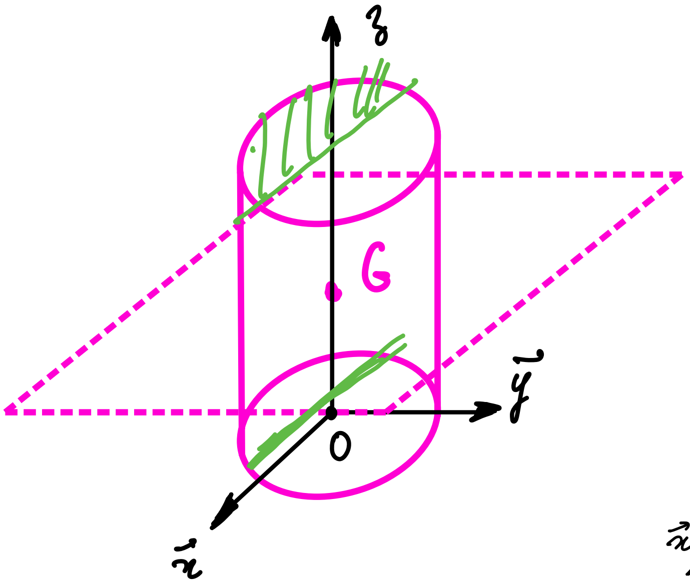
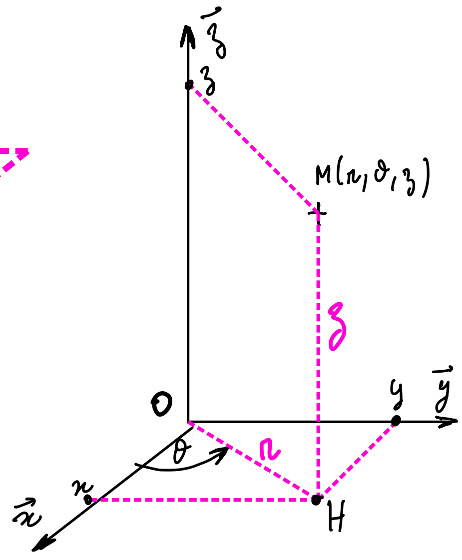
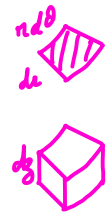
Opérateur d'inertie du cylindre d'axe $\vec{z}$, de rayon R et de hauteur H







$dS = r\,dr\,d\theta$

$dV = r\,dr\,d\theta\,dz$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

PS $xOz \rightarrow y_G = 0$ PS $yOz \Rightarrow x_G = 0$ $z_G = \frac{H}{2}$

$$J(O,S) = \begin{bmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & C_0 \end{bmatrix}_R$$

2PS $\Rightarrow D = E = F = 0$

$A = B$

$0 \leq \theta \leq \pi$

$$C_0 = I(S/Oz) = \iiint (x^2+y^2)\,dm = \iiint r^2\,dm = \iiint r^2 \eta\,dV$$

$$C_0 = \eta \iiint r^2\, r\,dr\,d\theta\,dz = \eta \int_0^R r^3\,dr \int_0^{2\pi} d\theta \int_0^H dz$$

$$C_0 = \eta \left[\frac{r^4}{4}\right]_0^R \left[\theta\right]_0^{2\pi} \left[z\right]_0^H = \eta \frac{R^4}{4}\, 2\pi\, H$$

$$\text{et } \eta = \frac{m}{V} = \frac{m}{\pi R^2 H} \Rightarrow C_0 = \frac{m}{\cancel{\pi} \cancel{R^2} \cancel{H}} \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \cancel{2\pi} \cancel{H}$$

$$\boxed{C_0 = m\frac{R^2}{2}}$$ en $kg\,m^2$

$$A_O = B_O = \iiint (y^2+z^2)\,dm = \iiint (x^2+z^2)\,dm$$

$$\mathrm{Trace}(J(O,S)) = 2A_O + C_O = 2\iiint (\underbrace{x^2+y^2}_{r^2}+z^2)\,dm$$

$$2A_O + C_O = 2C_O + 2\iiint z^2\,dm.$$

$$\Rightarrow 2A_O = C_O + 2\iiint z^2\,dm \Rightarrow A_O = \frac{C_O}{2} + \iiint z^2\,dm$$

$$A_O = \frac{mR^2}{4} + \iiint z^2\,dm$$

$$\iiint z^2\,dm = \iiint z^2 \eta\,dV = \eta \iiint r\,dr\,d\theta\, z^2\,dz$$

$$= \frac{m}{V} \int r\,dr \int d\theta \int z^2\,dz = \frac{m}{V}\left[\frac{r^2}{2}\right]_0^R \left[\theta\right]_0^{2\pi} \left[\frac{z^3}{3}\right]_0^H$$

$$= \frac{m}{\cancel{\pi}\cancel{R^2}\cancel{H}}\, \frac{\cancel{R^2}}{\cancel{2}}\, \cancel{2}\cancel{\pi}\, \frac{H^{\cancel{3}\,2}}{3} = \frac{mH^2}{3}$$

$$\boxed{A_O = \frac{mR^2}{4} + \frac{mH^2}{3}}$$

$$J(G,S) = ? = \begin{bmatrix} A_G & & \\ & A_G & \\ & & C_G \end{bmatrix}_R$$

$$C_G = C_O$$

$$\boxed{A_G = \frac{mR^2}{4} + \frac{mH^2}{12}}$$

$$\int_{-\frac{H}{2}}^{H/2} z^2 dz = \left[\frac{z^3}{3} \right]_{-\frac{H}{2}}^{H/2} = \frac{H^3}{24} + \frac{H^3}{24} = \frac{H^3}{12}$$

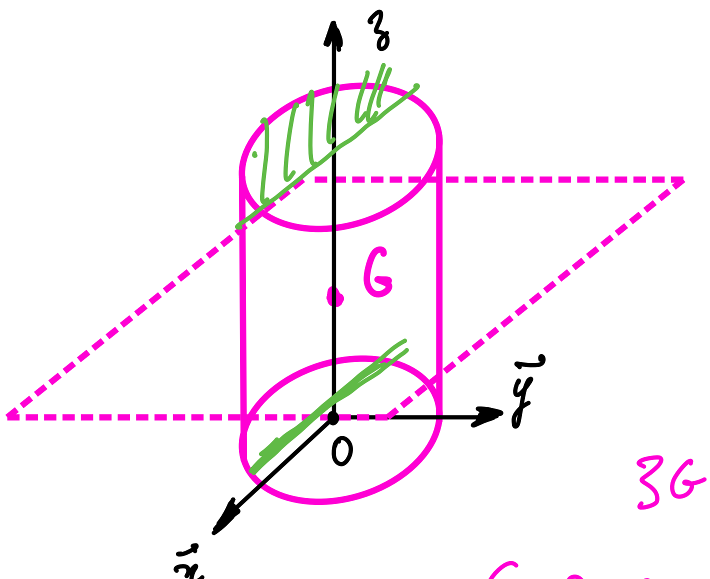
Huygens : $J(G,S) = J(O,S) - J(O,G(m))$

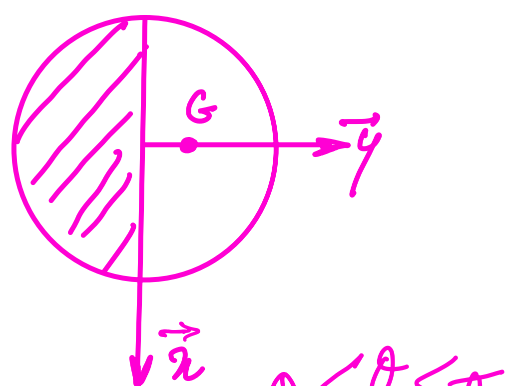
$$J(O,G(m)) = \begin{bmatrix} m\frac{H^2}{4} & 0 & 0 \\ 0 & m\frac{H^2}{4} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \vec{OG} \begin{vmatrix} 0 \\ 0 \\ \frac{H}{2} \end{vmatrix}$$

$$A_G = A_O - m\frac{H^2}{4} = \frac{mH^2}{3} - \frac{mH^2}{4} = \frac{mH^2}{12}$$

$$C_G = C_O - 0 = C_O$$

$\frac{1}{2}$ Cylindre d'axe (O,z) $y>0$





$z_G = \frac{H}{2}$

(yOz) PS $\Rightarrow x_G = 0$

$0 \leq \theta \leq \pi$

$y_G = \frac{4R}{3\pi}$

$$I(G,S) = \begin{bmatrix} A_G & & \\ & A_G & \\ & & C_G \end{bmatrix}_R$$

$$A_G = \frac{mR^2}{4} + \frac{mH^2}{12}$$

$$C_G = m\frac{R^2}{2}$$

$$I(O,S) = \begin{bmatrix} A_O & 0 & 0 \\ 0 & A_O & -D_O \\ 0 & -D_O & C_O \end{bmatrix}_R$$

$1PS\ (yOz)$

$$\Rightarrow E_O = F_O = 0$$

$$I(O,S) = I(G,S) + I(O,G(m))$$

$$\vec{OG} \begin{vmatrix} 0 \\ y_G \\ \frac{H}{2} \end{vmatrix}$$

$$I(O,G(m)) = \begin{bmatrix} m y_G^2 + m\frac{H^2}{4} & 0 & 0 \\ 0 & m\frac{H^2}{4} & -m y_G \frac{H}{2} \\ 0 & -m y_G \frac{H}{2} & m y_G^2 \end{bmatrix}$$

$$D_O = -m y_G \frac{H}{2} \neq 0$$

$$A_G = A_O - m\left(y_G^2 + \frac{H^2}{4}\right)$$