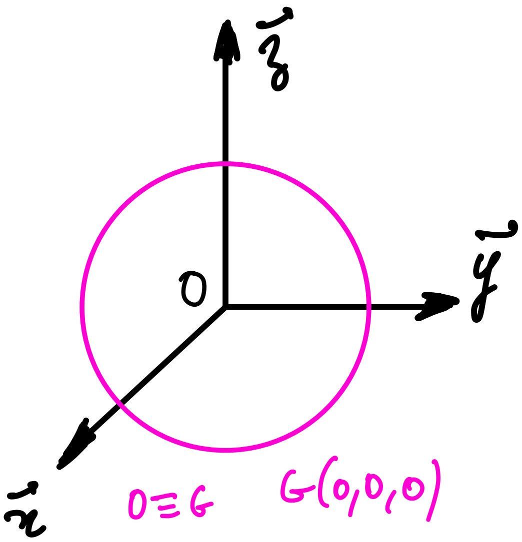
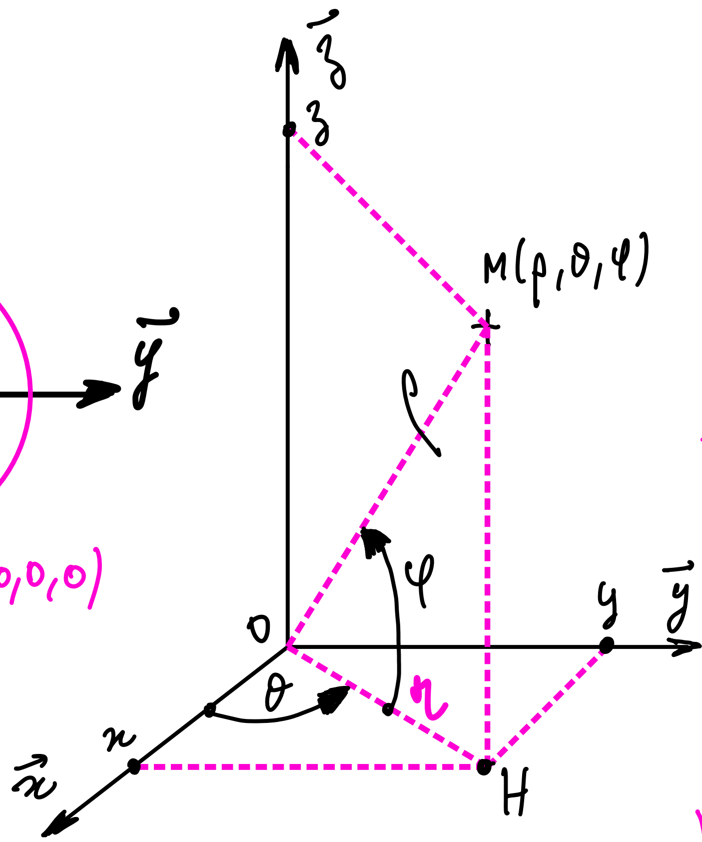
Opérateur d'inertie d'une boule de centre O et de rayon R





$dS = \rho\, d\varphi$

$dS = r\, dr\, d\theta$

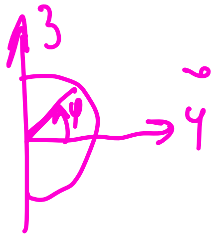
avec $r = \rho\cos\varphi$

$\Rightarrow dS = \cos\varphi\, d\varphi$
ρ

$\Rightarrow dV = \rho^2 d\rho \cos\varphi\, d\varphi\, d\theta$

$$\begin{cases} x = \rho\cos\varphi\cos\theta \\ y = \rho\cos\varphi\sin\theta \\ z = \rho\sin\varphi \end{cases}$$

$V = \frac{4}{3}\pi R^3$



2 PS $\Rightarrow E = D = F = 0$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & A_0 \end{bmatrix}_R$$

$$\mathrm{Trace}(J(O,S)) = 3A_0 = 2\iiint \underbrace{(x^2+y^2+z^2)}\, dm = 2\iiint \rho^2\, dm$$

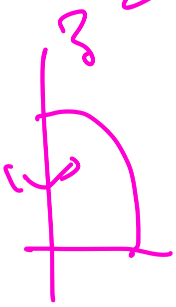
$$A_0 = \frac{2}{3}\iiint \rho^2\, dm = \frac{2}{3}\iiint \rho^2 \eta\, dV = \frac{2}{3}\eta \iiint \rho^4 d\rho \cos\varphi\, d\varphi\, d\theta$$

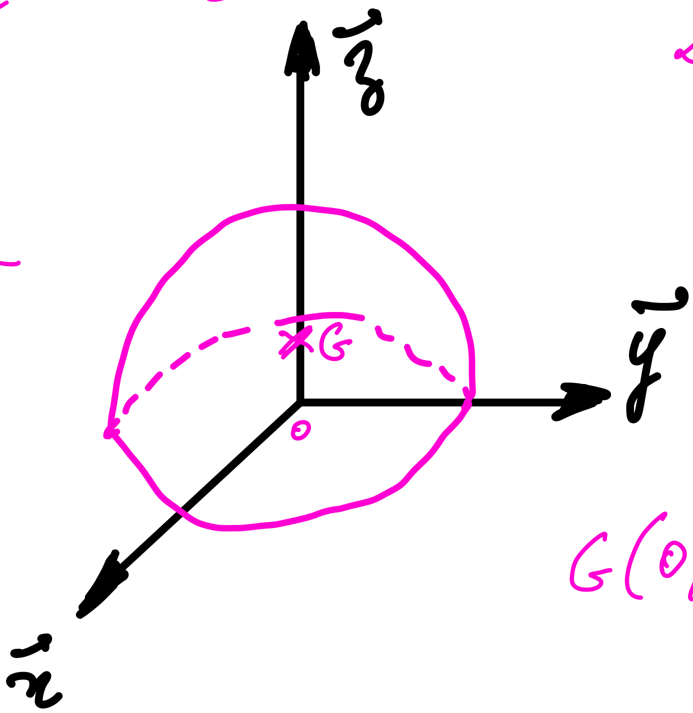
$$A_0 = \frac{2}{3}\eta \int_0^R \rho^4 d\rho \int_{-\frac{\pi}{2}}^{\pi/2} \cos\varphi\, d\varphi \int_0^{2\pi} d\theta = \frac{2}{3}\eta \left[\frac{\rho^5}{5}\right]_0^R \left[\sin\varphi\right]_{-\pi/2}^{\pi/2} \left[\theta\right]_0^{2\pi}$$

$$A_0 = \frac{2}{3}\eta \frac{R^5}{5}\left[2\right]\left[2\pi\right] = \frac{2}{3}\frac{m}{\frac{4}{3}\pi R^3}\frac{R^5}{5} 4\pi$$

$$\boxed{A_0 = \frac{2}{5} m R^2}$$

$\frac{1}{2}$ boule $z > 0$





2PS (yOz) et (xOz)

$\rightarrow D = E = F$

$$J(O,S) = \begin{bmatrix} A_0 & & \\ & A_0 & \\ & & C_0 \end{bmatrix}_R$$

$G(0,0,z_G)$ $\quad z_G = \frac{3}{8}R$

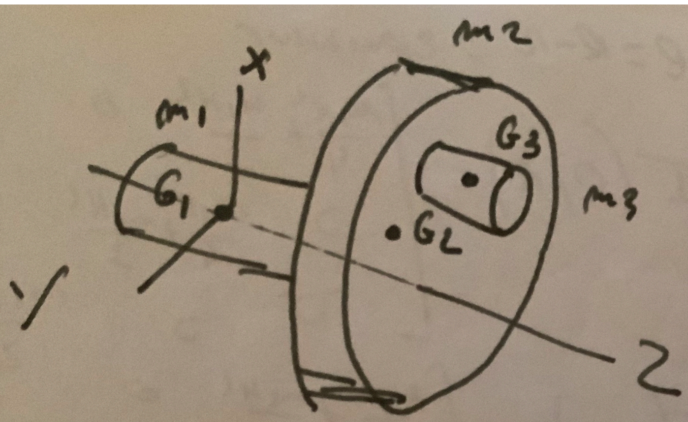
$\vec{OG}\begin{vmatrix} 0 \\ 0 \\ z_G \end{vmatrix}$

$$J(G,S) = J(O,S) - J(O,G(m))$$

$$J(O,G(m)) = \begin{bmatrix} m z_G^2 & & \\ & m z_G^2 & \\ & & 0 \end{bmatrix}$$

$$J(G,S) = \begin{bmatrix} A_G & 0 & 0 \\ 0 & A_G & 0 \\ 0 & 0 & C_G \end{bmatrix}$$

$C_G = C_0 \quad ; \quad A_G = A_0 - m z_G^2$



$$\overrightarrow{G_1G_2} = a_2\vec{x} + c_2\vec{z}$$
$$\overrightarrow{G_1G_3} = a_3\vec{x} + c_3\vec{z}$$

$$I(G_1, C_1) = \begin{pmatrix} A_1 & & 0 \\ & A_1 & \\ 0 & & C_1 \end{pmatrix}$$
$$I(G_2, C_2) = \begin{pmatrix} A_2 & & 0 \\ & A_2 & \\ 0 & & C_2 \end{pmatrix}$$
$$I(G_3, C_3) = \begin{pmatrix} A_3 & & 0 \\ & A_3 & \\ 0 & & C_3 \end{pmatrix}$$

$$\underline{\underline{I(G_1, (S))}}\ ?$$