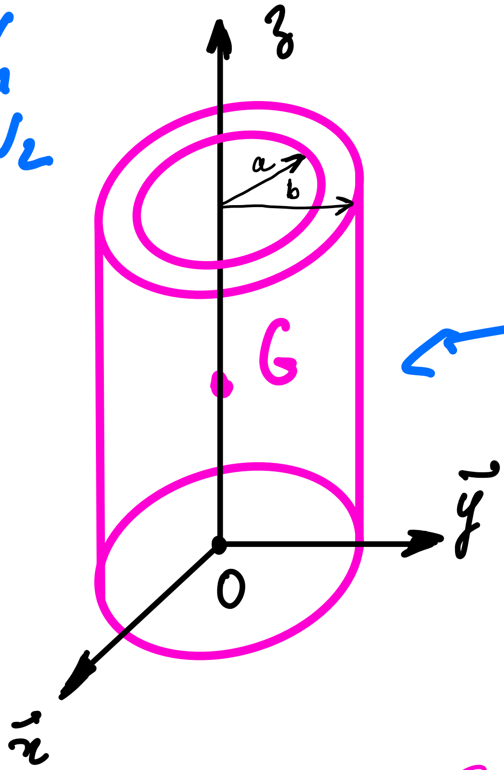
TUBE

$m_t = \rho V$

$m_2 = \rho V_2$

$$J(O, Tube) = \begin{bmatrix} A_0 & & \\ & A_0 & \\ & & C_0 \end{bmatrix}_R$$



$OG = \frac{H}{2}$

Calcul de C_0

S_1 : Cylindre plein de rayon b $\longleftrightarrow$ $C_1 = \cancel{\frac{m b^2}{2}}$ $\frac{m_1 b^2}{2}$

S_2 : Cylindre plein de rayon a $\longrightarrow$ $C_2 = \cancel{\frac{m a^2}{2}}$ $\frac{m_2 a^2}{2}$

Tube $S_1 - S_2$ $\Rightarrow C_0 = C_1 - C_2 = \cancel{\frac{m}{2}(b^2 - a^2)}$

$$C_0 = \frac{m_1 b^2}{2} - \frac{m_2 a^2}{2}$$

$$\rho = \frac{m}{\pi(b^2 - a^2)H} = \frac{m_1}{\pi b^2 H} = \frac{m_2}{\pi a^2 H}$$

$$m_1 = \frac{b^2}{b^2 - a^2} m \quad \text{et} \quad m_2 = \frac{a^2}{b^2 - a^2} m$$

$$C_0 = \frac{m b^4}{2(b^2 - a^2)} - \frac{m a^4}{2(b^2 - a^2)} = \frac{m}{2} \frac{b^4 - a^4}{b^2 - a^2} = \frac{m}{2}(b^2 + a^2)$$

$$C_0 = \eta \left[\frac{r^4}{4}\right]_0^R \left[\theta\right]_0^{2\pi} \left[z\right]_0^H = \eta \frac{R^4}{4} 2\pi H$$

et $\eta = \frac{m}{V} = \frac{m}{\pi R^2 H} \Rightarrow C_0 = \frac{m}{\cancel{\pi R^2 H}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \cancel{2\pi H}$

$$\boxed{C_0 = m\frac{R^2}{2}}$$ en $kg\,m^2$

$$C_0 = \eta \left[\frac{r^4}{4}\right]_a^b \left[\theta\right]_0^{2\pi} \left[z\right]_0^H$$

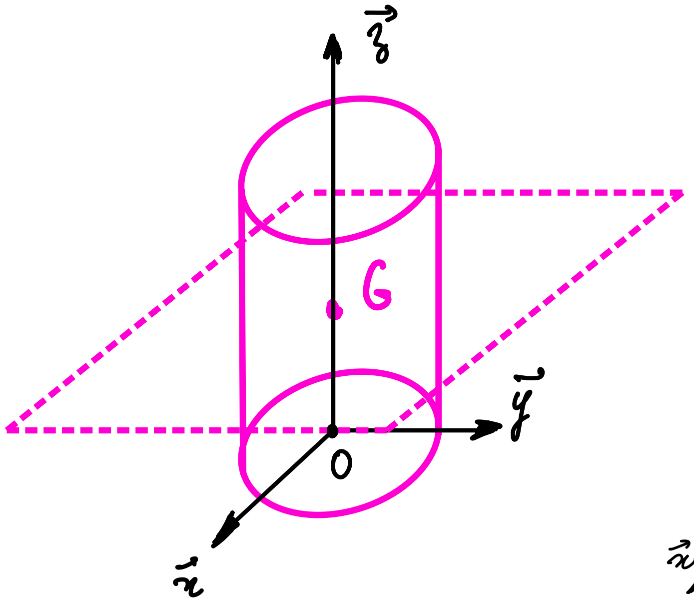
$$C_0 = \eta \frac{b^4 - a^4}{4} 2\pi H = \frac{m}{V} \frac{b^4 - a^4}{4} 2\pi H$$

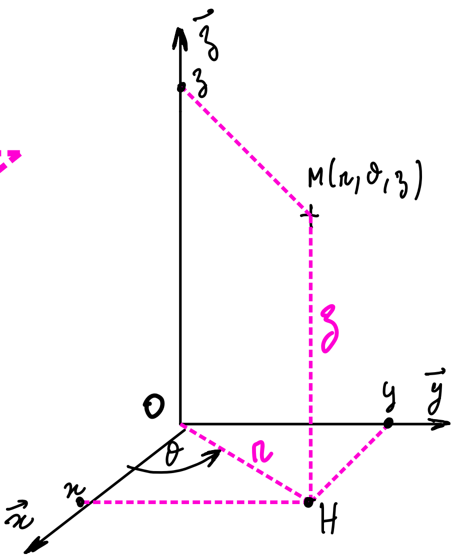
$$C_0 = \frac{m}{\cancel{\pi}(b^2 - a^2)\cancel{H}} \frac{b^4 - a^4}{\cancel{4}\,2} \cancel{2\pi}\,\cancel{H}$$

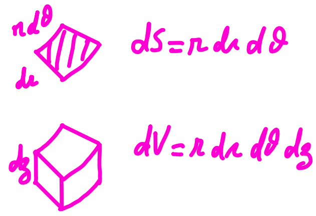
$$C_0 = \frac{m}{2} \frac{b^4 - a^4}{b^2 - a^2} = \frac{m}{2} \frac{\cancel{(b^2 - a^2)}(b^2 + a^2)}{\cancel{(b^2 - a^2)}}$$

$$\boxed{C_0 = \frac{m}{2}(b^2 + a^2)}$$

Opérateur d'inertie du cylindre d'axe z , de rayon R et de hauteur H







$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

PS $xOz \rightarrow y_G = 0$ PS $yOz \Rightarrow x_G = 0$ $z_G = \frac{H}{2}$

$$J(O, S) = \begin{bmatrix} A_O & 0 & 0 \\ 0 & A_O & 0 \\ 0 & 0 & C_O \end{bmatrix}_R$$

2PS $\Rightarrow D = E = F = 0$

$A = B$

$$C_O = I(S/Oz) = \iiint (x^2+y^2)\,dm = \iiint r^2\,dm = \iiint r^2 \eta\,dV$$

$$C_O = \eta \iiint r^2 r\,dr\,d\theta\,dz = \eta \int_0^R r^3 dr \int_0^{2\pi} d\theta \int_0^H dz$$

$$C_O = \eta \left[\frac{r^4}{4}\right]_0^R \left[\theta\right]_0^{2\pi} \left[z\right]_0^H = \eta \frac{R^4}{4} 2\pi H$$

et $\eta = \frac{m}{V} = \frac{m}{\pi R^2 H} \Rightarrow C_O = \frac{m}{\cancel{\pi R^2 H}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \cancel{2\pi}\,\cancel{H}$

$$\boxed{C_O = m\frac{R^2}{2}}$$ en $kg\,m^2$