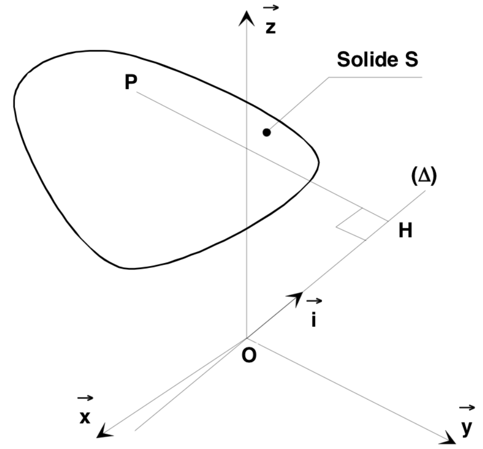
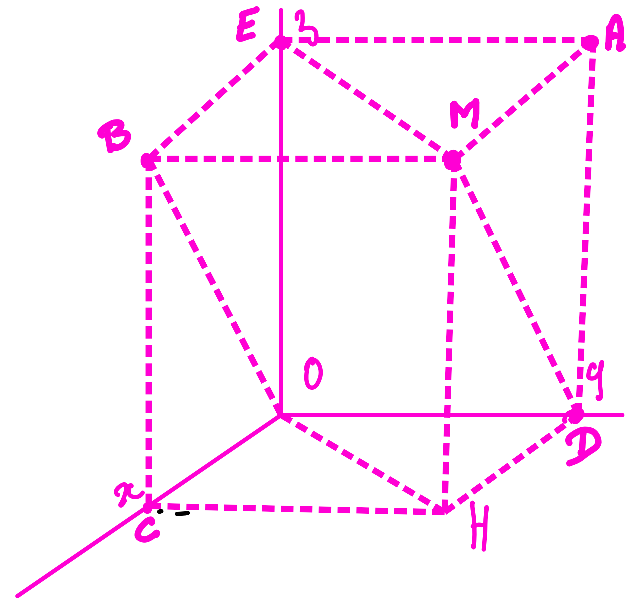
$I(\Delta,S)=\int_{P\in S}\overrightarrow{PH}^2 dm$

$I(\Delta,S)=\vec{i}\cdot\int_{P\in S}\overrightarrow{OP}\wedge(\vec{i}\wedge\overrightarrow{OP})dm$

$I(S/\Delta)=\int_{P\in S} HP^2 dm$

$I(S/\Delta)=\left[-\int \overrightarrow{OP}\wedge(\overrightarrow{OP}\wedge\vec{i})dm\right]\cdot\vec{i}$





$$\int_{P\in S}\overrightarrow{OP}\wedge(\vec{u}\wedge\overrightarrow{OP})\,dm=\begin{pmatrix}\int_{P\in S}(y^2+z^2)\,dm & -\int_{P\in S}xy\,dm & -\int_{P\in S}xz\,dm\\ -\int_{P\in S}xy\,dm & \int_{P\in S}(x^2+z^2)\,dm & -\int_{P\in S}yz\,dm\\ -\int_{P\in S}xz\,dm & -\int_{P\in S}yz\,dm & \int_{P\in S}(x^2+y^2)\,dm\end{pmatrix}\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}$$

$||$

$$-\int_{P\in S}\overrightarrow{OP}\wedge(\overrightarrow{OP}\wedge\vec{u})dm=J(O,S)\vec{u}$$

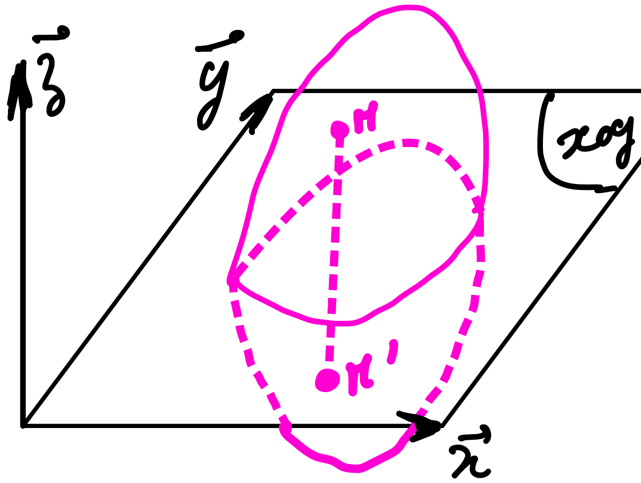
$$\overline{\overline{I}}(O,S)_R=\begin{pmatrix}A_O & -F_O & -E_O\\ -F_O & B_O & -D_O\\ -E_O & -D_O & C_O\end{pmatrix}_R \quad \text{avec } R=(O,\vec{x},\vec{y},\vec{z})$$

$J(O,S)$

$$\overline{\overline{I}}(O,S)_R\,\vec{u}=\int_{P\in S}\overrightarrow{OP}\wedge(\vec{u}\wedge\overrightarrow{OP})\,dm=\begin{pmatrix}A & -F & -E\\ -F & B & -D\\ -E & -D & C\end{pmatrix}_R\begin{pmatrix}u_1\\u_2\\u_3\end{pmatrix}_R$$

$$J(O,S)\vec{u}=-\int \overrightarrow{OP}\wedge(\overrightarrow{OP}\wedge\vec{u})dm$$

Un plan de symétrie (xoy)



$PS = (xoy) \Rightarrow D = E = 0$

$$\int_{P\in S} xz\,dm = \int_{z>0} xz\,dm + \int_{z<0} xz\,dm$$

$$= \int_{z>0} xz\,dm - \int_{z>0} xz\,dm$$

$$= 0$$

$$\int yz\,dm = 0$$

$$J(O,S) = \begin{bmatrix} A_0 & -F_0 & 0 \\ -F_0 & B_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}_R$$

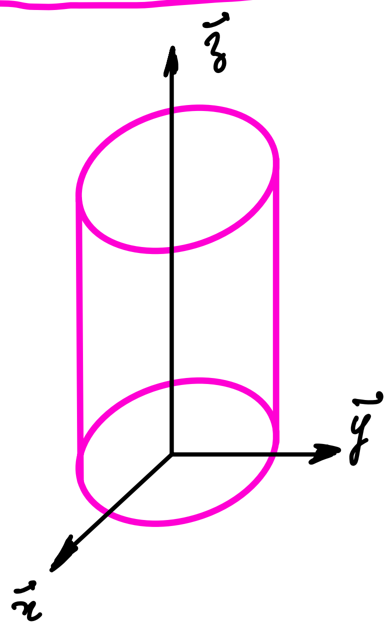
$1\,PS\,(yoz) \Rightarrow F_0 = E_0 = 0$ ou $D_0 \neq 0$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & B_0 & -D_0 \\ 0 & -D_0 & C_0 \end{bmatrix}_R$$

$1\,PS\,(xoz) \Rightarrow E_0 \neq 0$ ou $D_0 = F_0 = 0$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & -E_0 \\ 0 & B_0 & 0 \\ -E_0 & 0 & C_0 \end{bmatrix}_R$$

2 PLANS de Symétrie



PS : $yOz \rightarrow F = E = 0$

PS : $xOz \rightarrow D = F = 0$

$D_0 = E_0 = F_0$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}_R$$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}_R$$

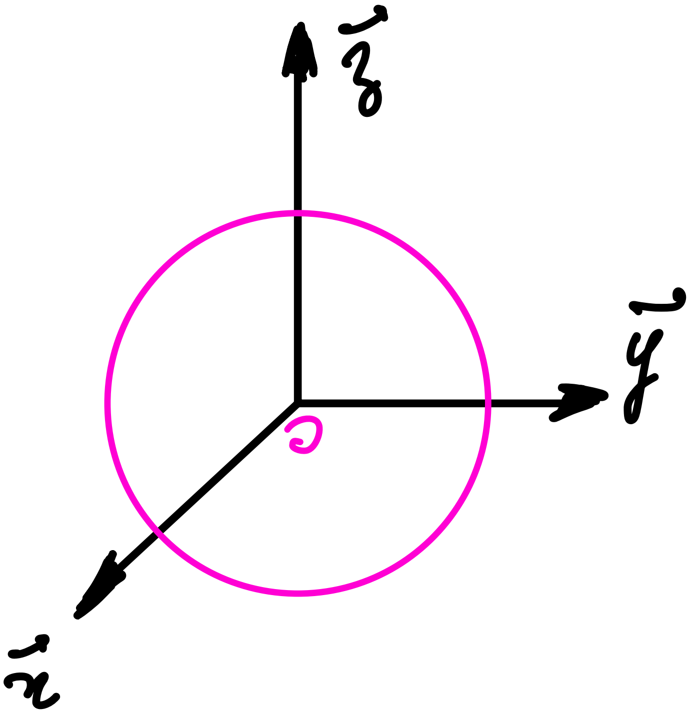
Axe Revolution $(O,z) \Rightarrow D = E = F = 0$

$A_0 = B_0$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}_R$$

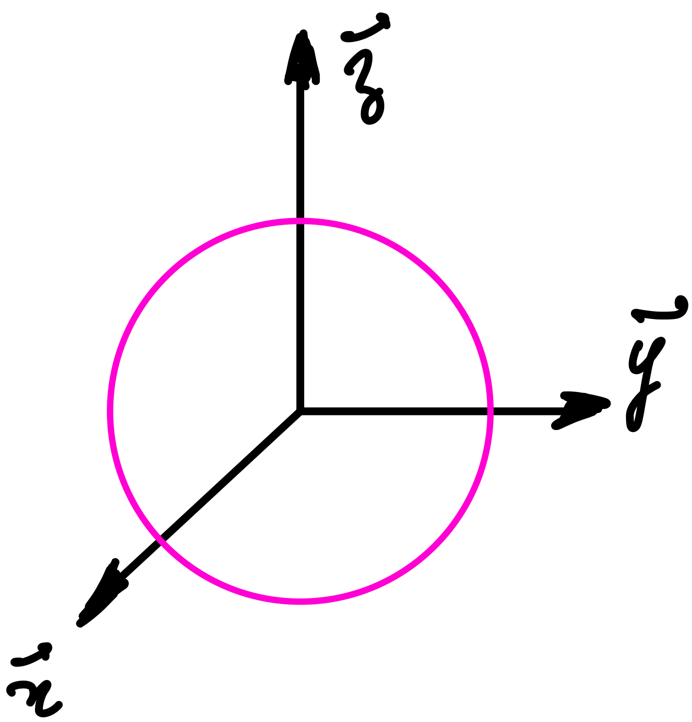
Axe Revolution (O,y) $\quad J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & A_0 \end{bmatrix}_R$

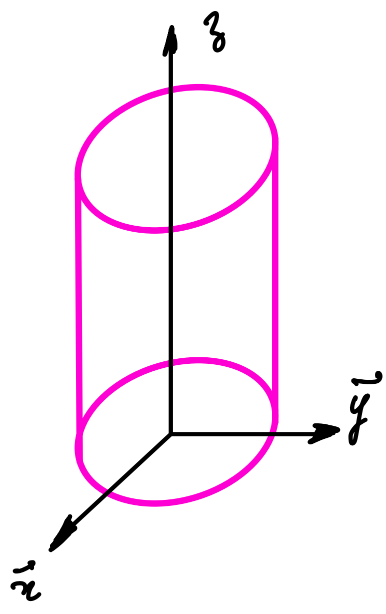
Axe Revolution (O,x) $\quad J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & B_0 \end{bmatrix}$



$$D_O = E_O = F_O$$

$$J(O,S) = \begin{bmatrix} A_0 & & 0 \\ & A_0 & \\ 0 & & A_0 \end{bmatrix}_R$$





$$I(S, O\vec{x}) = \left(J(O,S)\,\vec{x}\right)\cdot\vec{x}$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_R \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_R \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} A \\ -F \\ -E \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = A$$