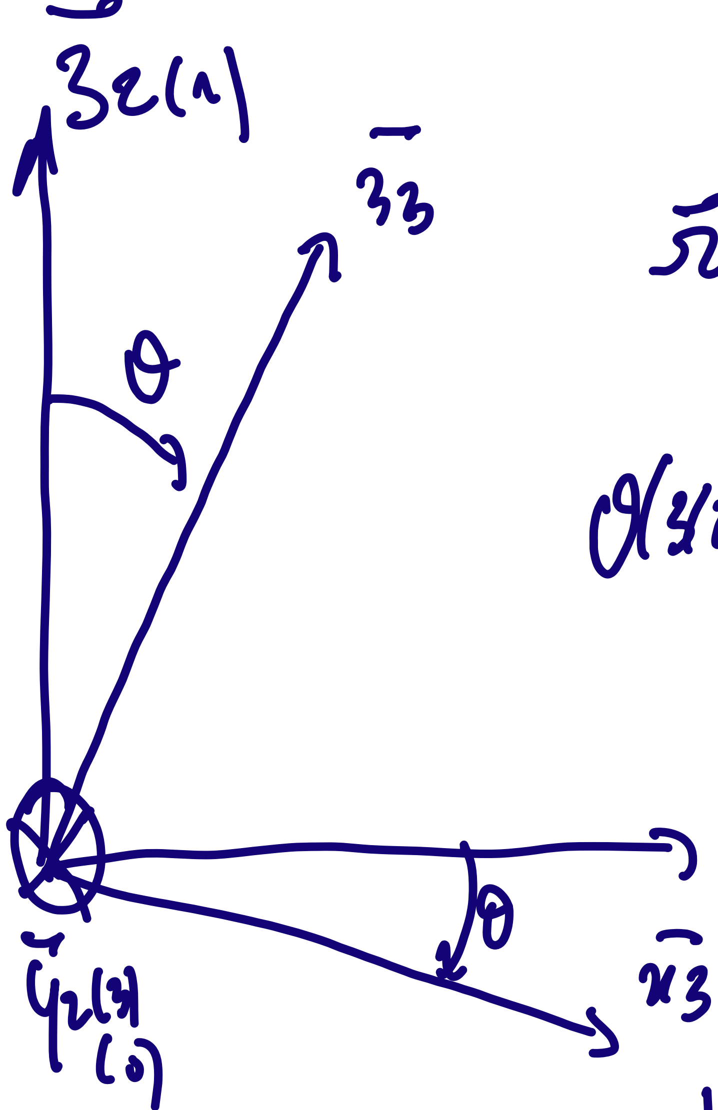
2) $\vec{V}(O,2/0) = \vec{V}(O,2/1) + \vec{V}(O,1/0)$

$$\dot{x}\,\vec{x}_0 = V_{x21}\,\vec{x}_0 + \vec{0}$$

$$\Rightarrow \boxed{V_{x21} = \dot{x}}$$

$$V_{x21} = +\frac{p}{2\pi}\omega_{x21} \Rightarrow \boxed{\dot{x} = -\frac{p}{2\pi}\dot{\varphi}}$$

$$\Rightarrow x = -\frac{p}{2\pi}\varphi + K$$



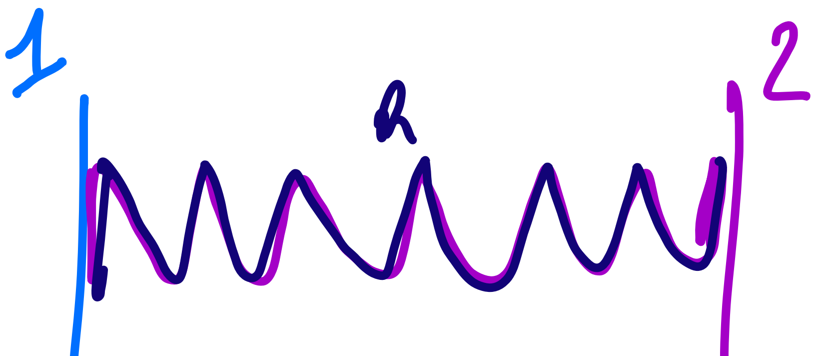
$$\vec{\Omega}(3/2) = \dot{\theta}\,\vec{y}_2(3)$$

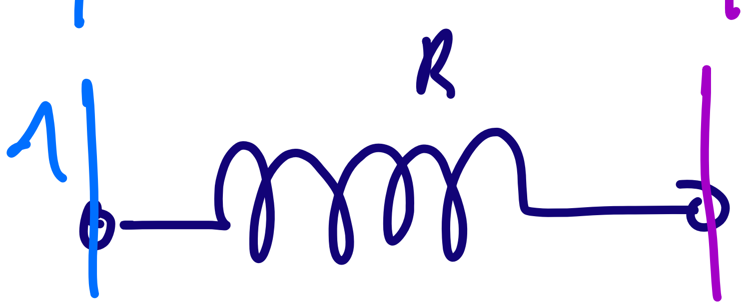
$$\mathcal{V}(3/2) = \left\{ \dot{\theta}\,\vec{y}_2(3),\ \vec{0} \right\}_P$$

$$\mathcal{F}(2\to3) = \left|\begin{matrix} X_{23} & L_{23} \\ Y_{23} & \cancel{M_{23}} \\ Z_{23} & N_{23} \end{matrix}\right|_P$$

$(-, \vec{y}_{2(3)}, -)$

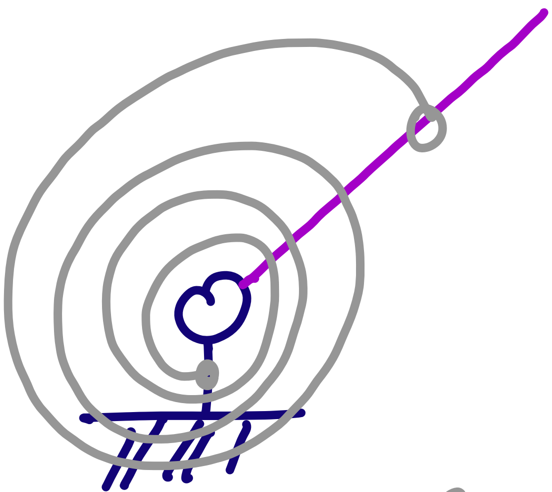
$$\mathcal{F}(R_{23}\to3) = \left\{ \vec{0},\ C_3\vec{y}_0 \right\}_{\forall M}$$





$$\text{"} \; F = k(l - \underbrace{l_0}) \; \text{"}$$

F: N k: N/mm l_0: mm



$$\text{"} \; C = k_T(\overbrace{\theta - \theta_0}^{\text{rad}}) \; \text{"}$$

C: Nm k_T: Nm/rad

$$\mathcal{T}(\text{Ressort} \to 3) = \left\{ \vec{0} \, , \, -k(\theta + \theta_0)\vec{x}_3 \right\}_{\forall P}$$

$$\mathcal{T}(\text{pes} \to i) = \left\{ \vec{P}_i \, , \, \vec{0} \right\}_{G_i}$$

$$\vec{P}_{1} = -m_i g \vec{z}_0$$

1ère eqⁿ du Mᵛᵗ : M.D. à $\boxed{3}$ en P / $\vec{y}_0$

$$\mathcal{F}(\bar{3} \to 3) = \mathcal{D}(3/0)$$

$$\mathcal{F}(pes \to 3) + \mathcal{F}(r \to 3) + \mathcal{F}(M_{23} \to 3) + \mathcal{F}(2 \to 3) = \mathcal{D}(3/0)$$

M.D en P / $\vec{y}_0$: $\vec{M}(P, \vec{P}_3) + \vec{M}(P, r \to 3) + C_3 \vec{y}_0 + (-f_3 \dot{\theta} \vec{y}_0) = \vec{\delta}(P, 3/0)$

$$\boxed{\underbrace{(\vec{PG_3} \wedge \vec{P}_3) \cdot \vec{y}_0} - k(\theta + \theta_0) + C_3 - f_3 \dot{\theta} = \vec{\delta}(P, 3/0) \cdot \vec{y}_0}$$

$$\vec{\sigma}(G_3, 3/0) = J(G_3, 3)\,\vec{\Omega}(3/0)$$

$$\vec{\delta}(G_3, 3/0) = \left[\frac{d\vec{\sigma}(G_3, 3/0)}{dt}\right]_{(0)} \leftarrow$$

$$\vec{\delta}(P, 3/0) = \vec{\delta}(G_3, 3/0) + \overrightarrow{PG_3} \wedge m_3 \vec{A}(G_3, 3/0)$$

$$\vec{\delta}(P, 3/0) \cdot \vec{y}_0 = \vec{\delta}(G_3, 3/0) \cdot \vec{y}_0 + \left[\overrightarrow{PG_3} \wedge m_3 \vec{A}(G_3, 3/0)\right] \cdot \vec{y}_0$$

$$= \frac{d\vec{\sigma}(G_3, 3/0) \cdot \vec{y}_0}{dt} + \underline{\qquad\qquad}$$

$$\vec{V}(G_3, 3/0) = \left[\frac{d\overrightarrow{OG_3}}{dt}\right]_{(0)}$$

$$\overrightarrow{OG_3} = \overrightarrow{OP} + \overrightarrow{PG_3} = x\vec{x}_0 + r\vec{x}_3 \leftarrow$$

$$\vec{A}(G_3, 3/0) = \left(\frac{d\vec{V}(G_3, 3/0)}{dt}\right)_{(0)}$$

$$\left(\frac{d\overrightarrow{OG_3}}{dt}\right)_{(0)} = \dot{x}\,\vec{x}_0 + r\left(\frac{d\vec{x}_3}{dt}\right)_{(0)}$$

$$= \dot{x}\,\vec{x}_0 + r\;\vec{\Omega}(3/0)\wedge\vec{x}_3$$

$$\left(\vec{\Omega}(3/2) + \cancel{\vec{\Omega}(2/0)}\right)\wedge\vec{x}_3$$

$$\dot{\theta}\,\vec{y}_3$$

$$= \dot{x}\,\vec{x}_0 - r\dot{\theta}\,\vec{z}_3$$

$$\vec{V}(G_3, 3/0) = \vec{V}(G_3, 3/2) + \vec{V}(G_3, 2/0)$$

$$= \overrightarrow{G_3P}\wedge\vec{\Omega}(3/2) + \dot{x}\,\vec{x}_0$$

$$= -r\vec{x}_3\wedge\dot{\theta}\,\vec{y}_3 + \dot{x}\,\vec{x}_0$$

$$= -r\dot{\theta}\,\vec{z}_3 + \dot{x}\,\vec{x}_0 \quad \leftarrow$$

$$\vec{A}(G_3, 3/0) = \ddot{x}\,\vec{x}_0 - r\ddot{\theta}\,\vec{z}_3 - r\dot{\theta}\underbrace{\left(\frac{d\vec{z}_3}{dt}\right)_{(0)}}$$

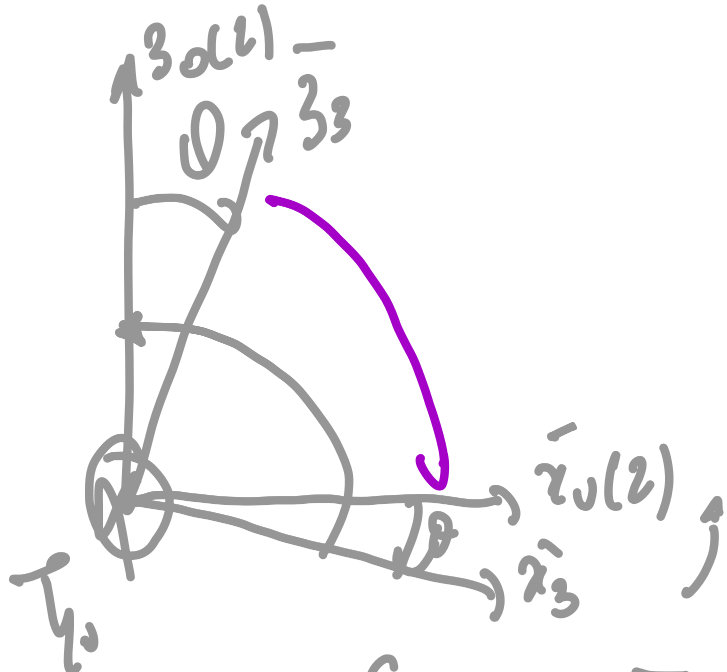
$$\left(\frac{d\vec{z}_3}{dt}\right)_{(0)} = \vec{\Omega}(3/0)\wedge\vec{z}_3 = \dot{\theta}\,\vec{y}_3\wedge\vec{z}_3 = \dot{\theta}\,\vec{x}_3$$

$$\vec{A}(G_3, 3/0) = \ddot{x}\,\vec{x}_0 - r\ddot{\theta}\,\vec{z}_3 - r\dot{\theta}^2\,\vec{x}_3$$

$$\left(\overrightarrow{PG_3} \wedge m_3 \vec{A}(G_3, 3/0)\right) \cdot \vec{y}_0$$

$$m_3 \left(r \vec{x}_3 \wedge \ddot{x}\, \vec{x}_0\right) \cdot \vec{y}_0 - m_3 \left(\overbrace{r \vec{x}_3 \wedge r \ddot{\theta}\, \vec{z}_3}^{-r\ddot{\theta}\vec{y}_3}\right) \cdot \vec{y}_0$$

$$- m_3 \left(\underbrace{\cancel{r \vec{x}_3 \wedge r \dot{\theta}^2 \vec{x}_3}}_{\vec{0}}\right) \cdot \vec{y}_0$$



$$r m_3 \ddot{x} \underbrace{\left(\vec{x}_3 \wedge \vec{x}_0\right)}_{-\sin\theta\, \vec{y}_0} \cdot \vec{y}_0 + \underbrace{m_3 r^2 \ddot{\theta}}\ \underbrace{\vec{y}_3 \cdot \vec{y}_0}_{\cos\theta}$$

$$- r m_3 \ddot{x} \sin\theta + m_3 r^2 \ddot{\theta}$$

$$\left(\overrightarrow{PG_3} \wedge \vec{P}_3\right) \cdot \vec{y}_0 = \left(r \vec{x}_3 \wedge - m_3 g \vec{z}_0\right) \cdot \vec{y}_0$$

$$= -r m_3 g \, (\underbrace{\vec{x}_3 \wedge \vec{z}_0}_{-\cos\theta \vec{y}_0}) \cdot \vec{y}_0 = r m_3 g \cos\theta$$

$$\vec{\sigma}(G_3, 3/0) + \vec{\Omega}(3/0) = \dot{\theta}\,\vec{y}_0(3)$$

$$\vec{\sigma}(G_3, 3/0) = J(G_3, 3)\, \vec{\Omega}(3/0)$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_{R_3} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}_{R_3} = \begin{bmatrix} -F\dot{\theta} \\ B\dot{\theta} \\ -D\dot{\theta} \end{bmatrix}_{R_3}$$

$$= -F\dot{\theta}\,\vec{x}_3 + B\dot{\theta}\,\vec{y}_{3(0)} - D\dot{\theta}\,\vec{z}_3$$

$$\vec{\sigma}(G_3, 3/0) \cdot \vec{y}_0 = B\dot{\theta} \qquad \text{"}J\omega\text{"}$$

$$\frac{d\, \vec{\sigma}(G_3, 3/0) \cdot \vec{y}_0}{dt} = B\ddot{\theta} \qquad \text{"}J \frac{d\omega}{dt}\text{"}$$

$$(E) : (2 + 3 + r_{23} + R)$$

$$\Sigma(\bar{E} \to E) = \mathcal{D}(E/0)$$

$$\mathcal{F}(pes \to 3) + \mathcal{F}(pes \to 2) + \mathcal{F}(0 \to 2) + \mathcal{F}(1 \to 2)$$

$$= \mathcal{D}(2/0) + \mathcal{D}(3/0) + \cancel{\mathcal{D}(\Pi_{23}/0)} + \cancel{\mathcal{D}(\ldots)}$$

R.D ā E / $\vec{x}_0$

$$\cancel{\vec{P}_3 \cdot \vec{x}_0} + \cancel{\vec{P}_2 \cdot \vec{x}_0} + \left(-f_2 \dot{x}\right) + X_{12}$$

$$-m_3 g \overset{0}{\vec{z}_0 \cdot \vec{x}_0}$$

$$= m_2 \vec{A}(G_2, 2/0) \cdot \vec{x}_0 + m_3 \vec{A}(G_3, 3/0) \cdot \vec{x}_0$$

$$\vec{V}(G_2, 2/0) = \dot{x}\,\vec{x}_0 \to \vec{A}(G_2, 2/0) = \ddot{x}\,\vec{x}_0$$

$$-f_2 \dot{x} + X_{12} = m_2 \ddot{x} + m_3 \vec{A}(G_3, 3/0) \cdot \vec{x}_0$$

$$\vec{A}(G_3, 3/0) \cdot \vec{x}_0 = \left(\ddot{x}\,\vec{x}_0 - l\,\ddot{\theta}\,\vec{z}_3 - l\,\dot{\theta}^2\,\vec{x}_3\right) \cdot \vec{x}_0$$

$$= \ddot{x} - r\ddot{\theta}\, \underbrace{\vec{z}_3 \cdot \vec{x}_0}_{\sin\theta} - r\dot{\theta}^2\, \underbrace{\vec{x}_3 \cdot \vec{x}_0}_{\cos\theta}$$

On isole $\boxed{1}$: MD en O / $\vec{x}_0$

$$\mathcal{F}(\bar{1} \to 1) = \mathcal{D}(1/0)$$

$$\mathcal{F}(M_{0} \to 1) + \mathcal{F}(0 \to 1) + \mathcal{F}(\underset{\smile}{2} \to 1) + \mathcal{F}(pes \to 1)$$
$$= \mathcal{D}(1/0)$$

MD à $\boxed{1}$ en O / $\vec{x}_0$

$$C_1 + (-f_1\dot{\varphi}) + L_{21} + \cancel{\left(\underset{\substack{\parallel \\ OG_1 x_0}}{\vec{OG_1}} \wedge \underset{-m_1 g \vec{y}_0}{\vec{P}_1^{\,0}}\right) \cdot \vec{x}_0}$$

$$= \vec{\delta}(0, 1/0) \cdot \vec{x}_0 = J_1 \ddot{\varphi}$$

$$L_{12} = C_1 - f_1 \dot\varphi - J_1 \ddot\varphi$$