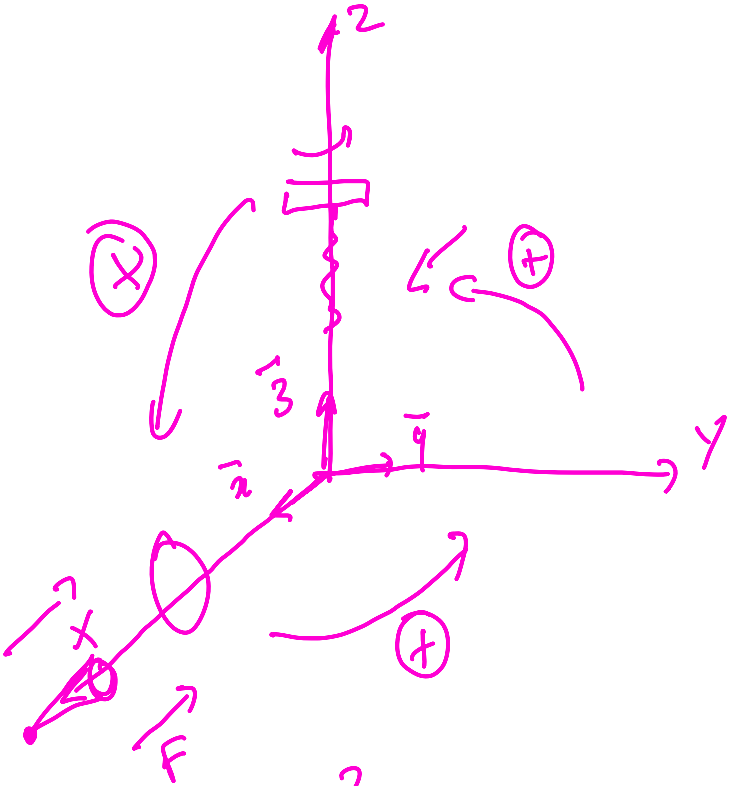
# Cinétique

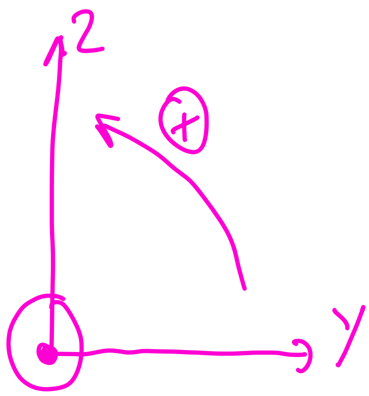


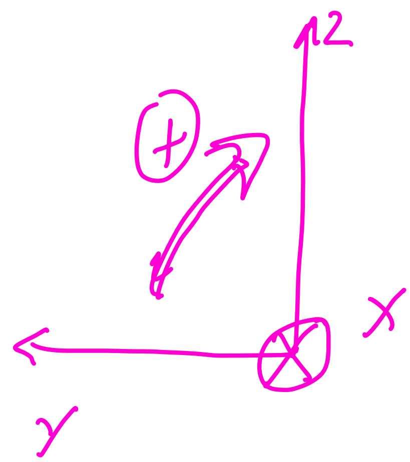

ortho $\vec x \cdot \vec y = 0$ ; $\vec y \cdot \vec z = 0$ ; $\vec z \cdot \vec x = 0$

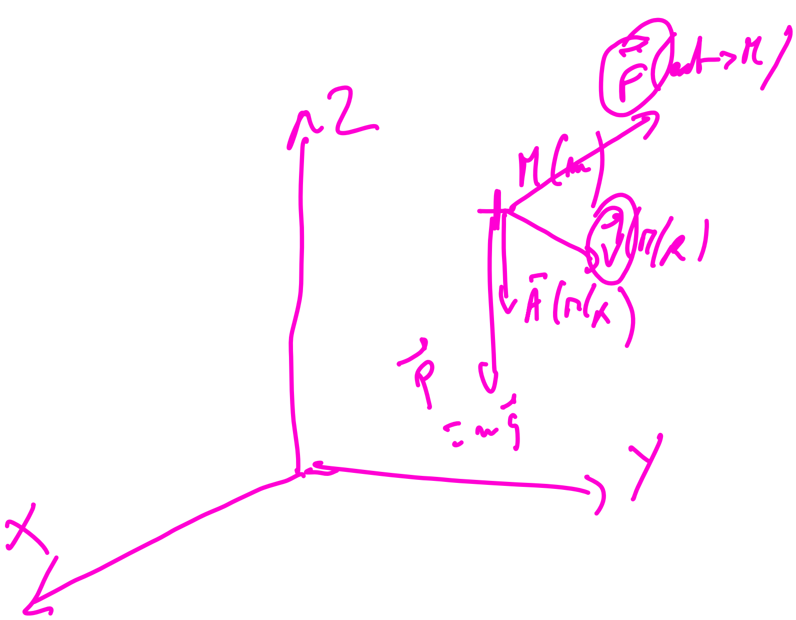
Normée : $\|\vec x\| = \|\vec y\| = \|\vec z\| = 1$

Directe

| | | |
|---|---|---|
| XY | $\vec x \wedge \vec y = \vec z$ | $\vec y \wedge \vec x = -\vec z$ |
| YZ | $\vec y \wedge \vec z = \vec x$ | $\vec z \wedge \vec y = -\vec x$ |
| ZX | $\vec z \wedge \vec x = \vec y$ | $\vec x \wedge \vec z = -\vec y$ |



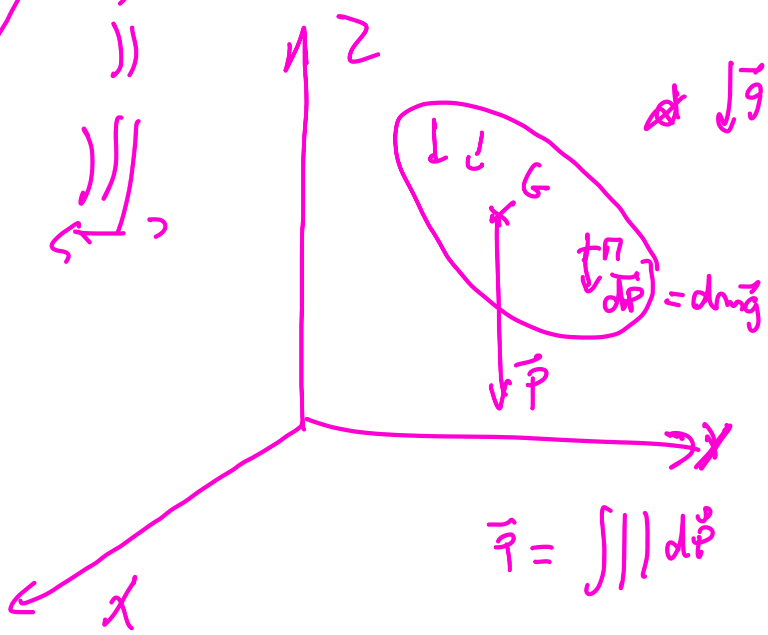




$\int$

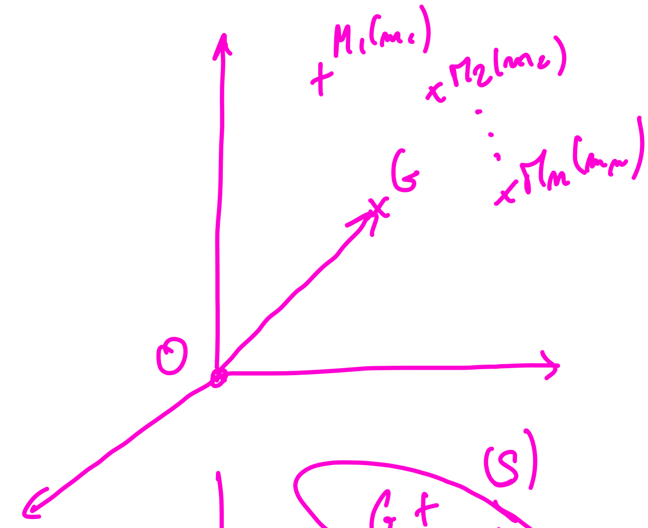
$\iint$

$\iiint$




$\vec P = \iiint d\vec P$

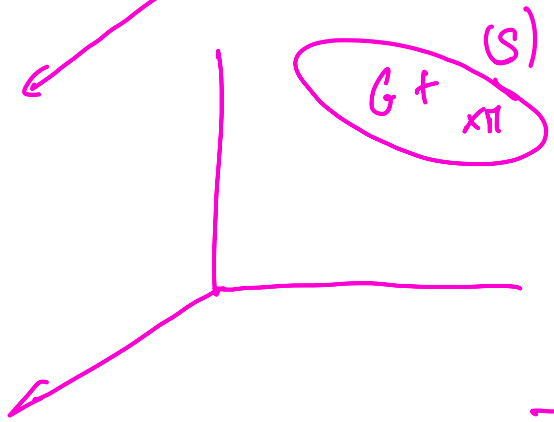
# Centre de gravité (Inertie)




$$\sum_{i=1}^{n} m_i \overrightarrow{GM_i} = \vec{0} \quad \propto$$

$$\left(\sum_{i=1}^{n} m_i\right)\overrightarrow{GO} + \sum_{i=1}^{n} m_i \overrightarrow{OM_i} = \vec{0}$$

$$\rightarrow \left(\sum_{i=1}^{n} m_i\right)\overrightarrow{OG} = \sum_{i=1}^{n} m_i \overrightarrow{OM_i}$$




$$\iiint_{M\in S} \overrightarrow{GM}\,dm = \vec{0} \quad \propto$$

$$\left(\iiint dm\right)\overrightarrow{OG} = \iiint \overrightarrow{OM}\,dm$$

$$m\,\overrightarrow{OG} = \quad '' \quad '' \quad ''$$

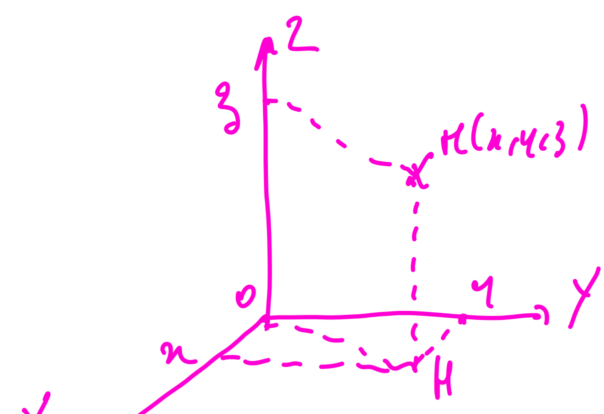
$\rightarrow /\vec{x}$ : $m x_G = \iiint x\,dm$

$\rightarrow /\vec{y}$ : $m y_G = \iiint y\,dm$

$\rightarrow /\vec{z}$ : $m z_G = \iiint z\,dm$

P.S. $\Rightarrow$ $x_G$ ou $y_G$ ou $z_G$ $= 0$

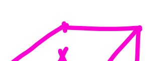
| | | | |
|---|---|---|---|
| $\iiint$ | $dm = \rho\, dV$ | $\rho = \frac{m}{V}$ | $kg/m^3$ |
| $\iint$ | $dm = \sigma\, dS$ | $\sigma = \frac{m}{S}$ | $kg/m^2$ |
| $\int$ | $dm = \lambda\, dl$ | $\lambda = \frac{m}{l}$ | $kg/m$ |

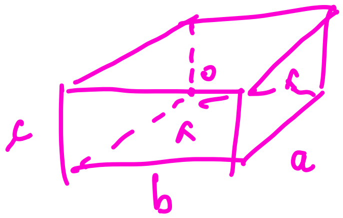


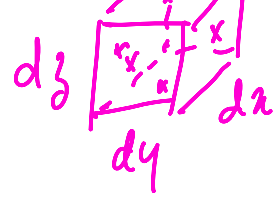

$dl$ : $dx \,/\, dy \,/\, dz$

$dS$ : $dxdy \,/\, dxdz \,/\, dydz$

$dV$ : $dx.dy.dz$

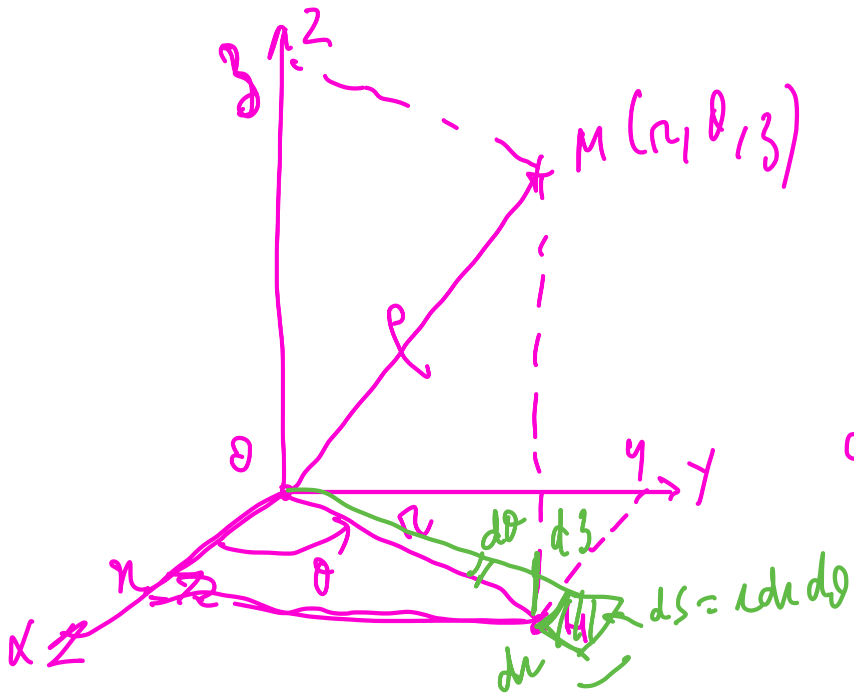





$$V = \iiint dV = \iiint dx\,dy\,dy = \int_0^b dx \int_0^a dy \int_0^c dz = bac$$

Cylindrique

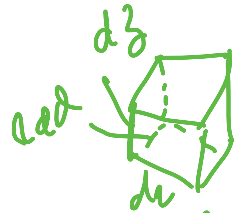



$r = OH \quad \theta = (\vec{x}, \vec{OH})$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$dl \quad dr \,/\, r\,d\theta \,/\, dz$

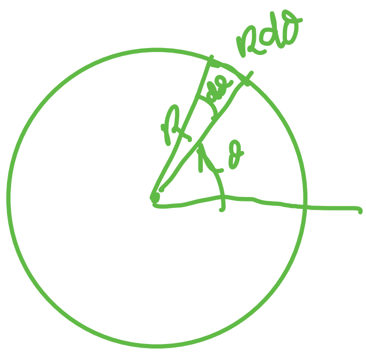
$dV \quad \overbrace{r\,dr\,d\theta\,dz}$



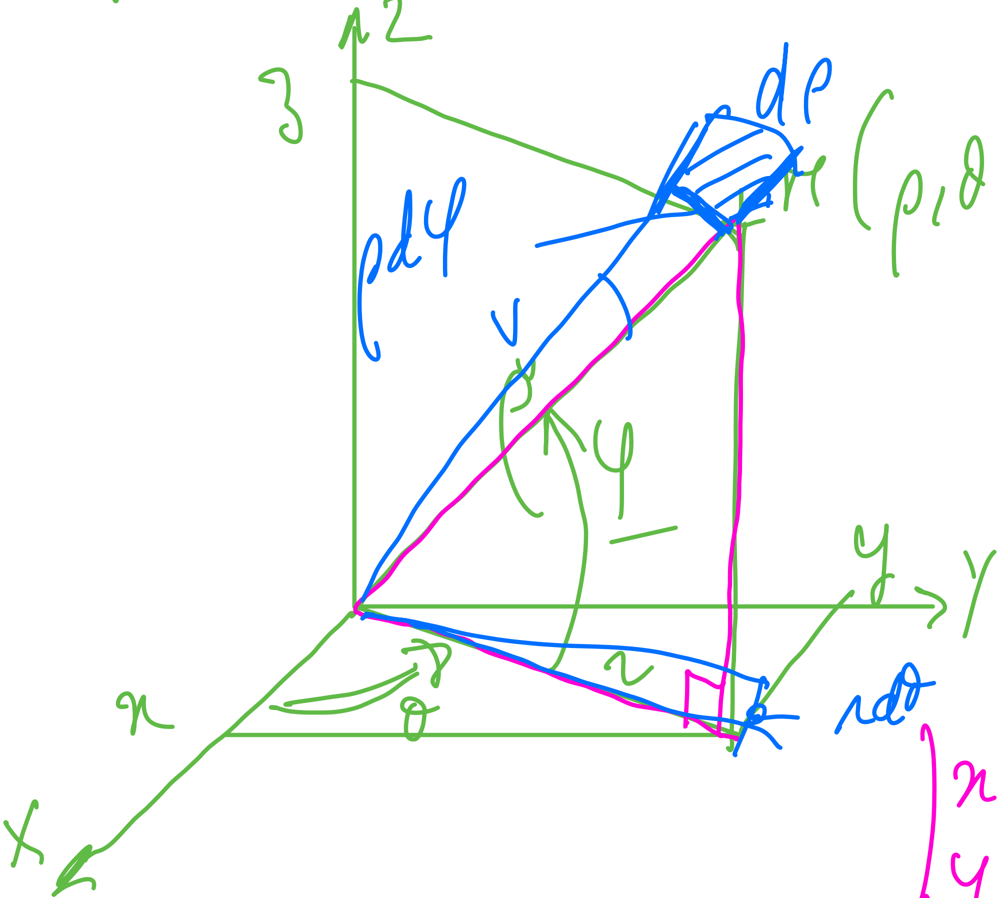

$$S = \iint dS = \iint r\,dr\,d\theta = \int_0^R r\,dr \int_0^{2\pi} d\theta = \frac{R^2}{2} 2\pi = \pi R^2$$

$dl = R\,d\theta$

$$L = \int_0^{2\pi} R\,d\theta = 2\pi R$$



Sphérique




$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$r = \rho\cos\varphi$$

$$\begin{cases} x = \rho\cos\varphi\cos\theta \\ y = \rho\cos\varphi\sin\theta \\ z = \rho\sin\varphi \end{cases}$$

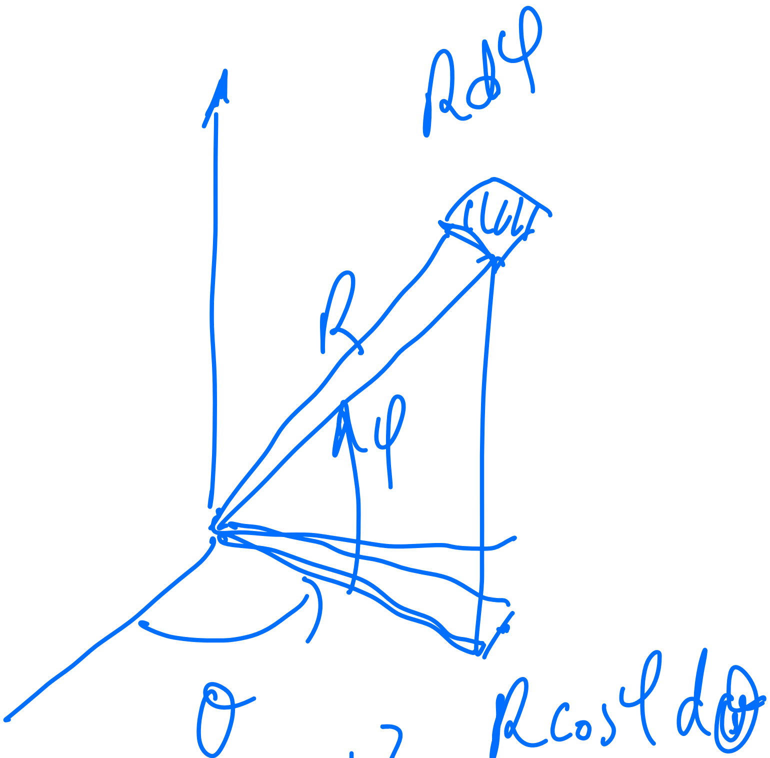
$dl$ / $d\rho$ / $\rho\, d\varphi$ / $r\, d\theta = \rho\cos\varphi\, d\theta$

$$V = \iiint dV = \iiint \rho^2 d\rho \cos\varphi\, d\varphi\, d\theta$$

$$= \int_0^R \rho^2 d\rho \int_{-\pi/2}^{\pi/2} \cos\varphi\, d\varphi \int_0^{2\pi} d\theta$$

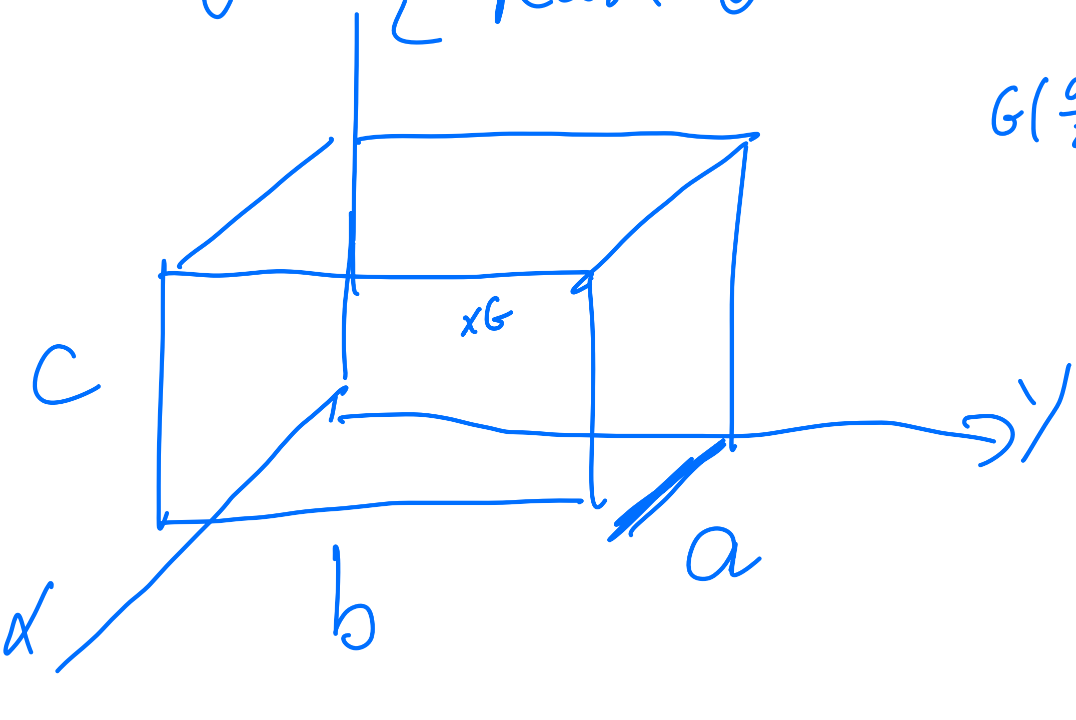
$$= \left[\frac{\rho^3}{3}\right]_0^R \left[\sin\varphi\right]_{-\pi/2}^{\pi/2} \left[\theta\right]_0^{2\pi}$$

$$= \frac{R^3}{3}\, 2 \cdot 2\pi = \frac{4}{3}\pi R^3$$



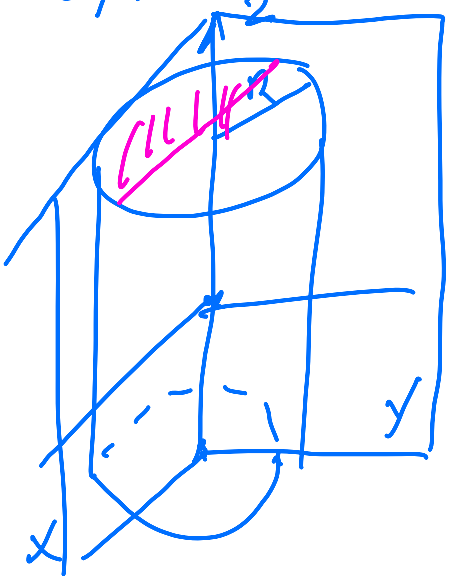

$$dS = R^2 \cos\varphi\, d\varphi\, d\theta$$

$$\Rightarrow S = 4\pi R^2$$




$$G\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$$

Cylindre

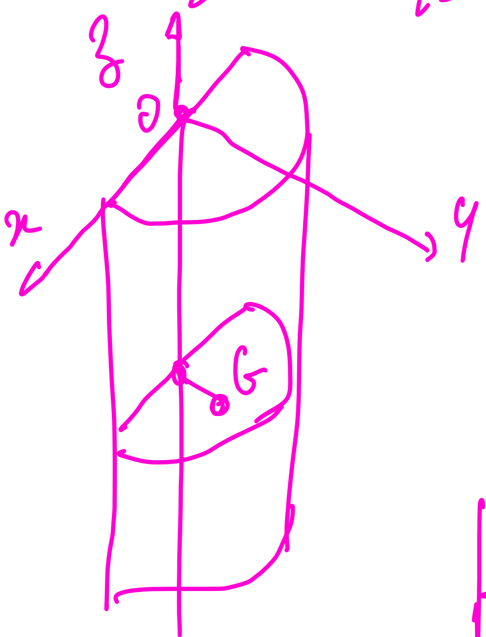



$$\left.\begin{array}{l}(xOz)\ PS \Rightarrow y_G = 0 \\ (yOz)\ PS \Rightarrow x_G = 0\end{array}\right\}\ G \in (Oz)$$

$$(xGy)\ \ PS \Rightarrow z_G = \frac{H}{2}$$

$$G\left(0, 0, \frac{H}{2}\right)$$

$\frac{1}{2}$ cylindre $y > 0$

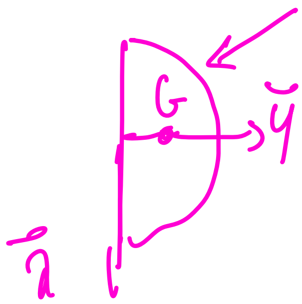



$$(xOz)\ \cancel{PS} \Rightarrow y_G \neq 0$$

$$(yOz)\ PS \Rightarrow x_G$$

$$(xGz)\ PS \Rightarrow z_G = \frac{H}{2}$$

$$G\left(0, y_G, -\frac{H}{2}\right)$$




$$\rho = \frac{m}{V}$$

$$m y_G = \iiint y \, dm$$

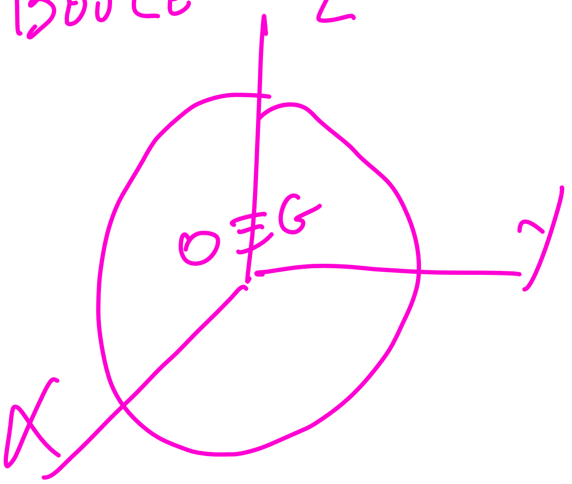
$$m y_G = \iiint y \rho \, dV$$

$$y_G = \frac{\rho}{m} \iiint y \, dV = \frac{1}{V} \iiint r \sin\theta \; r\,dr\,d\theta\,dz$$

$$y_G = \frac{1}{V}\int_0^R r^2 dr \int_0^{\pi} \sin\theta \, d\theta \int_0^H dz = \frac{1}{V}\,\frac{R^3}{3}\,2\,H$$

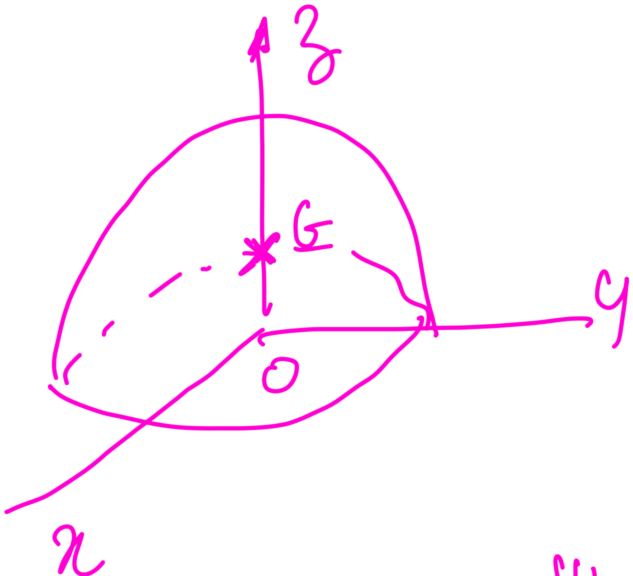
$$y_G = \frac{2}{\pi R^{\cancel{2}} \cancel{H}}\,\frac{R^{\cancel{3}}}{3}\,2\,\cancel{H} = \frac{4R}{3\pi}$$

Boule




$O \equiv G$

$\frac{1}{2}$ Boule $z > 0$




$(xOz)$ PS $\Rightarrow y_G = 0$
$(yOz)$ PS $\Rightarrow x_G = 0$
$\rightarrow G(0, 0, z)$

$z_G$ ?

$$m z_G = \iiint z \, dm = \iiint z \, \rho \, dV$$

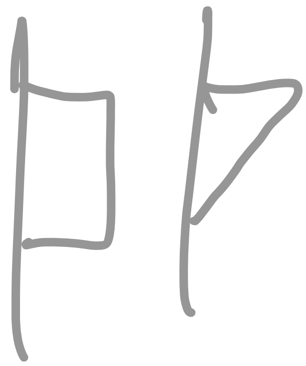
$$z_G = \frac{\rho}{m}\iiint z \, dV = \frac{1}{V}\iiint \rho \sin\varphi \, \rho^2 d\rho \cos\varphi \, d\theta \, d\theta$$

$$z_G = \frac{1}{V}\int_0^R \rho^3\,d\rho \int_0^{\pi/2} \underbrace{\sin\varphi\cos\varphi}_{\frac{\sin 2\varphi}{2}}\,d\varphi \int_0^{2\pi} d\theta$$

$$= \frac{1}{V}\,\frac{R^4}{4}\left[\frac{-\cos 2\varphi}{4}\right]_0^{\pi/2} 2\pi$$

$$= \frac{3}{\cancel{2\pi} R^{\cancel{3}}}\,\frac{R^{\cancel{4}}}{4}\,\frac{1}{2}\,\cancel{2\pi}$$

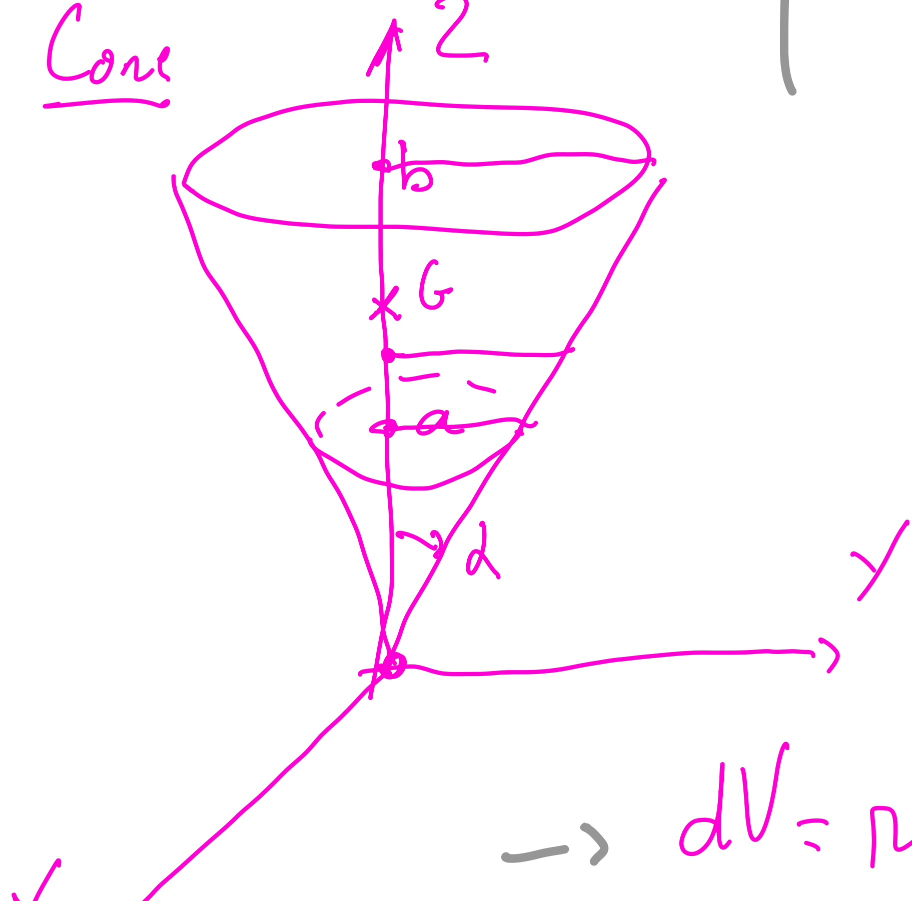
$$= \frac{3}{8}R$$



$0 \le \theta \le 2\pi$

$a \le z \le b$

Cone





$\tan\alpha = \frac{r}{z}$

$r = z\tan\alpha$

$\rightarrow dV = r\,dr\,d\theta\,dz$

$$\to V = \iiint dV = \iiint r\,dr\,d\theta\,dz$$

$$V = \int_0^{2\pi} d\theta \int_a^b \underbrace{\left( \int_0^{z\tan\alpha} r\,dr \right)} dz$$

$$= \cancel{2}\pi \int_a^b \frac{z^2 \tan^2\alpha}{\cancel{2}}\, dz \Big)$$

$$= \pi \tan^2\alpha \left( \frac{z^3}{3} \right|_a^b = \underbrace{\frac{\pi\tan^2\alpha}{3}\left(b^3 - a^3\right)}$$

$$m z_G = \iiint z\,dm = \iiint z\,\rho\,dV$$

$$z_G = \frac{\rho}{m} \int z\,dV = \frac{1}{V}\,\pi \int_a^b z^3 \tan^2\alpha\, dz$$

$$z_G = \frac{\pi}{V} \tan^2\alpha \left( \frac{z^4}{4} \right|_a^b = \frac{\pi}{V} \tan^2\alpha\, \frac{b^4 - a^4}{4}$$

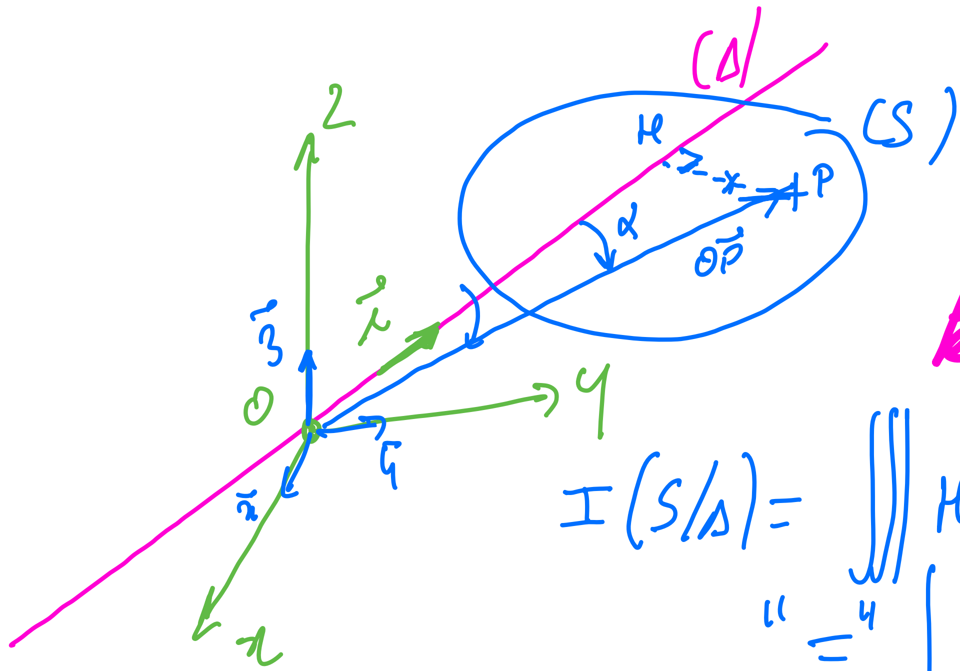
$$z_G = \frac{\cancel{\pi\tan^2\alpha}\; 3}{\cancel{\pi\tan^2\alpha}\left(b^3 - a^3\right)}\, \frac{b^4 - a^4}{4} = \frac{3}{4}\, \frac{b^4 - a^4}{b^3 - a^3}$$

$a = 0 \Rightarrow 3G = \frac{3}{4} b$

Operateur d'inertie d'un solide

$J \frac{d\omega}{dt} = \sum m = C_m - C_R - J \frac{d\omega}{dt}$

Moment d'inertie d'un solide /r à un axe




$\sin\alpha = \frac{MP}{OP}$

kg.m²

$I(S/\Delta) = \iiint MP^2 \, dm$

$" = \int MP^2 \, dm$

$\|\vec{i} \wedge \vec{OP}\| = \cancel{\|\vec{i}\|}^{\,1} \, \|\vec{OP}\| \, |\sin(\vec{i}, \vec{OP})| = OP \, |\sin\alpha| = MP$

$MP^2 = \underset{(\vec{u} \wedge \vec{v})\cdot}{(\vec{i} \wedge \vec{OP})} \cdot (\underbrace{\vec{i} \wedge \vec{OP}}_{\vec{w}}) = \left[ (\vec{i} \wedge \vec{OP}) \wedge \cancel{\vec{u}} \right] \cdot \vec{OP}$

$= \left[ \vec{OP} \wedge (\vec{i} \wedge \vec{OP}) \right] \cdot \vec{i}$

$= - \left[ \vec{OP} \wedge (\vec{OP} \wedge \vec{i}) \right] \cdot \vec{i}$

$I(S/\Delta) = \left( - \int \vec{OP} \wedge (\vec{OP} \wedge \vec{i}) \, dm \right) \cdot \vec{i} = \int MP^2 \, dm$

$= \left( \underbrace{J(O,S)} \, \vec{i} \right) \cdot \vec{i}$

$$\overrightarrow{OP} \wedge \vec{i} = \begin{vmatrix} x \\ y \\ z \end{vmatrix}_{(b)} \wedge \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix}_{(b)} = \begin{vmatrix} y i_3 - z i_2 \\ z i_1 - x i_3 \\ x i_2 - y i_1 \end{vmatrix}_{(b)}$$

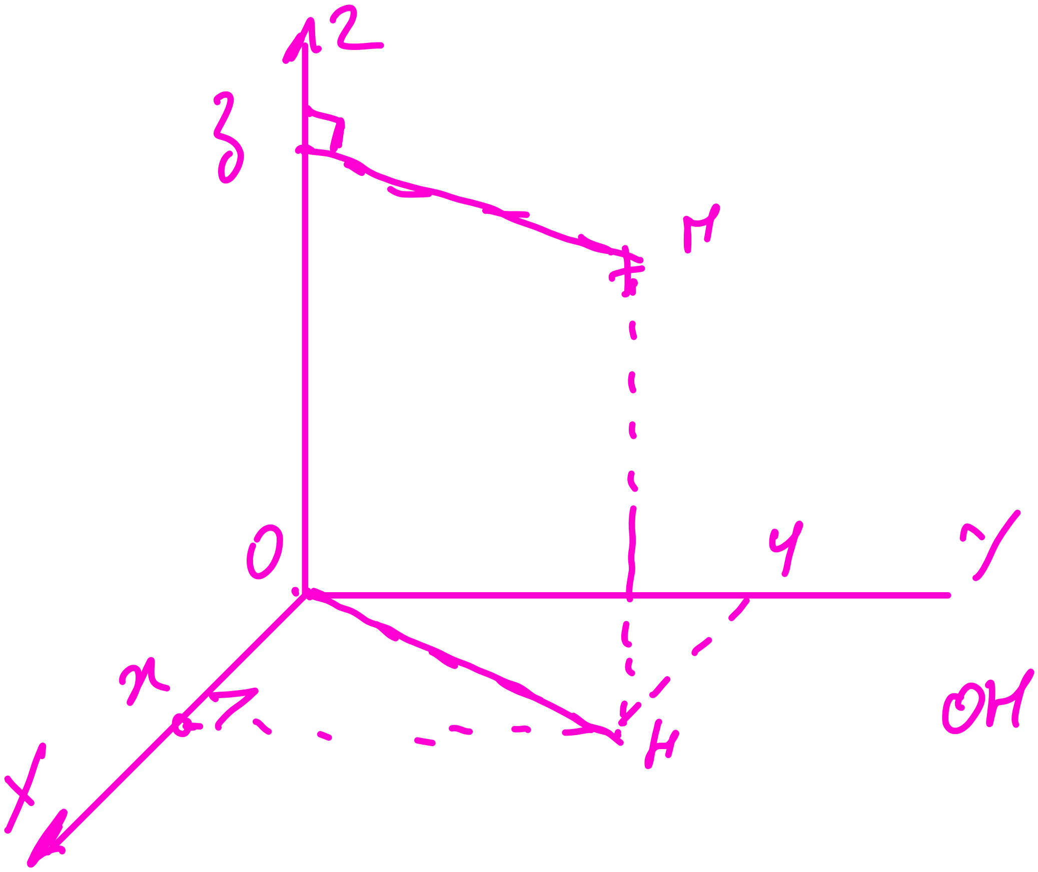
$$\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix}$$

$$\overrightarrow{OP} \wedge (\overrightarrow{OP} \wedge \vec{i})$$

$$\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} = \begin{bmatrix} -(y^2+z^2) & xy & xz \\ xy & -(x^2+z^2) & yz \\ xz & yz & -(y^2+x^2) \end{bmatrix}$$

$$\underset{G}{J}(0, S) = \begin{bmatrix} \int (y^2+z^2)\, dm & -\int xy\, dm & -\int xz\, dm \\ -\int xy\, dm & \int (x^2+z^2)\, dm & -\int yz\, dm \\ -\int xz\, dm & -\int yz\, dm & \int (y^2+x^2)\, dm \end{bmatrix}$$

$$\longrightarrow \begin{bmatrix} A_0 & -F_0 & -E_0 \\ -F_0 & B_0 & -D_0 \\ -E_0 & -D_0 & C_0 \end{bmatrix}_R$$




$OH^2 = x^2 + y^2$

$$I(S/O\vec{z}) = \left(J(O,S)\,\vec{z}\right)\cdot\vec{z}$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_R \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_R \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -E \\ -D \\ C \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = C$$

$$I(S/O\vec{y}) = \left(J(O,S)\,\vec{y}\right)\cdot\vec{y}$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{pmatrix} -F \\ B \\ -D \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = B$$

$$I(S/O\vec{x}) = \left( J(O,S)\, \vec{x} \right) \cdot \vec{x}$$

$$= \begin{pmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} A \\ -F \\ -E \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = A$$

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$J_x \omega_x$

$d \quad J_x \dfrac{d\omega_x}{dt}$

$$\begin{bmatrix} J_x = A & -F & -E \\ -F & J_y = B & -D \\ -E & -D & J_z = C \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_x \omega_x \\ J_y \omega_y \\ J_z \omega_z \end{bmatrix}$$

$A, B, C$

T.M.C $\quad \Sigma M = J \dfrac{d\omega}{dt}$

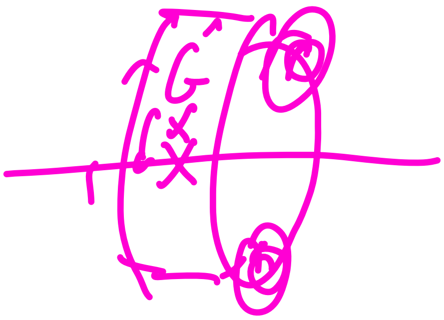
$$\sum m - J\frac{d\omega}{dt} = 0$$

$> 0$

$< 0$

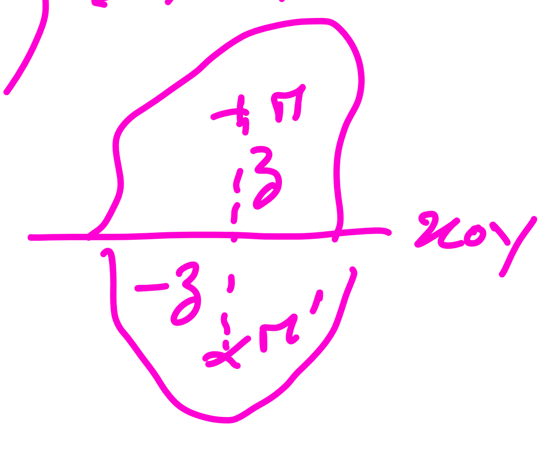
$J_x \omega_x$

$$\begin{bmatrix} A & -F & -E \\ -F & A & -D \\ -E & -D & C \end{bmatrix} \begin{bmatrix} \omega_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A\omega_x \\ \cancel{-F\omega_x} \\ \cancel{-E\omega_x} \end{bmatrix}$$




Simplification de l'OP d'inertie grâce aux propriétés de symétrie

1 Plan de Symétrie → (xoy) → 2 produits nuls

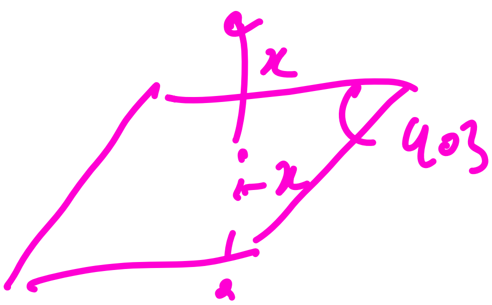



$$J(O,S) = \begin{bmatrix} A & -F & 0 \\ -F & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

$$-E = \int_{M \in S} xz\,dm = \int_{z>0} xz\,dm + \underbrace{\int_{z<0} xz\,dm}_{\to\ -\int_{z>0} xz\,dm} = 0$$

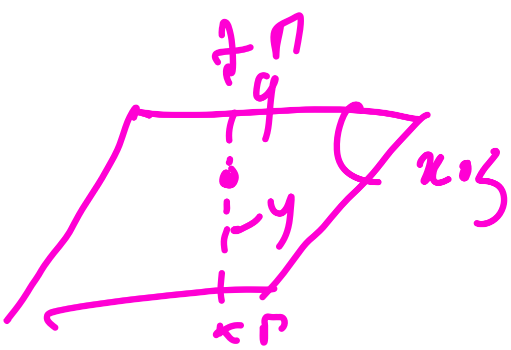
$$-D = \int yz\,dm = 0$$

PS $(yOz)$ $\Rightarrow$ $F = E = 0$ ou $D \neq 0$



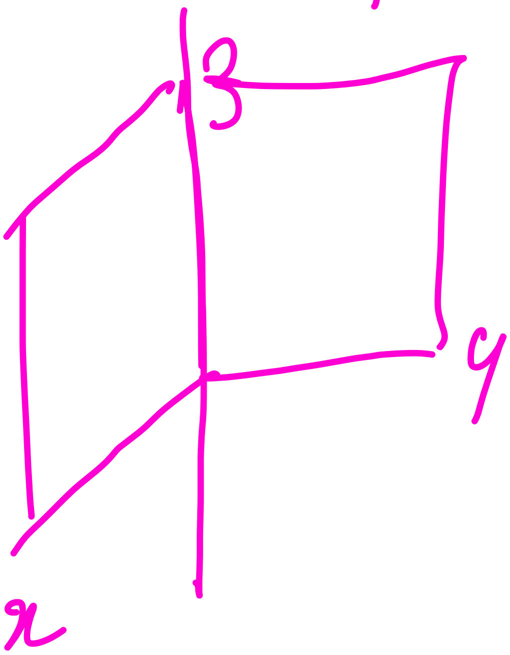

$$J(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & -D \\ 0 & -D & C \end{bmatrix}$$

PS $(xOz)$ $\Rightarrow$ $D = F = 0$ $\quad E \neq 0$




$$J(O,S) = \begin{bmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{bmatrix}$$

## 2 Plans de Symétrie




(yOz) PS

(xOz) PS

$\rightarrow \vec{Oz}$ est axe de symétrie

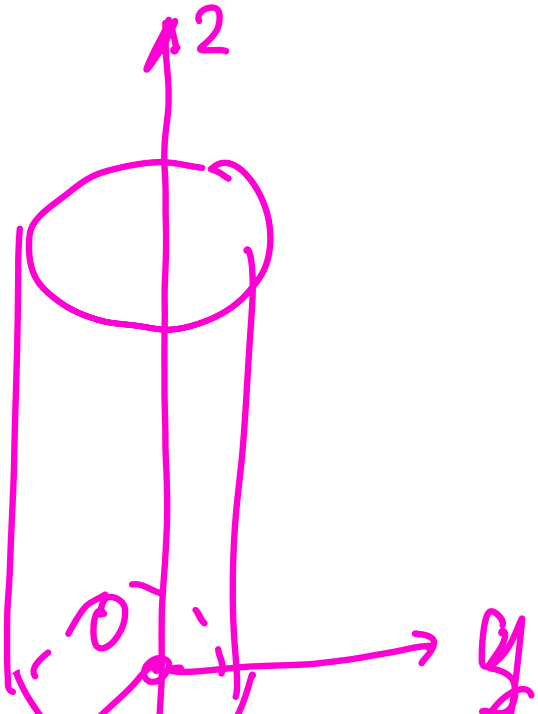
$D = E = F = 0$

$$J(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}_R$$

$R \rightarrow R_{solide}$

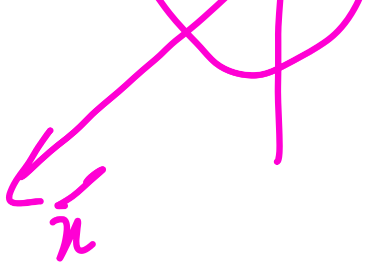
$$\begin{bmatrix} A & & \\ & B & \\ & & C \end{bmatrix}$$

## Axe de Revolution $(O, z)$




$(Oxz)$ et $(Oyz)$ sont PS

$$J(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{bmatrix}$$




$$\int x^2 dm = \int y^2 dm$$

$A = \int (y^2+z^2) dm$

$B = \int (x^2+z^2) dm \rightarrow A = B$

Cylindre d'axe $(O,\vec{y})$

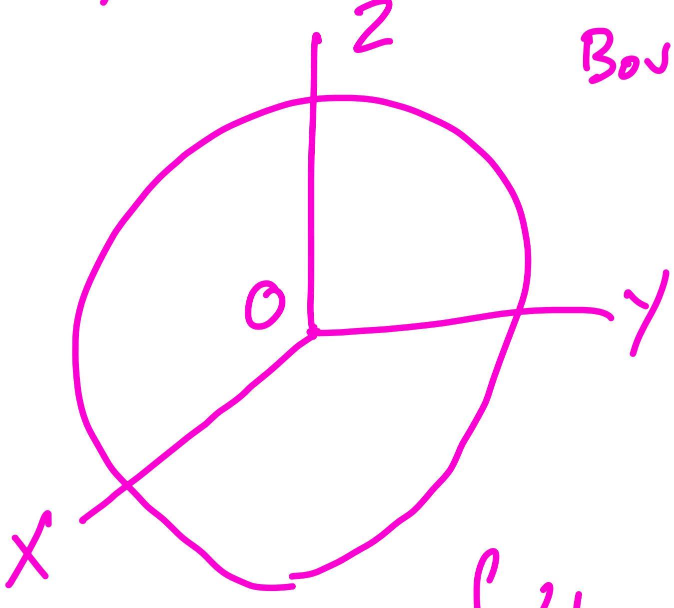
$$I(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{bmatrix}_{R_S}$$

Cylindre d'axe $(O,\vec{z})$

$$I(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}_{R_S}$$

Symétrie autour d'un point

Boule



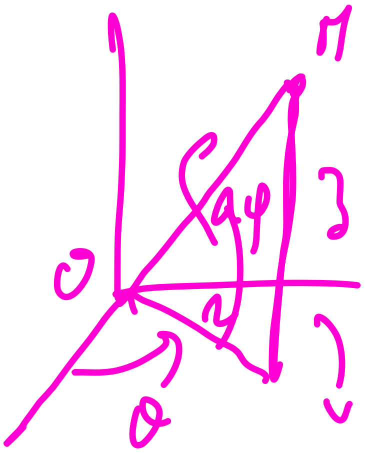

au moins 2 PS $\rightarrow D = E = F = 0$

$$I(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}_{R_S}$$

$$\int x^2 dm = \int y^2 dm = \int z^2 dm.$$

$$\mathrm{Trace}\left(J(O,S)\right) = A+B+C = 2\iiint (x^2+y^2+z^2)\,dm$$

$$A+B+C = 2\iiint \rho^2\,dm$$




BOULE $\Rightarrow 3A = 2\iiint \rho^2\,dm$

$$\Rightarrow A = \frac{2}{3}\iiint \rho^2\,dm$$

$x = r\cos\theta = \rho\cos\varphi\cos\theta$

$y = r\sin\theta = \rho\cos\varphi\sin\theta$

$z = \rho\sin\varphi$

$r = \rho\cos\varphi$

$$dV = \rho^2\,d\rho\cos\varphi\,d\varphi\,d\theta$$

$$A = \frac{2}{3}\iiint \rho^2\,\eta\,dV = \frac{2}{3}\eta\iiint \rho^4\,d\rho\cos\varphi\,d\varphi\,d\theta$$

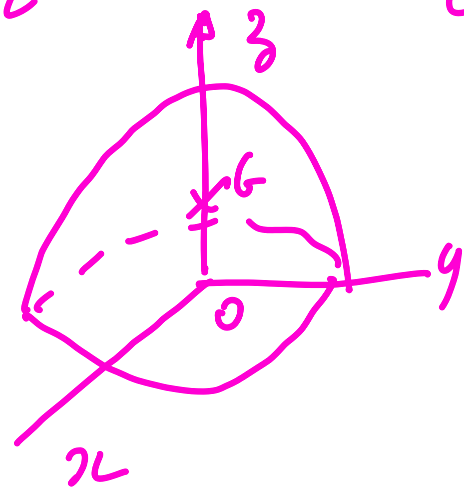
$$A = \frac{2}{3}\eta\left[\frac{\rho^5}{5}\right]_0^R\left[\sin\varphi\right]_{-\pi/2}^{\pi/2}\left[\theta\right]_0^{2\pi}$$

$$A = \frac{2}{3}\eta\,\frac{R^5}{5}\,2\,2\pi \qquad \eta = \frac{m}{V}$$

$$A = \frac{2}{\cancel{3}}\,\frac{m\,\cancel{3}}{\cancel{4\pi R^3}}\,\frac{\cancel{R^5}^{\,2}}{5}\,\cancel{2\,2\pi} = \frac{2}{5}mR^2$$

kg m²

$\frac{1}{2}$ boule $z \geq 0$




$G(0,0,z_G)$ $z_G = \frac{3}{8}R$

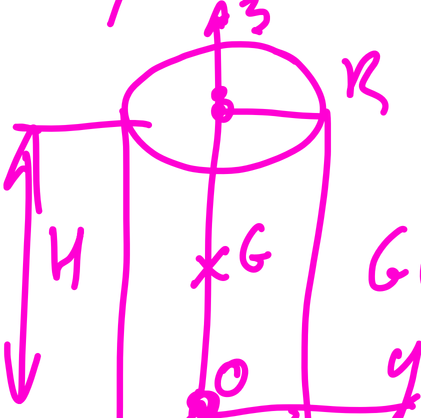
$$J(O,\text{boule}) = \begin{bmatrix} A & & \\ & A & \\ & & C \end{bmatrix}$$

$$C = \frac{2}{3}\frac{m}{V}\left[\frac{\rho^5}{5}\right]_0^R \left[\sin\varphi\right]_{\cancel{-\pi/2}\ 0}^{\pi/2} \left[\theta\right]_0^{2\pi}$$

$$C = \frac{2}{3}\frac{m\,3}{2\pi R^3}\left(\frac{R^5}{2}\right)\left[1\right]\left[2\pi\right]$$

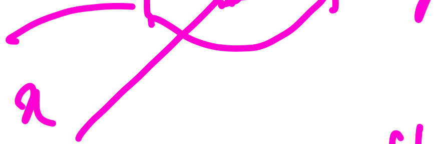
$$C = \frac{2}{5}mR^2$$

Cylindre $M, R$

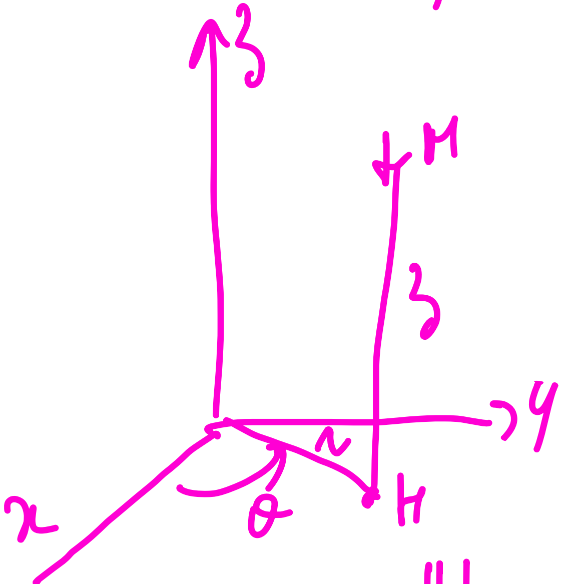



$G(0,0,H/2)$

$$J(O,C_y) = \begin{bmatrix} A_O & & \\ & A_O & \\ & & C_O \end{bmatrix}$$




$$C = \iiint (x^2+y^2)\,dm = \iiint r^2\,dm = \rho\iiint r^2\,dV$$




$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases} \qquad x^2+y^2 = r^2$$

$$dV = r\,dr\,d\theta\,dz$$

$$C = \rho\iiint r^3\,dr\,d\theta\,dz = \frac{m}{V}\left[\frac{r^4}{4}\right]_0^R \left[\theta\right]_0^{2\pi} \left[z\right]_0^H$$

$$C = \frac{m}{\cancel{\pi} \cancel{R^2} \cancel{H}} \frac{\cancel{R^4}^{\,2}}{4} 2\cancel{\pi}\cancel{H} = \frac{mR^2}{2}$$

$$A = \iiint (\underbrace{y^2}+\underbrace{z^2})\,dm = \iiint (\underbrace{x^2}+\underbrace{z^2})\,dm$$

$$\text{Trace} = 2A+C = 2\iiint (x^2+y^2+z^2)\,dm.$$

$$2A+C = \underbrace{2\iiint (x^2+y^2)\,dm} + 2\iiint z^2\,dm.$$

$$2A+C = 2C + 2\iiint z^2 dm$$

$$2A = C + 2\iiint z^2 dm \Rightarrow A = \frac{C}{2} + \underbrace{\iiint z^2 dm}$$

$$\iiint z^2 dm = \iiint z^2 \rho\, dV = \rho \iiint r\, dr\, d\theta\, z^2 dz$$

$$= \frac{m}{V} \left[\frac{r^2}{2}\right]_0^R \left[\theta\right]_0^{2\pi} \left[\frac{z^3}{3}\right]_0^H$$

$$= \frac{m}{\cancel{\pi} \cancel{R^2} \cancel{H}} \frac{\cancel{R^2}}{\cancel{2}} \cancel{2\pi} \frac{H^{\cancel{3}\,2}}{3} = \frac{mH^2}{3}$$

$$A_O = \frac{mR^2}{4} + \frac{mH^2}{3}$$

en G

$$\iiint z^2 dm = \frac{m}{\pi R^2 H} \frac{R^2}{2} 2\pi \left[\frac{z^3}{3}\right]_{-H/2}^{H/2}$$

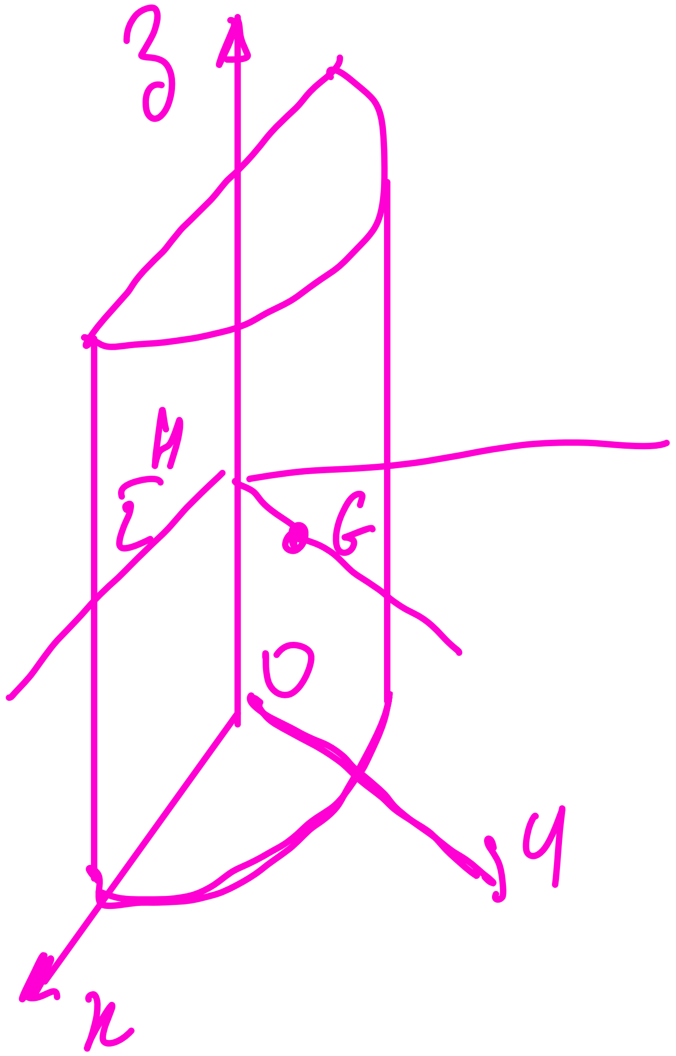
$$= \frac{m}{\cancel{\pi} \cancel{R^2} \cancel{H}} \frac{\cancel{R^2}}{\cancel{2}} \cancel{2\pi} \frac{H^{\cancel{3}}}{12} = \frac{mH^2}{12}$$

$$A_G = \frac{mR^2}{4} + \frac{mH^2}{12}$$

Minimum en G

→ Matrice "minimale" en G

$\frac{1}{2}$ Cylindre $y>0$




$G\left(0, y_G, \frac{H}{2}\right)$ $\quad y_G = \frac{4R}{3\pi}$

$$J\left(O, \tfrac{1}{2}Cyl\right) = \begin{bmatrix} A_O & 0 & 0 \\ 0 & B_O & -D_O \\ 0 & -D_O & C_O \end{bmatrix}_{R_S}$$

1PS $yOz$ $\Rightarrow$ $F = E = 0$

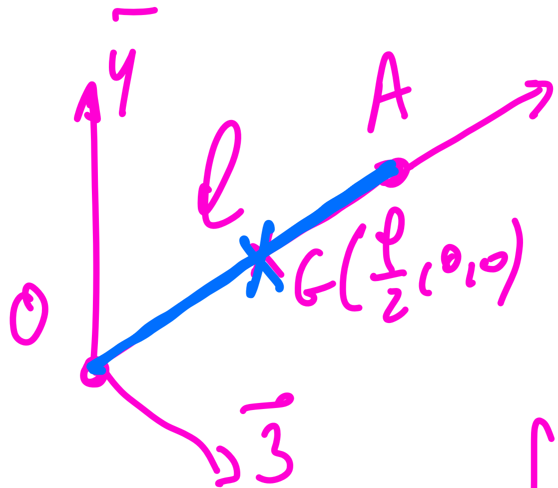
$$J\left(G, \tfrac{1}{2}Cyl\right) = \begin{bmatrix} A_G & & 0 \\ & B_G & \\ 0 & & C_G \end{bmatrix}_{R_S}$$

$$J\left(O, \tfrac{1}{2}Cyl\right) = J\left(G, \tfrac{1}{2}Cyl\right) + J\left(O, G(m)\right)$$

$x_G = 0$
$y_G$
$z_G$

$$J(O, G(m)) = \begin{bmatrix} m(y_G^2+z_G^2) & 0 & 0 \\ 0 & m z_G^2 & -m y_G z_G \\ 0 & -m y_G z_G & m y_G^2 \end{bmatrix}$$

Barre de longueur $\ell$ $\rightarrow O, \vec{x}$




EPS $\Rightarrow D = F = 0$

$(y = z = 0) \Rightarrow A = 0$

$B = C$

$$\mathcal{I}(O, \text{barre}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & B_0 \end{bmatrix}$$

$$B_0 = C_0 = \int x^2 \, dm = \lambda \int_0^{\ell} x^2 \, dx = \lambda \left[\frac{x^3}{3}\right]_0^{\ell}$$

$dm = \lambda d\ell$

$$= \lambda \frac{\ell^3}{3} = \frac{m}{\ell}\frac{\ell^3}{3} = \frac{m\ell^2}{3}$$

$$B_G = C_G = \frac{m}{\ell}\left[\frac{x^3}{3}\right]_{-\ell/2}^{\ell/2} = \frac{m\ell^2}{12}$$

Huygens $\quad O \leftrightarrow G \quad O \rightarrow A$

$$\left(\mathcal{J}(O,S)\,\vec{u}\right)\vec{u}$$

$$\left(\int \vec{OP}\wedge(\vec{OP}\wedge\vec{u})\,dm\right)\cdot\vec{u}$$

$$-\int \vec{OP}\wedge(\vec{OP}\wedge\vec{u})\,dm \qquad \vec{OP}=\vec{OG}+\vec{GP}$$

$$-\int (\vec{OG}+\vec{GP})\wedge\left((\vec{OG}+\vec{GP})\wedge\vec{u}\right)$$

$$-\int \vec{OG}\wedge(\vec{OG}\wedge\vec{u})\,dm-\cancel{\int \vec{OG}\wedge(\vec{GP}\wedge\vec{u})\,dm}$$

$$-\underbrace{\cancel{\int \vec{GP}\wedge(\vec{OG}\wedge\vec{u})\,dm}}_{\vec{0}}-\int \vec{GP}\wedge(\vec{GP}\wedge\vec{u})\,dm$$

$$\int \vec{OG}\wedge(\vec{GP}\wedge\vec{u})\,dm=\vec{OG}\wedge\left(\underbrace{\left(\int \vec{GP}\,dm\right)}_{\vec{0}}\wedge\vec{u}\right)$$

$$\int \vec{GP}\wedge(\vec{OG}\wedge\vec{u})\,dm=\left(\underbrace{\cancel{\int \vec{GP}\,dm}}_{\vec{0}}\right)\wedge(\vec{OG}\wedge\vec{u})$$

$$-\int \vec{OP} \wedge (\vec{OP} \wedge \vec{i})\,dm = -\int \vec{OG} \wedge (\vec{OG} \wedge \vec{i})\,dm$$

$$\underbrace{-\int \vec{GP} \wedge (\vec{GP} \wedge \vec{i})\,dm}_{J(G,S)\vec{i}}$$

$$J(O,S)\vec{i} = J(G,S)\vec{i} - \left(\int dm\right) \vec{OG} \wedge (\vec{OG} \wedge \vec{i})$$

$$- m\, \vec{OG} \wedge (\vec{OG} \wedge \vec{i})$$

$$J(O,S)\vec{i} = J(G,S)\vec{i} + \underbrace{J(O,G(m))}\,\vec{i}$$

$$\vec{GO} \begin{vmatrix} -x_G \\ -y_G \\ -z_G \end{vmatrix} \quad \vec{OG} \begin{vmatrix} x_G \\ y_G \\ z_G \end{vmatrix}$$

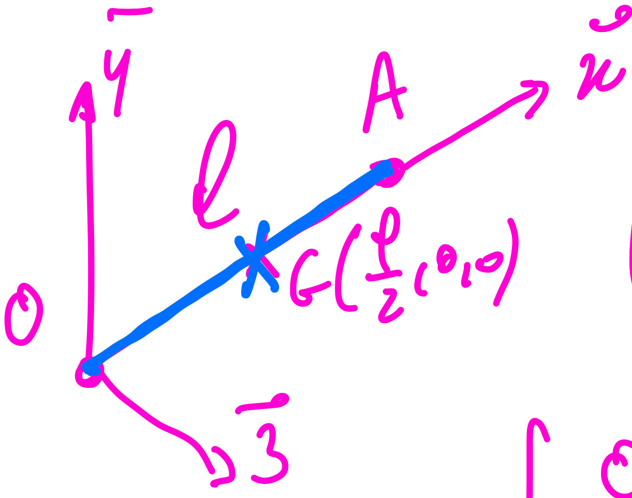
$$J(O,G(m)) = \begin{bmatrix} m(y_G^2+z_G^2) & -m x_G y_G & -m x_G z_G \\ -m x_G y_G & m(x_G^2+z_G^2) & -m y_G z_G \\ -m x_G z_G & -m y_G z_G & m(x_G^2+y_G^2) \end{bmatrix}$$

$$J(O,S) = J(G,S) + J(O,G(m))$$

HUYGENS

$$J(G,S) = J(O,S) - J(O,G(m))$$

Barre de longueur $\ell$ → $(O, \vec{x})$




EPS ⇒ $D = F = 0$

$(y = z = 0)$ ⇒ $A = 0$
$B = C$

$$\mathcal{J}(O, \text{barre}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0 & 0 \\ 0 & 0 & B_0 \end{bmatrix}$$

$$B_0 = C_0 = \int x^2 \underset{d\,d\ell}{dm} = d\int_0^\ell x^2 dx = d\left[\frac{x^3}{3}\right]_0^\ell$$

$$= d\,\frac{\ell^3}{3} = \frac{m}{\ell}\,\frac{\ell^3}{3} = \frac{m\ell^2}{3}$$

$\ell/2$

$$J(O, \text{barre}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0 = m\ell^2/3 & 0 \\ 0 & 0 & B_0 = m\ell^2/3 \end{bmatrix}$$

$$\mathcal{J}(G,\text{barre}) = \mathcal{J}(O,\text{barre}) - \mathcal{J}(O, G|m)$$

$$\vec{OG} \begin{vmatrix} \ell/2 \\ 0 \\ 0 \end{vmatrix}$$

$$\mathcal{J}(O, G|m) = \begin{bmatrix} 0 & & \\ & \frac{m\ell^2}{4} & \\ & & \frac{m\ell^2}{4} \end{bmatrix}$$

$$B_O = \frac{m\ell^2}{3}$$

$$B_G = B_O - \frac{m\ell^2}{4} = \frac{m\ell^2}{3} - \frac{m\ell^2}{4}$$

$$= \frac{m\ell^2}{12}$$