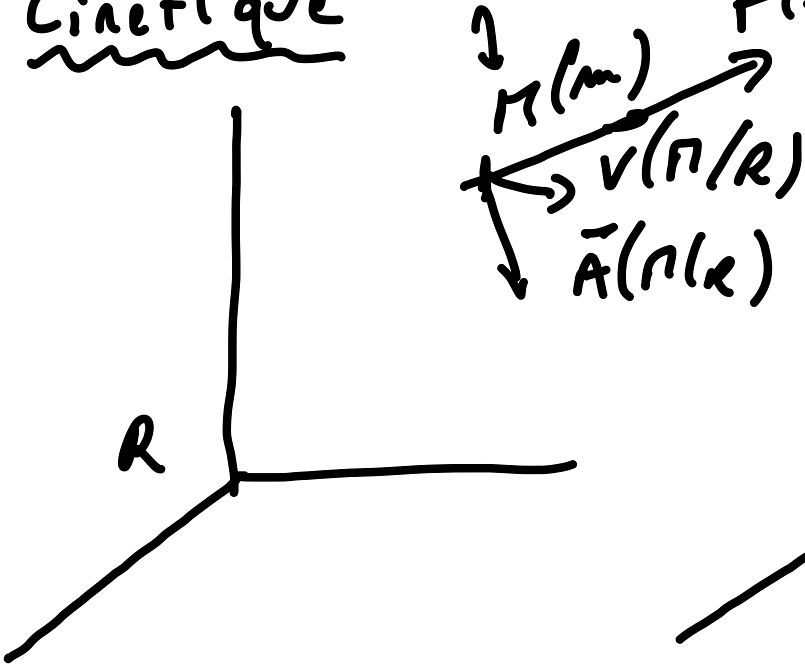
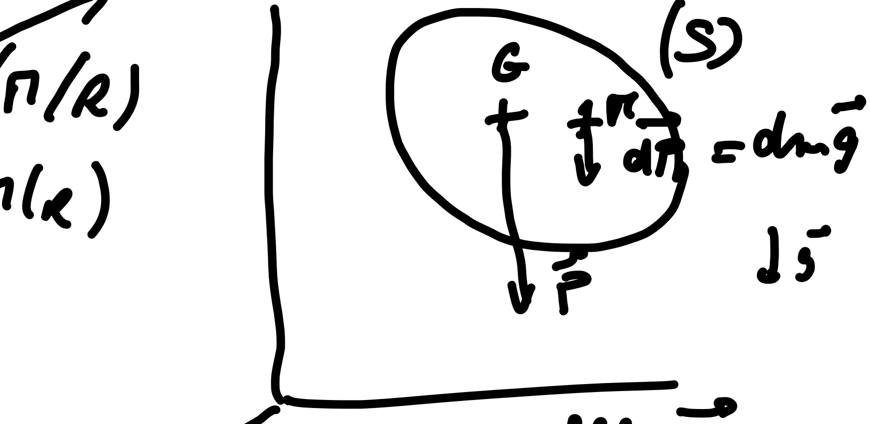
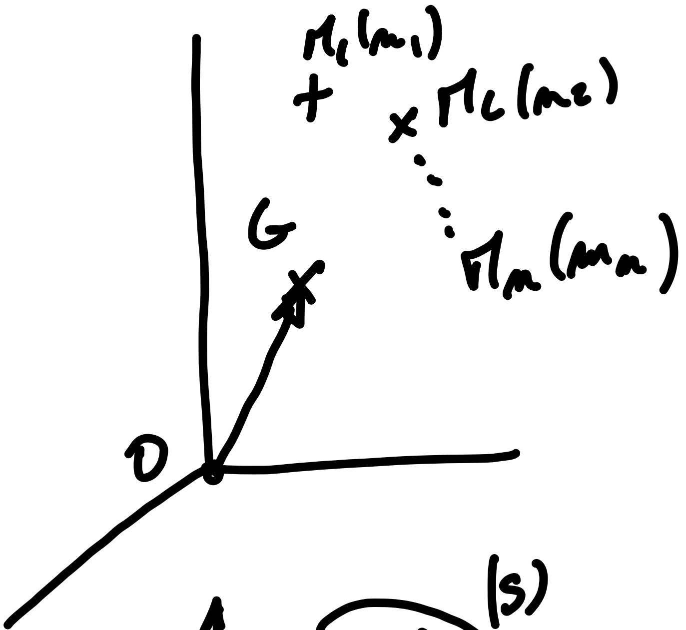
Cinétique





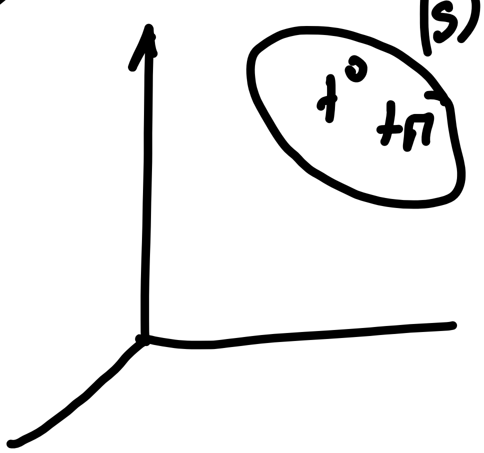
$$\vec P = \iiint d\vec P$$

Centre de Gravité d'un solide



$$\rightarrow \sum_{i=1}^{n} m_i \overrightarrow{GM_i} = \vec 0$$

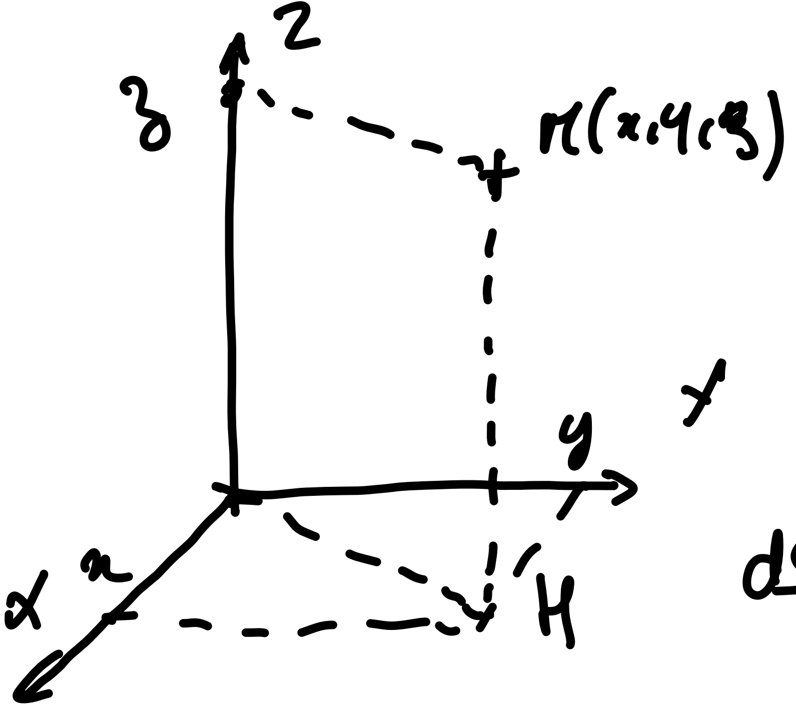
$$\underbrace{\left(\sum_{i=1}^{n} m_i\right)}_{m} \overrightarrow{OG} = \sum_{i=1}^{n} m_i \overrightarrow{OM_i}$$



$$\iiint_{M \in S} \overrightarrow{GM}\,dm = \vec 0$$

$$\underbrace{\left(\iiint dm\right)}_{m} \overrightarrow{OG} = \iiint \overrightarrow{OM}\,dm$$

$$m\,\overrightarrow{OG} = \iiint \overrightarrow{OM}\,dm.$$



$$dV = dx\,dy\,dz$$

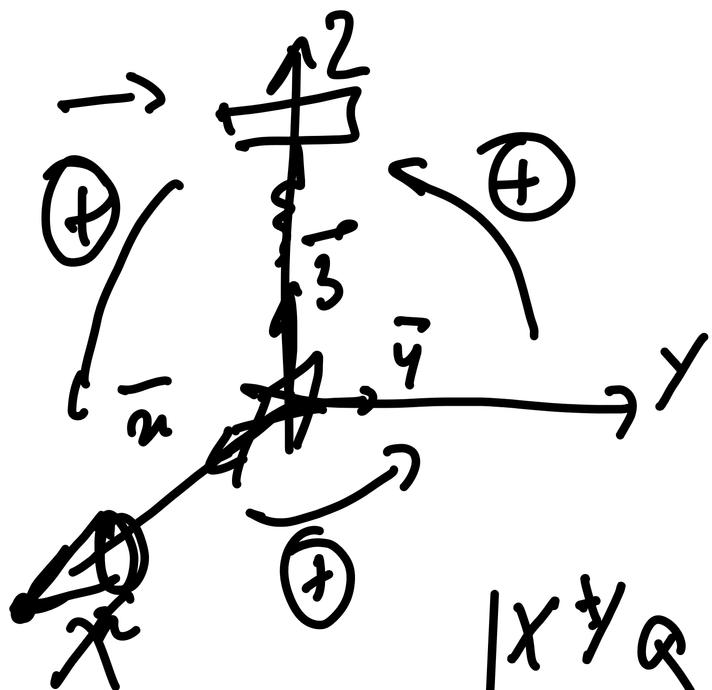
dl $\quad dx \,/\, dy \,/\, dz$

dS $\quad dx\,dy \,/\, dx\,dz \,/\, dy\,dz$

$(O, \vec{x}, \vec{y}, \vec{z})$

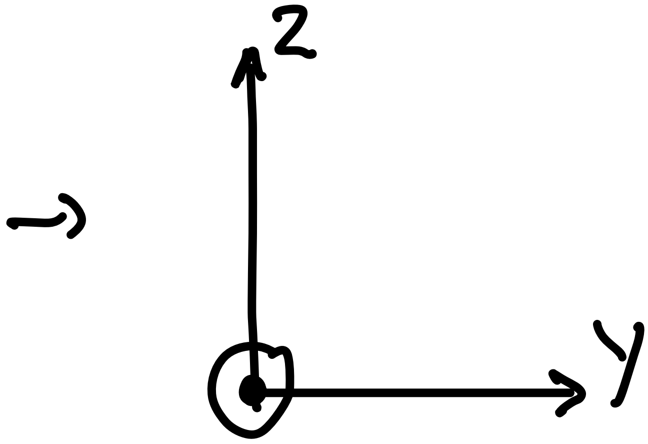
$$\left.\begin{aligned} \vec{x}\cdot\vec{y} &= 0 \\ \vec{y}\cdot\vec{z} &= 0 \\ \vec{z}\cdot\vec{x} &= 0 \end{aligned}\right\} \text{Ortho}$$

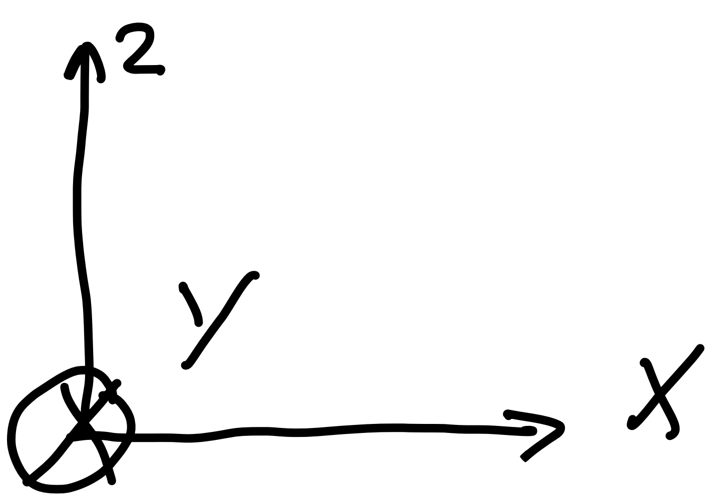
$$\|\vec{x}\| = \|\vec{y}\| = \|\vec{z}\| = 1$$



XY
YZ
ZX

$$\left\{\begin{aligned} \vec{x}\wedge\vec{y} &= \vec{z} \\ \vec{y}\wedge\vec{z} &= \vec{x} \\ \vec{z}\wedge\vec{x} &= \vec{y} \end{aligned}\right.$$





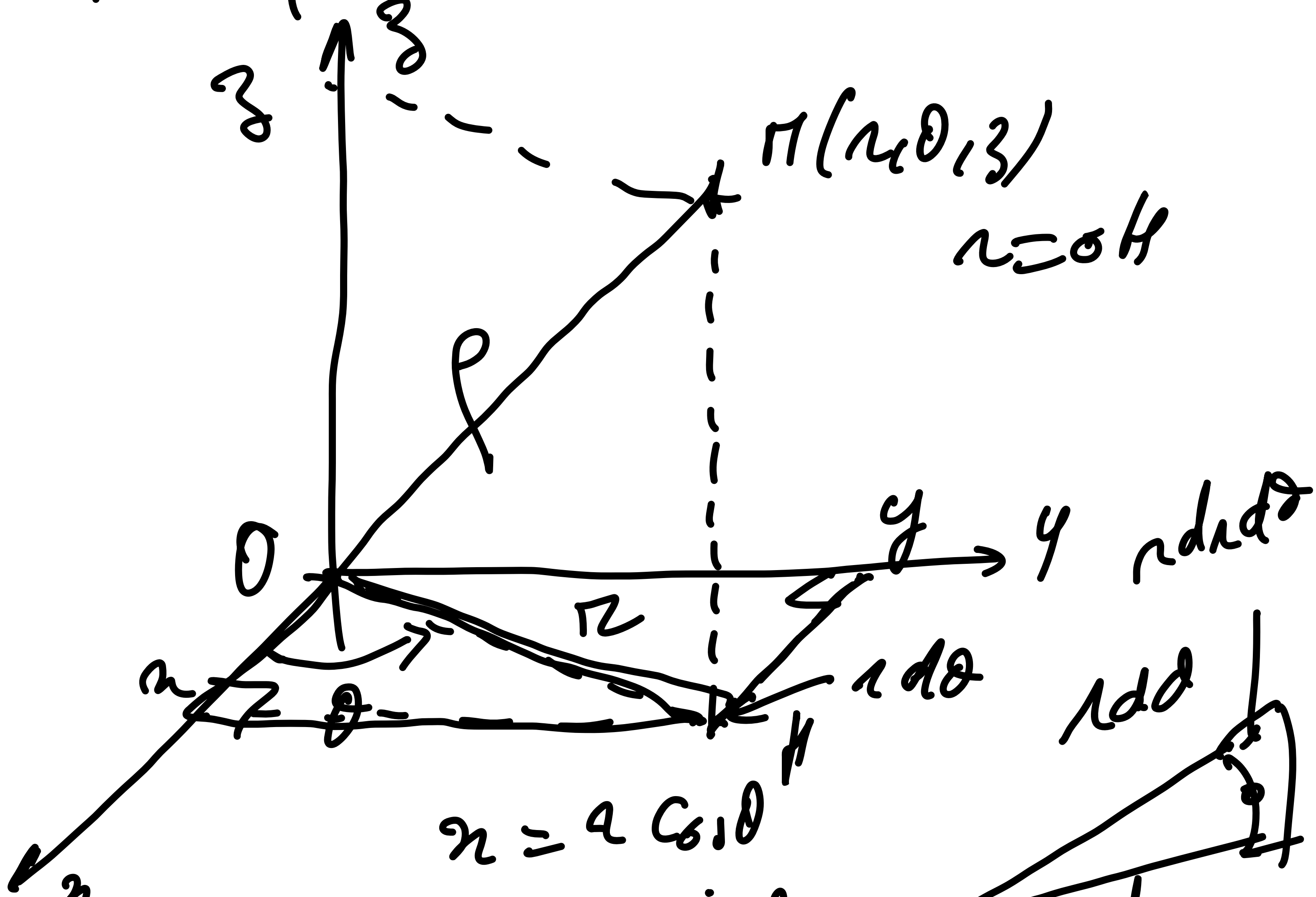
$0 \leq x \leq a$
$0 \leq y \leq b$
$0 \leq z \leq c$

$$V = \iiint dV$$

$$V = \iiint dx\,dy\,dz$$

$$V = \int_0^a dx \int_0^b dy \int_0^c dz = abc$$

Cylindrique

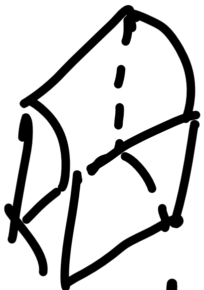


$x = r \cos\theta$
$y = r \sin\theta$

$$z = z$$

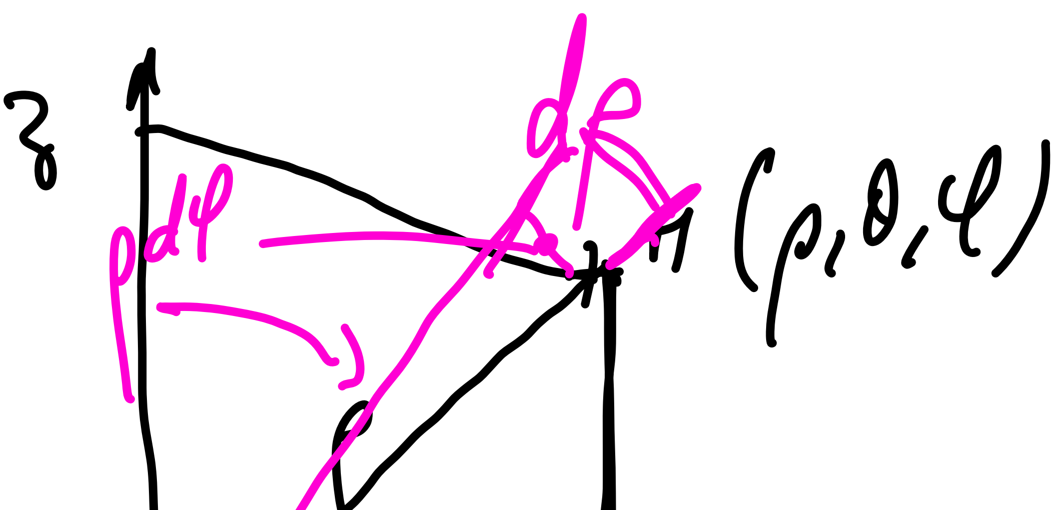
$$dl \quad \bigg/ dr \quad \bigg/ r\,d\theta \quad \bigg/ dz$$

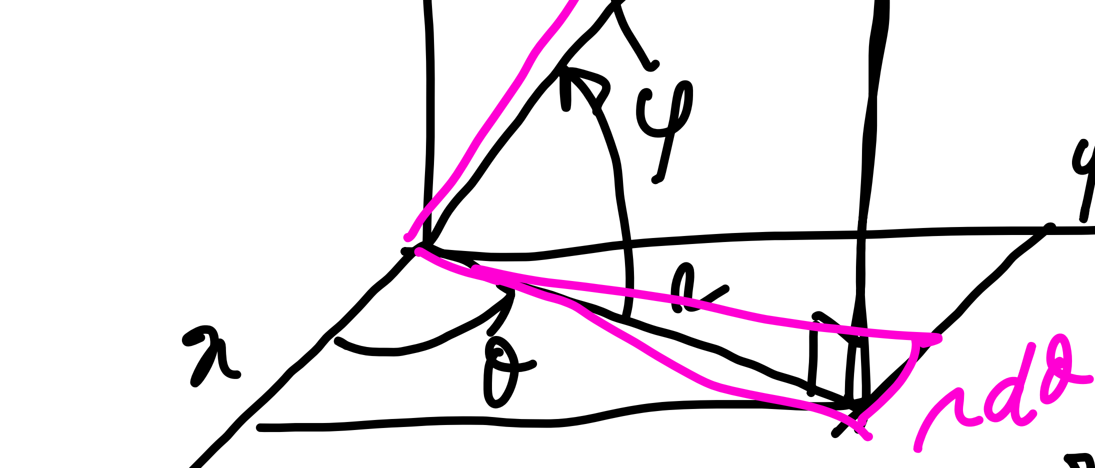
$$dV = r\,dr\,d\theta\,dz$$



$$V = \iiint dV = \int_0^R r\,dr \int_0^{2\pi} d\theta \int_0^H dz$$

$$= \frac{R^2}{\cancel{2}}\,\cancel{2}\pi\,H$$





$r = \rho \cos \varphi$

$x = r \cos \theta$

$y = r \sin \theta$

$$\begin{cases} x = \rho \cos\varphi \cos\theta \\ y = \rho \cos\varphi \sin\theta \\ z = \rho \sin\varphi \end{cases}$$

$d\ell \quad |d\rho| \quad \rho\, d\varphi \,|\, \overset{r\,d\theta}{\rho \cos\varphi\, d\theta}$

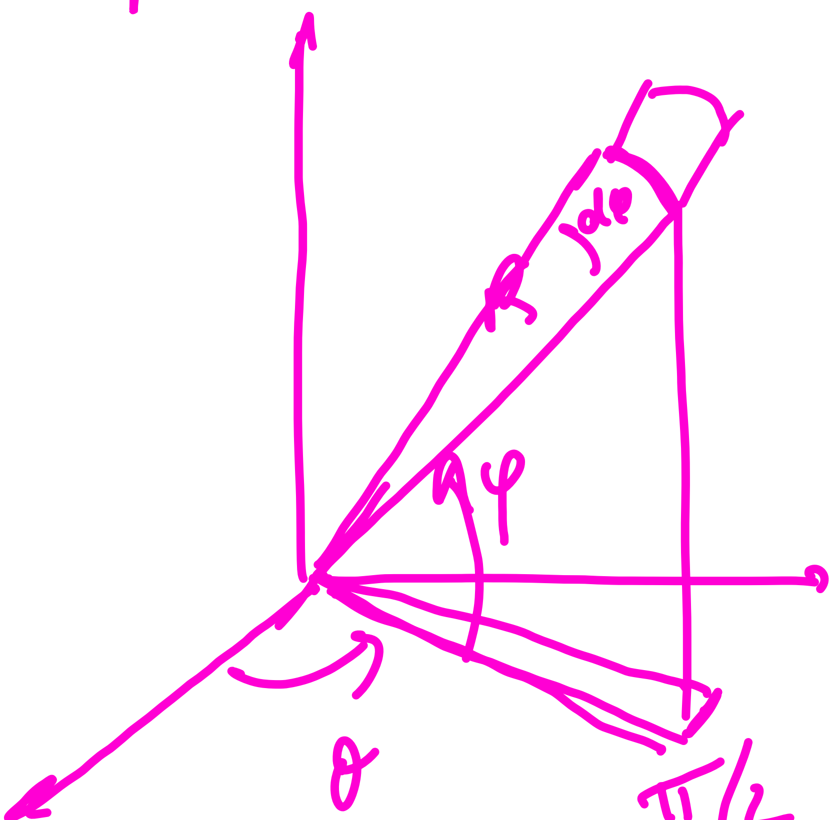
$$dV = \rho^2 d\rho \cos\varphi\, d\varphi\, d\theta$$

$$V = \iiint dV = \int_0^R \rho^2 d\rho \int_{-\pi/2}^{\pi/2} \cos\varphi\, d\varphi \int_0^{2\pi} d\theta$$

$$= \left[\frac{r^3}{3}\right] \left[\sin\varphi\right]_{-\pi/2}^{\pi/2} \left[\theta\right]_0^{2\pi}$$

$$= \frac{r^3}{3} \quad 2 \quad 2\pi = \frac{4}{3}\pi R^3$$

Sphère de Rayon R



$$dS = R\,d\varphi\, R\cos\varphi\, d\theta$$
$$= R^2 \cos\varphi\, d\varphi\, d\theta$$

$$S = \iint dS$$

$$S = R^2 \int_{-\pi/2}^{\pi/2} \cos\varphi\, d\varphi \int_0^{2\pi} d\theta$$

$$= R^2 \quad 2 \quad 2\pi = 4\pi R^2$$

Cylindre de hauteur H et de Rayon R

$$m\vec{OG} = \iiint \vec{OP}\, dm$$

$$\begin{cases} m x_G = \iiint x\, dm \\ m y_G = \iiint y\, dm. \\ m z_G = \iiint z\, dm \end{cases}$$

$$\left.\begin{array}{l} (xOz)\ PS \to y_G = 0 \\ (yOz)\ PS \to x_G = 0 \end{array}\right\} G \in (Oz)$$

G à $\frac{H}{2}$ $\quad (Gxz)$ PS $\to z_G = H/2$

$\frac{1}{2}$ cylindre $y > 0$

$$\begin{cases} (yOz)\ PS \to x_G = 0 \\ (Gxy)\ PS \to z_G = H/2 \end{cases}$$

$$y_G \neq 0$$

x

$$m y_G = \iiint y \, dm$$

$$\left(\begin{array}{ll} \iiint & dm = \eta \, dV \\ \iint & dm = \sigma \, dS \\ \int & dm = \lambda \, dl \end{array} \right) \qquad \eta = \frac{m}{V}$$

$$m y_G = \iiint y \, \eta \, dV$$

$$y_G = \frac{\eta}{m} \iiint y \, dV$$

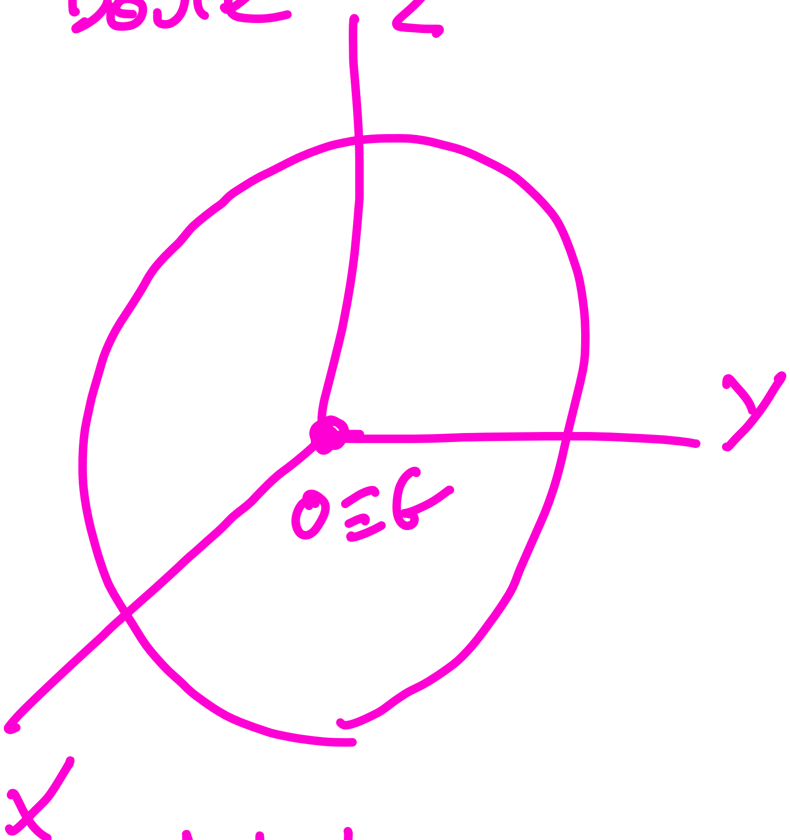
$$y_G = \frac{1}{V} \iiint r \sin\vartheta \, r \, dr \, d\vartheta \, dz$$

$$y_G = \frac{1}{V} \int_0^R r^2 dr \int_0^{\pi} \sin\vartheta \, d\vartheta \int_0^H dz$$

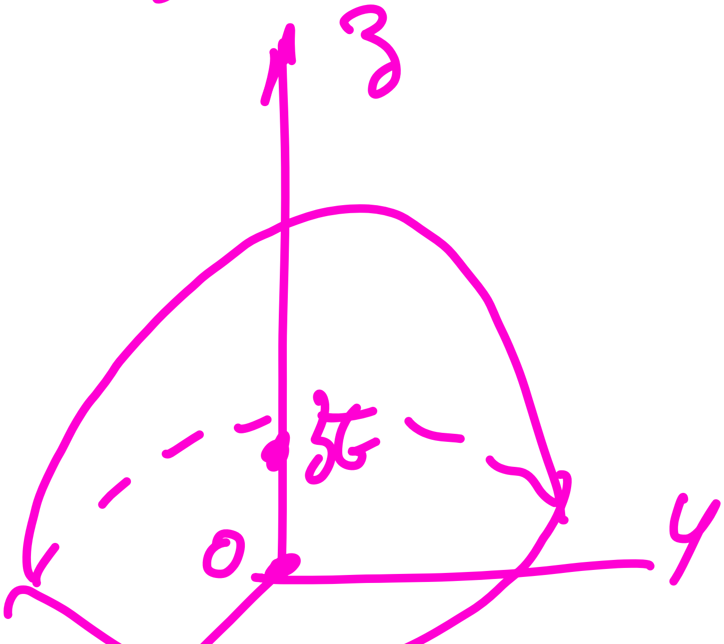
$$y_G = \frac{1}{V} \frac{R^3}{3} 2 H = \frac{2}{\pi R^2 \cancel{H}} \frac{2R^3 \cancel{H}}{3}$$

$$= \frac{4R}{3\pi}$$

Boule



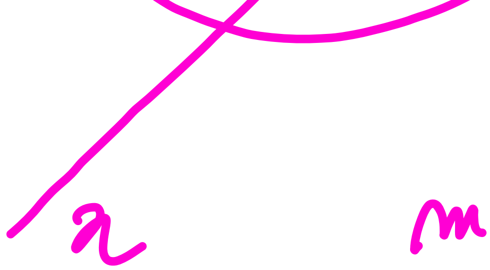
$\frac{1}{2}$ boule $z > 0$



(zOx) PS $\rightarrow y_G = 0$

(zOy) PS $\rightarrow x_G = 0$

$\rightarrow G \in (O, z)$



$$m z_G = \iiint z\, dm = \iiint z \cancel{\rho}\, dV$$

$$z_G = \frac{\eta}{m} \iiint z\, \rho^2 d\rho \cos\varphi\, d\varphi\, d\theta$$

$$z = \rho \sin\varphi$$

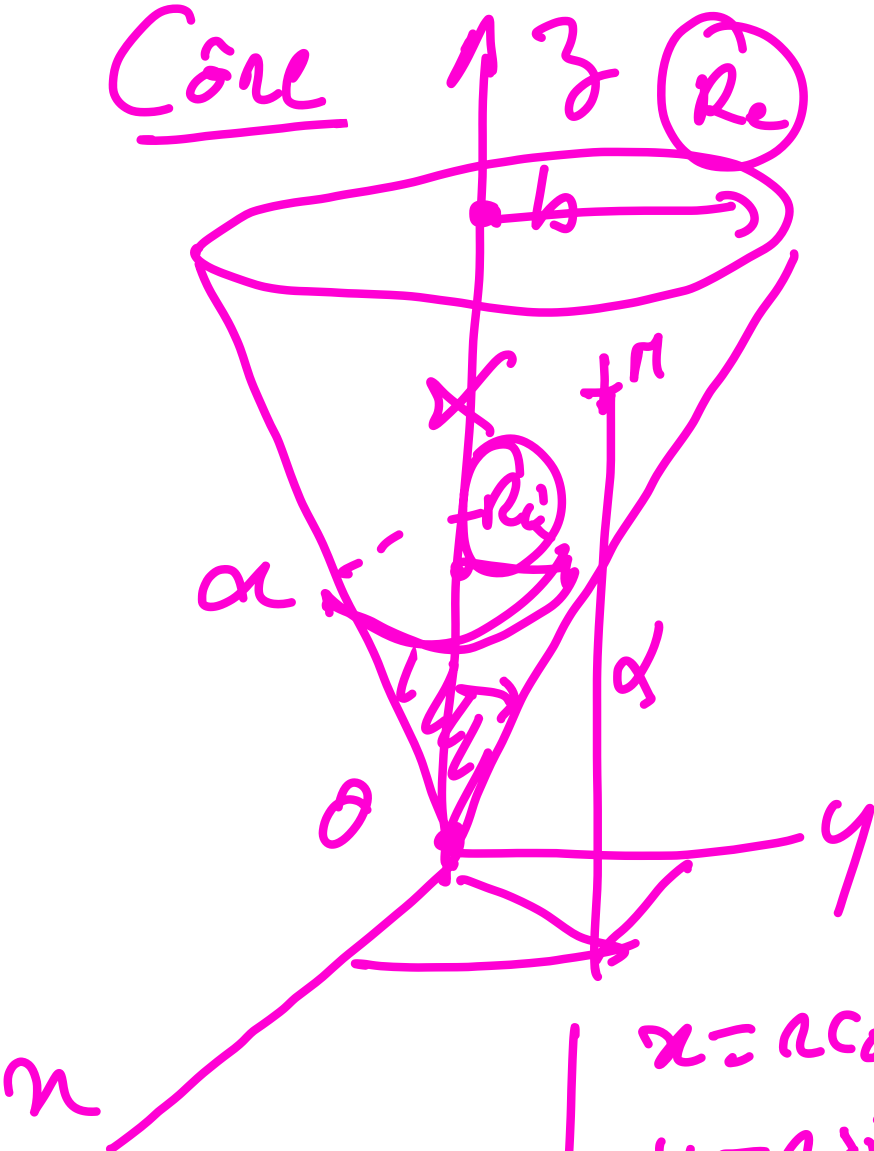
$$z_G = \frac{1}{V} \int_0^R \rho^3 d\rho \int_0^{\pi/2} \underbrace{\sin\varphi \cos\varphi}_{\frac{\sin 2\varphi}{2}}\, d\varphi \int_0^{2\pi} d\theta$$

$$= \frac{1}{V} \frac{R^4}{4} \left[-\frac{\cos 2\varphi}{4} \right]_0^{\pi/2} \left[2\pi \right]$$

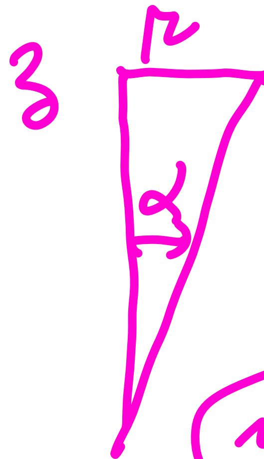
$$= \frac{3}{\cancel{2\pi} R^3} \frac{R^4}{4} \frac{1}{2} \cancel{2\pi}$$

$$= \frac{3}{8} R$$

Cône



$a \leq z \leq b$



$\tan\alpha = \frac{r}{z}$

$r = z\tan\alpha$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$dV = r\,dr\,d\theta\,dz$

$= d\theta\; r\,dr\,dz$

$V = \iiint dV = \iiint r\,dr\,d\theta\,dz$

$V = \int_0^{2\pi} d\theta \int_a^b \left(\int_0^{z\tan\alpha} r\,dr \right) dz$

$$= 2\pi \int_a^b \frac{z^2 \tan^2\alpha}{2} \, dz$$

$$= \pi \tan^2\alpha \left[\frac{z^3}{3} \right]_a^b$$

$$= \frac{\pi \tan^2\alpha (b^3 - a^3)}{3}$$

$G(0, 0, z_G)$

$$m z_G = \iiint z \, dm = \iiint z \eta \, dV$$

$$z_G = \frac{\eta}{m} \iiint z \, dV$$

$$z_G = \frac{1}{V} 2\pi \int_a^b \frac{z^2 \tan^2\alpha}{2} z \, dz$$

$$= \frac{1}{V}\pi\tan^2\alpha\left[\frac{z^4}{4}\right]_a^b$$

$$= \frac{1}{V}\pi\tan^2\alpha\,\frac{b^4-a^4}{4}$$

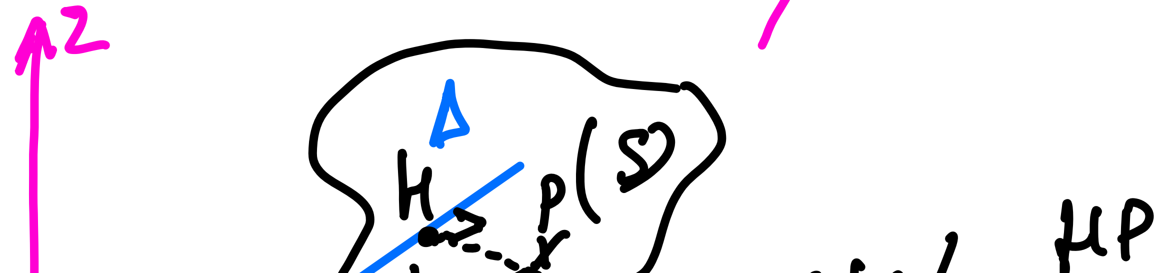
$$= \frac{3}{\cancel{\pi\tan^2\alpha}\,(b^3-a^3)}\,\cancel{\pi\tan^2\alpha}\,\frac{b^4-a^4}{4}$$

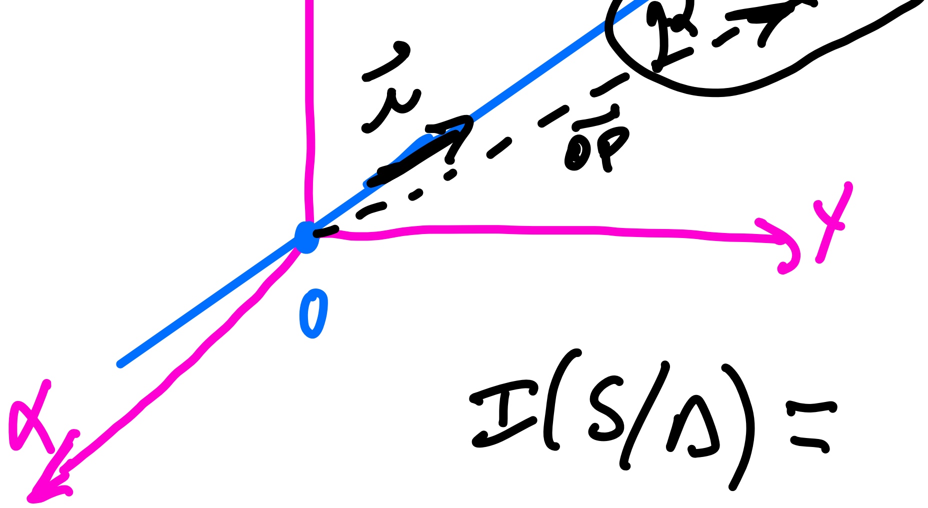
$$= \frac{3}{4}\,\frac{b^4-a^4}{b^3-a^3}$$

si $a=0$ $\qquad z_G = \frac{3}{4}b$

II] Opérateur d'inertie d'un solide

Moment d'inertie : $I(S/\Delta)$





$\sin\alpha = \frac{\quad}{OP}$

$$I(S/\Delta) = \int_{P\in S} HP^2\,dm$$

$$\|\vec u \wedge \vec{OP}\| = \overset{1}{\cancel{\|\vec u\|}}\,\|\vec{OP}\|\,\left|\sin(\vec u, \vec{OP})\right|$$

$$= OP.\sin\alpha = HP$$

$$HP^2 = (\vec u \wedge \vec{OP})\cdot(\vec u \wedge \vec{OP})$$

$$= \left((\vec u \wedge \vec{OP})\wedge \vec u\right)\cdot \vec{OP}$$

$$= \left(\vec{OP}\wedge(\vec u \wedge \vec{OP})\right)\cdot \vec u$$

$$= -\left(\vec{OP}\wedge(\vec{OP}\wedge \vec u)\right)\cdot \vec u$$

$$\underset{(b)}{\vec{OP}}\left|\begin{matrix} x \\ y \\ z \end{matrix}\right. \qquad \underset{(b)}{\vec u}\left|\begin{matrix} u_1 \\ u_2 \\ u_3 \end{matrix}\right. \qquad \vec{OP}\wedge\vec u\left|\begin{matrix} y u_3 - z u_2 \\ z u_1 - x u_3 \\ x u_2 - y u_1 \end{matrix}\right.$$

$$\begin{bmatrix} 0 & -z & y \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}$$

$$\begin{bmatrix} & & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}\begin{bmatrix} \\ i_2 \\ i_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}\begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix}$$

$$= - \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & y^2+x^2 \end{bmatrix}$$

$$I(S/\Delta) = \int_{P \in S} HP^2 \, dm$$

$$= - \int \left(\vec{OP} \wedge \left(\vec{OP} \wedge \vec{i} \right) \right) \cdot \vec{i} \, dm \, .$$

$$= \left(-\int \vec{OP} \wedge (\vec{OP} \wedge \vec{i})\, dm\right) \cdot \vec{i}$$

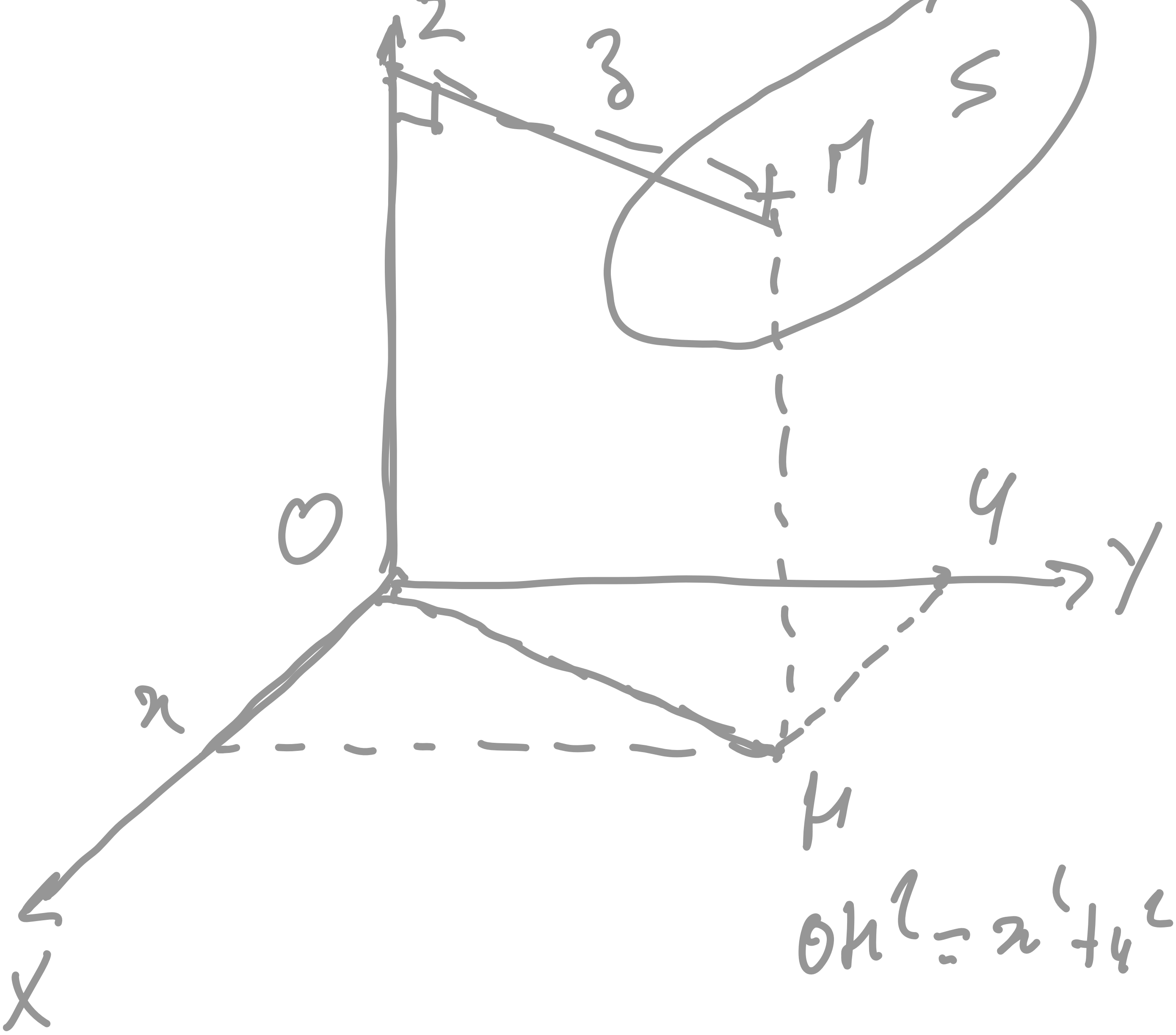
$$I(S/\Delta) = \left(\mathcal{J}(O,S)\, \vec{i}\right) \cdot \vec{i}$$

Operateur d'inertie

$$\mathcal{J}(O,S) = \begin{bmatrix} \int (y^2+z^2)\, dm & -\int xy\, dm & -\int xz\, dm \\ -\int xy\, dm & \int (x^2+z^2)\, dm & -\int yz\, dm \\ -\int xz\, dm & -\int yz\, dm & \int (x^2+y^2)\, dm \end{bmatrix}$$

$$\mathcal{J}(O,S) = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}_{R_S}$$

A, B, C



$OH^2 = x^2 + y^2$

$$I(S/(O,\vec{x})) = \left[\bar{J}(O,S)\,\vec{x}\right]\cdot\vec{x}$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$$= \begin{bmatrix} A \\ -F \\ -E \end{bmatrix}\cdot\begin{bmatrix}1\\0\\0\end{bmatrix} = A$$

$$I(S/(O,\vec{y})) = \left(J(O,S)\,\vec{y}\right)\cdot\vec{y}$$

$$= \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -F \\ B \\ -D \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = B$$

$$I(S/(O,\vec{z})) = \left(J(O,S)\,\vec{z}\right)\cdot\vec{z} = C$$

$$J\omega \qquad J_x \frac{d\omega_x}{dt} \qquad J_y \frac{d\omega_y}{dt}$$

$$J_z \frac{d\omega_z}{dt}$$

Simplification de $J(O,S)$ en f$^{\text{ncti}}$ des propriétés de symétrie

1 Plan de Symétrie

$$\int xz\,dm = \int_{z>0} xz\,dm + \int_{z<0} xz\,dm = 0$$

$$\int yz\,dm = 0$$

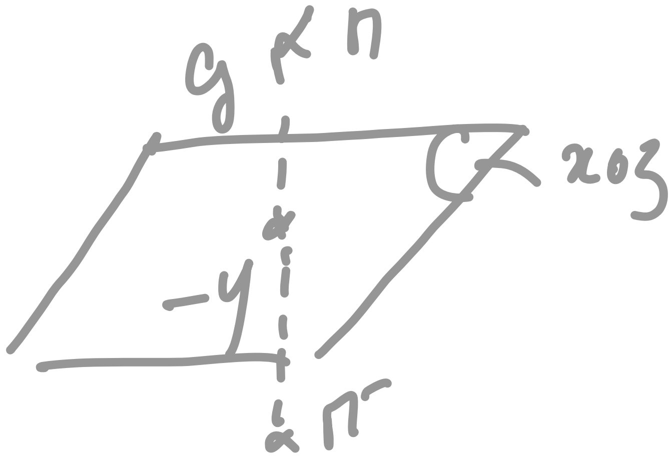
$$J(O,S) = \begin{bmatrix} A & -F & 0 \\ -F & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

PS(xoy) $\Rightarrow E = D = 0$

PS(yoz) $\Rightarrow E = F = 0$

$$J(O,S) = \begin{pmatrix} A & 0 & 0 \\ 0 & B & -D \\ 0 & -E & C \end{pmatrix}$$

PS (xOz) $\Rightarrow$ $F = D = 0$



$$\mathcal{I}(O,S) = \begin{pmatrix} A & 0 & -E \\ 0 & B & 0 \\ -E & 0 & C \end{pmatrix}$$

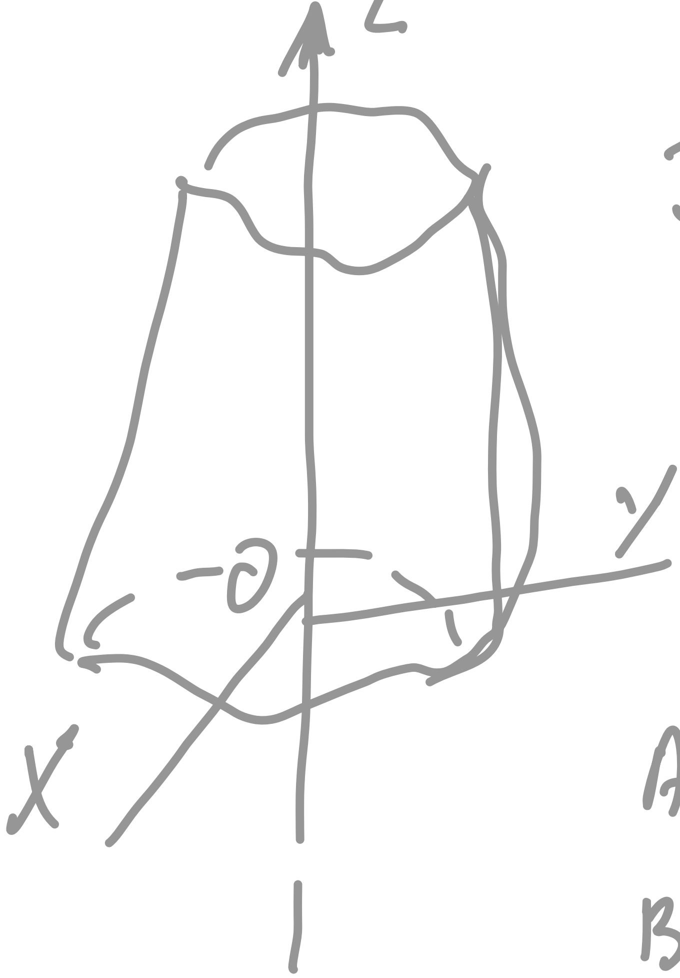
---

2 Plans de Symétrie $D = E = F = 0$

$$\mathcal{I}(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

$$= \begin{bmatrix} A & & 0 \\ & B & \\ 0 & & C \end{bmatrix}$$

<u>Axe de révolution</u> d'axe (O,z)



$$J(O,S) = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix}$$

2 PS (yOz) et (xOz)

$A = \int (y^2 + z^2)\, dm$.

$B = \int (x^2 + z^2)\, dm$

$C = \int (y^2 + x^2)\, dm$.

$\int x^2\, dm = \int y^2\, dm$. $\Rightarrow$ $A = B$

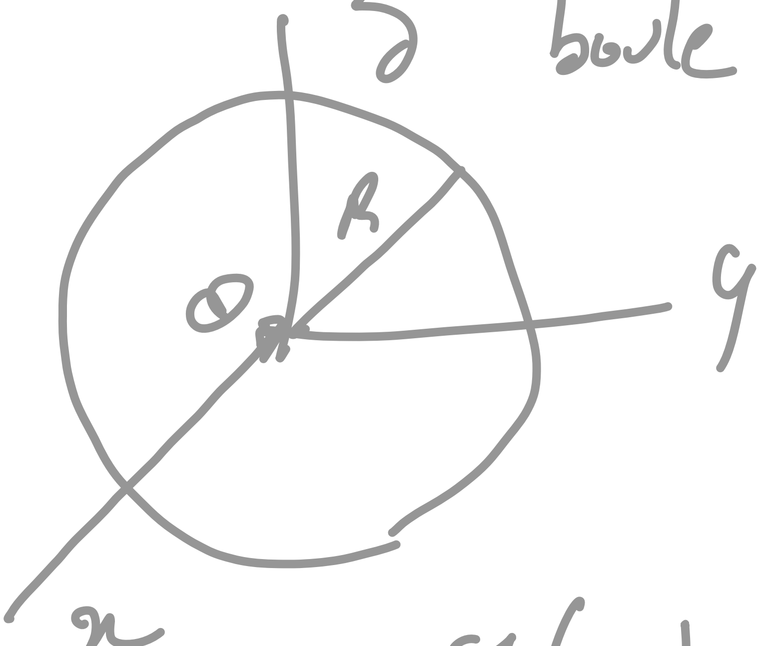
Revolution d'axe (O,y) $\quad A = C$

$$J(O,S) = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & A \end{pmatrix}$$

Revolution d'axe (O,x) $\quad B = C$

$$J(O,S) = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{bmatrix}$$

Symétrie sphérique / à un point



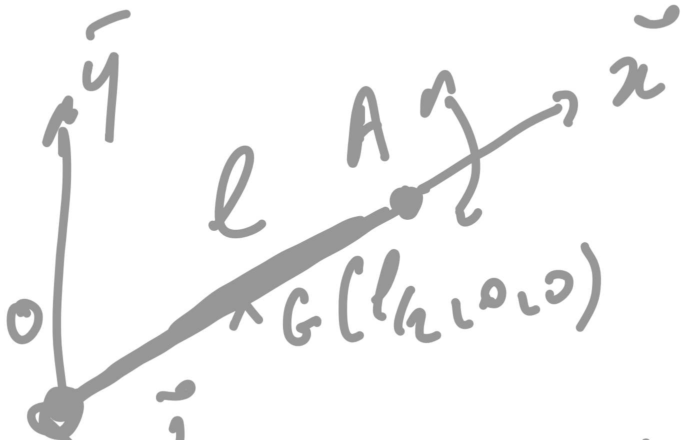
$$A = \frac{2}{5} m R^2$$

$$J(O, \text{boule}) = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{bmatrix}$$

$$\mathrm{Trace}\big(J(O,S)\big) = 3A = 2\iiint \underbrace{(x^2+y^2+z^2)}_{} dm$$

$$A = \frac{2}{3}\iiint \rho^2 dm \longleftarrow = 2\iiint \rho^2 dm.$$

barre de longueur $l \longrightarrow (O, \vec{x})$



$y = z = 0$

$\vec{z}$

$$J(O, \text{barre}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_O & 0 \\ 0 & 0 & B_O \end{bmatrix}$$

$$B_O = C_O = \int x^2 \, dm$$

$$dm = \lambda \, dl = \lambda \, dx$$

$$B_O = \int x^2 \, dm = \int x^2 \lambda \, dx = \lambda \int_0^l x^2 \, dx$$

$$= \frac{m}{l} \left[\frac{x^3}{3} \right]_0^l = \frac{ml^2}{3}$$

$$B_G = \frac{m}{l} \left[\frac{x^3}{3} \right]_{-l/2}^{l/2} = \frac{ml^2}{12}$$

Barre $\rightarrow (O, \vec{y})$

$x = z = 0$

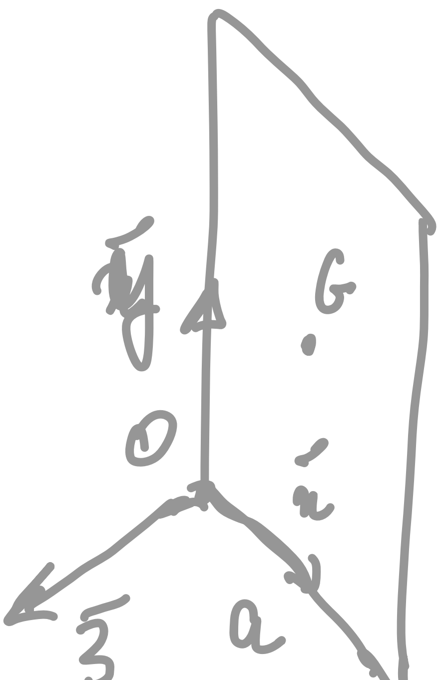
$$J(O,S) = \begin{bmatrix} A_0 & & 0 \\ & 0 & \\ 0 & & A_0 \end{bmatrix}$$

Barre $\rightarrow (O, \vec{z})$

$y = x = 0$

$$J(O,S) = \begin{bmatrix} A_0 & & 0 \\ & A_0 & \\ 0 & & 0 \end{bmatrix}$$

Plaque epaisseur nulle



$$J(G,S) = \begin{bmatrix} A & & 0 \\ & B & \\ 0 & & C \end{bmatrix}$$

PS $\rightarrow (xGy)$ (xGz) (yGz)

$z = 0$

$A = \int y^2 dm$

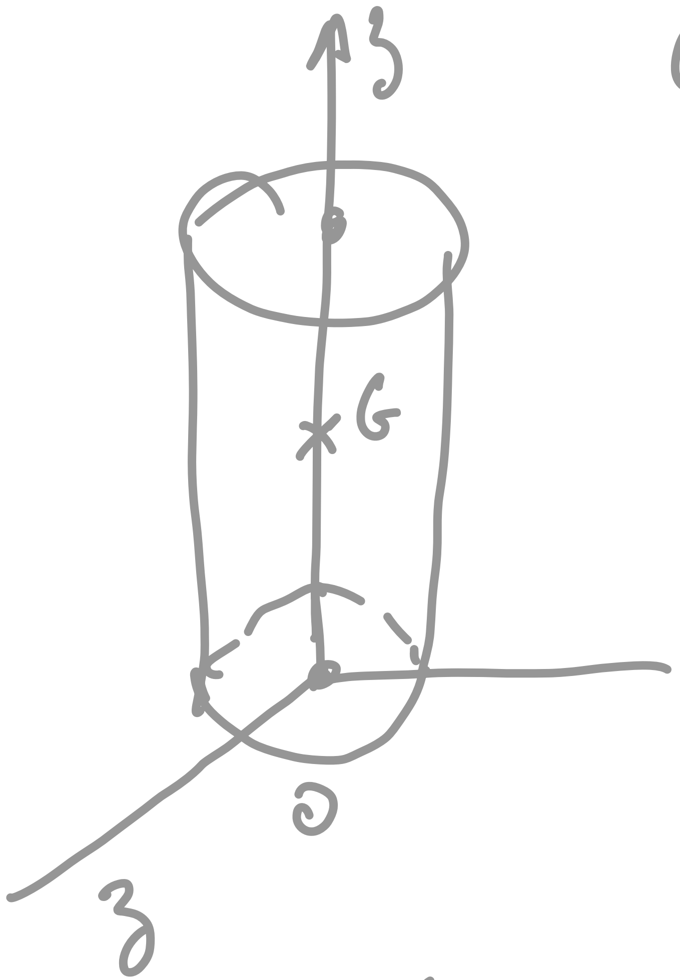
$B = \int x^2 dm$

$C = \int (x^2 + y^2) dm$

$C = A + B$

Cylindre de rayon R et de hauteur H

d'axe $(O, \vec{z})$

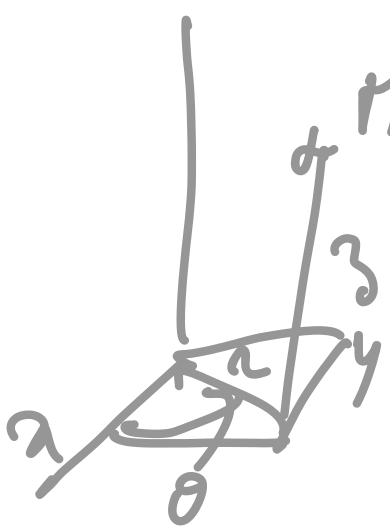


$G(0, 0, H/2)$

$$I(O, S) = \begin{bmatrix} A & & 0 \\ & A & \\ 0 & & C \end{bmatrix}$$

2 PS → xOz
→ yOz

$$C = \iiint (x^2+y^2)\,dm = \iiint r^2\,dm.$$



$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

$dV = r\,dr\,d\theta\,dz$

$dV = r\,dr\,d\theta\,dz$

$$C = \iiint r^2 \eta \, dV = \eta \iiint r^3 \, dr\, d\theta\, dz$$

$$= \eta \left[\frac{r^4}{4}\right]_0^R \left[\theta\right]_0^{2\pi} \left[z\right]_0^H$$

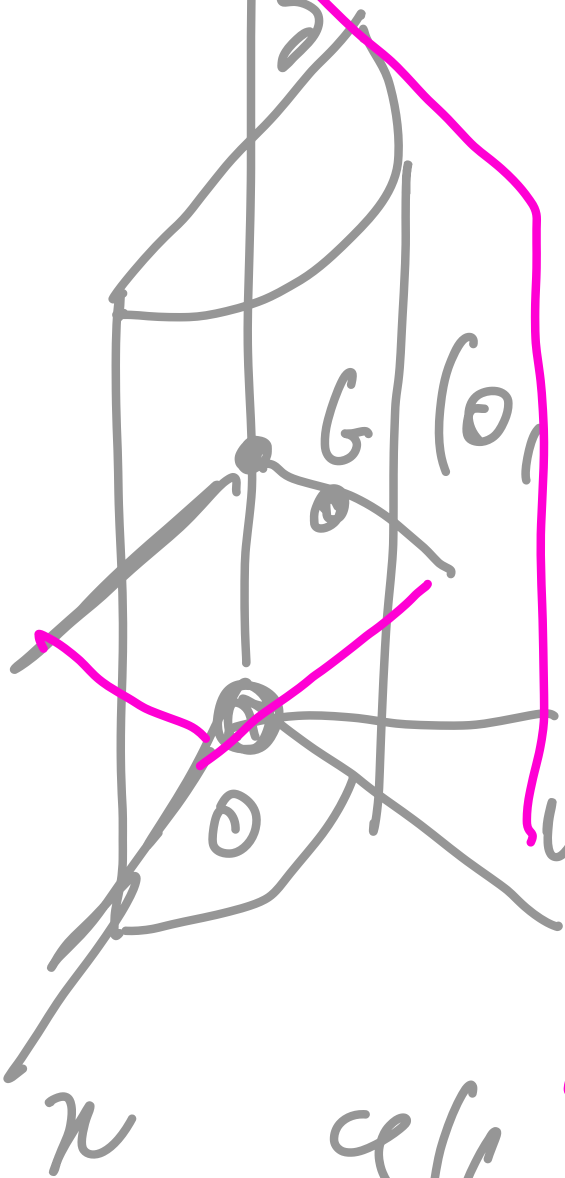
$$= \frac{m}{\cancel{\pi} \cancel{R^2} \cancel{H}} \, \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \, \cancel{2\pi} \, \cancel{H} = \frac{mR^2}{2}$$

$$A = \iiint (y^2 + z^2)\, dm.$$

$$B = \iiint (x^2 + z^2)\, dm.$$

$A = B$ car $\int x^2 dm = \int y^2 dm$

$$\mathrm{Trace}(J) = 2A + C = C + 2\int (x^2 + z^2)\, dm.$$



$\frac{1}{2}$ cylindre

$y > 0$

$G\left(0, y_G, \frac{y}{2}\right)$

$\mathcal{I}\left(O, \frac{1}{2}cyl\right) = \begin{pmatrix} A_O & 0 & 0 \\ 0 & B_O & -D_O \\ 0 & -D_O & C_O \end{pmatrix}$

1 PS (yOz) $\rightarrow$

$\mathcal{I}\left(G, \frac{1}{2}cyl\right) = \begin{pmatrix} A_G & & \\ & B_G & \\ & & C_G \end{pmatrix}$

$\left.\begin{matrix} (G, yz) \text{ PS} \\ (G, xy) \text{ PS} \end{matrix}\right)$ 2 PS

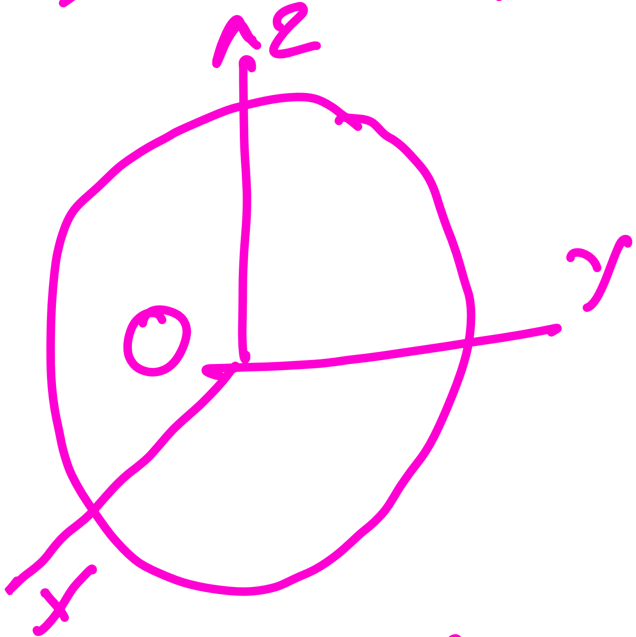
$C = \frac{mR^2}{2}$

$$C = \iiint r^2 \eta \, dV = \eta \iiint r^3 \, dr \, d\theta \, dz$$

$$= \eta \left[\frac{r^4}{4} \right]_0^R \left[\theta \right]_0^{\cancel{2\pi}\ \pi} \left[z \right]_0^H$$

$$= \frac{m \, 2}{\cancel{\pi} \cancel{R^2} \cancel{H}} \, \frac{R^{\cancel{4}\,2}}{\cancel{4}\,2} \, \cancel{2\pi} \, \cancel{H} = \frac{m R^2}{2}$$

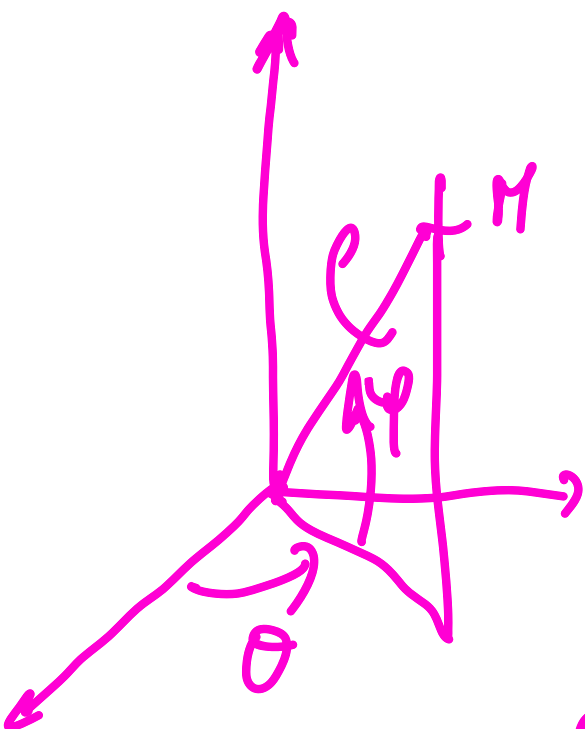
BOULE de rayon R



$$J(O,\text{boule}) = \begin{pmatrix} A & & 0 \\ & A & \\ 0 & & A \end{pmatrix}$$

α $3A = 2\iiint \rho^2\, dm$

$r = \rho \cos\varphi$



$$\begin{cases} x = \rho\cos\varphi\cos\theta \\ y = \rho\cos\varphi\sin\theta \\ z = \rho\sin\varphi \end{cases}$$

$dV = \rho^2\, d\rho \cos\varphi\, d\varphi\, d\theta$

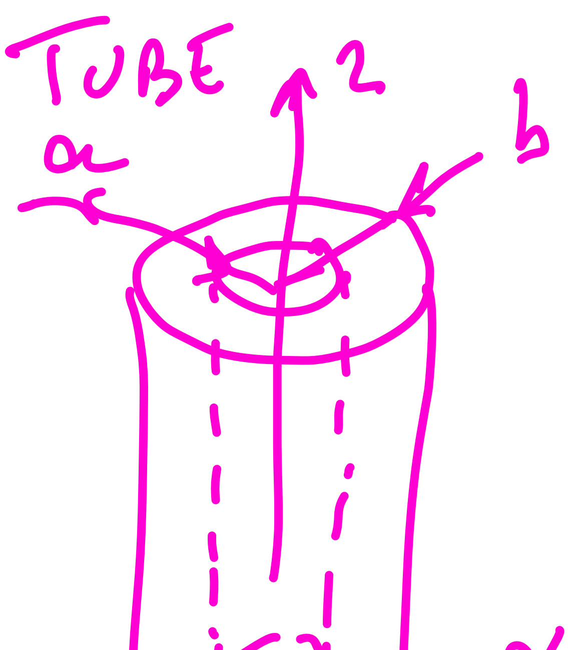
$A = \frac{2}{3} \iiint \rho^2\, \eta\, dV$

$$A = \frac{2}{3}\eta \iiint \rho^4 \, d\rho \cos\varphi \, d\varphi \, d\theta$$

$$A = \frac{2}{3}\frac{m}{V}\left[\frac{\rho^5}{5}\right]_0^R \left[\sin\varphi\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\theta\right]_0^{2\pi}$$

$$A = \frac{2}{\cancel{3}}\,\frac{m\,\cancel{3}}{4\cancel{\pi}\cancel{R^3}}\,\frac{R^{\cancel{5}\,2}}{5}\,\cancel{2}\,\cancel{2\pi}$$

$$A = \frac{2}{5}mR^2 \quad kg\,m^2$$



$$I(O, tube) = \begin{pmatrix} A_0 & & \\ & A_0 & \\ & & C_0 \end{pmatrix}$$

$$C_0 = \iiint r^2\, dm$$

$$C = \iiint r^2 \eta\, dV = \eta \iiint r^3\, dr\, d\theta\, dz$$

$$= \eta \left[\frac{r^4}{4}\right]_a^b \left[\theta\right]_0^{2\pi} \left[z\right]_0^H$$

$$= \frac{m}{\cancel{\pi}(b^2-a^2)\cancel{H}} \, \frac{b^4-a^4}{\cancel{4}\,2} \, \cancel{2\pi} \, \cancel{H}$$

$$= \frac{m}{2} \, \frac{b^4-a^4}{b^2-a^2} = \frac{m}{2} \, \frac{(b^2+a^2)\cancel{(b^2-a^2)}}{\cancel{(b^2-a^2)}}$$

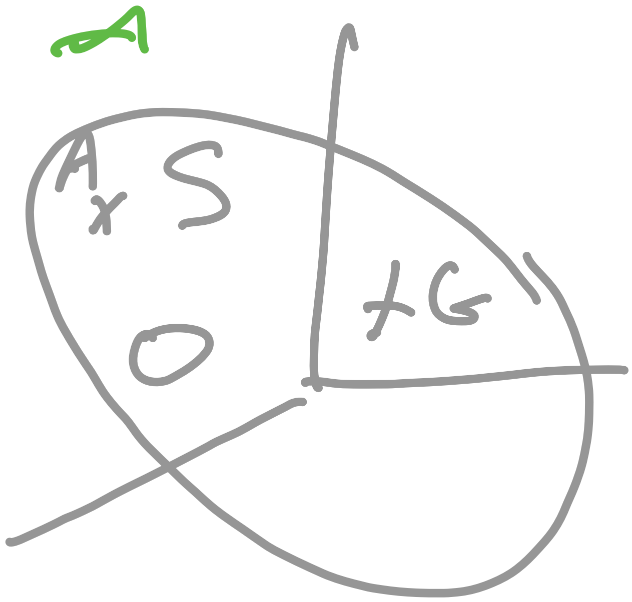
$$= m \, \frac{b^2+a^2}{2}$$

$$C = \frac{m_1 b^2}{L} - \frac{m_2 a^2}{L}$$

$$\eta = \frac{m}{\pi(b^2 - a^2)H} = \frac{m_1}{\pi b^2 H} = \frac{m_2}{\pi a^2 H}$$

$$m_1 = m \frac{b^2}{b^2 - a^2}$$

$$m_2 = m \frac{a^2}{b^2 - a^2}$$



Huygens

$$\overset{A}{\underset{}{}} \qquad \overset{A}{}$$

$$G \qquad G$$

$$\mathcal{J}(O,S)\,\vec{u} = -\int \overrightarrow{OP} \wedge \left(\overrightarrow{OP} \wedge \vec{u}\right) dm.$$

$$\overrightarrow{OP} = \overrightarrow{OG} + \overrightarrow{GP}$$

$$\vec{OP} = \vec{OG} + \vec{GP}$$

$$= -\int \left(\vec{OG}+\vec{GP}\right)\wedge\left(\left(\vec{OG}+\vec{GP}\right)\wedge\vec{u}\right)dm$$

$$= \boxed{-\int \vec{OG}\wedge\left(\vec{OG}\wedge\vec{u}\right)dm} - \int \vec{OG}\wedge\cancel{\left(\vec{GP}\wedge\vec{u}\right)}^{\vec{0}}dm$$

$$- \underbrace{\int \vec{GP}\wedge\cancel{\left(\vec{OG}\wedge\vec{u}\right)}dm}_{\vec{0}} - \underbrace{\int \vec{GP}\wedge\left(\vec{GP}\wedge\vec{u}\right)dm}_{J(G,S)\vec{u}}$$

$$-\left(\left(\underbrace{\cancel{\int \vec{GP}\,dm}}_{\vec{0}}\right)\wedge\vec{OG}\right)\wedge\vec{u}$$

$$-\int_{P\in S} \vec{OG}\wedge\left(\vec{GP}\wedge\vec{u}\right)dm = -\vec{OG}\wedge\left(\left(\underbrace{\int \vec{GP}\,dm}_{\vec{0}}\right)\wedge\vec{u}\right)$$

$$J(O,S)\vec{u} = J(G,S)\vec{u} - \int \vec{OG}\wedge\left(\vec{OG}\wedge\vec{u}\right)dm$$

$$-\int \vec{OG}\wedge\left(\vec{OG}\wedge\vec{u}\right)dm = -\left(\int dm\right)\vec{OG}\wedge\left(\vec{OG}\wedge\vec{u}\right)$$

$$= -m\,\vec{OG}\wedge\left(\vec{OG}\wedge\vec{u}\right)$$

$$\Rightarrow \underbrace{J(O, G(m))}\,\vec{u}$$

$$\vec{OG}\begin{vmatrix} x_G \\ y_G \\ z_G \end{vmatrix} \qquad \vec{GO}\begin{vmatrix} -x_G \\ -y_G \\ -z_G \end{vmatrix}$$

$$m\,\vec{OG} \wedge (\vec{OG} \wedge \vec{u}) = \begin{bmatrix} m(y_G^2+z_G^2) & & \\ & m(x_G^2+z_G^2) & \\ & & m(x_G^2+y_G^2) \end{bmatrix}$$

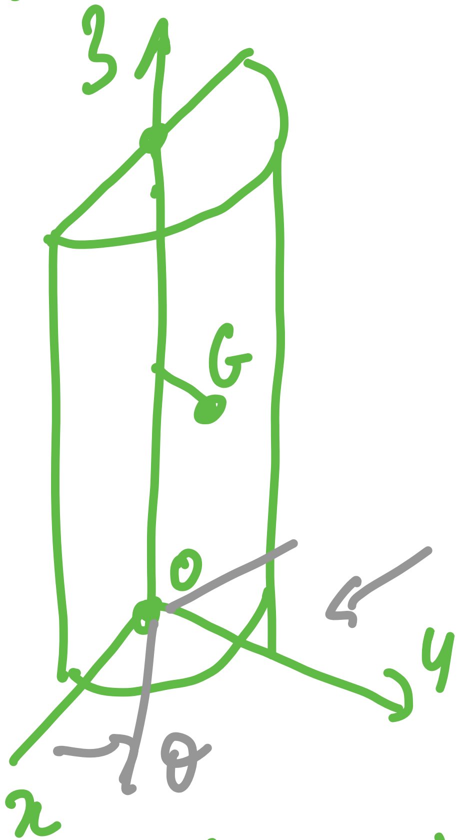
$$\begin{bmatrix} \underbrace{m(y_G^2+z_G^2)} & -m\,x_G y_G & -m\,x_G z_G \\ -m\,y_G x_G & m(x_G^2+z_G^2) & -m\,y_G z_G \\ -m\,x_G z_G & -m\,y_G z_G & m(x_G^2+y_G^2) \end{bmatrix}$$

Minimale

$$\boxed{J(O,S) = J(G,S) + J(O, G(m))}$$

$$J(G,S) = J(O,S) - J(O,G(m))$$

$\frac{1}{2}$ Cylindre $y>0$ $\quad G\left(0, y_G, \frac{H}{2}\right) \quad \left(y_G = \frac{4R}{3\pi}\right)$



$$J\left(G, \tfrac{1}{2} Cyl\right) = \begin{bmatrix} A_G & 0 & 0 \\ 0 & B_G & 0 \\ 0 & 0 & C_G \end{bmatrix}_{R_S}$$

$$J\left(O, \tfrac{1}{2} Cyl\right) = J\left(G, \tfrac{1}{2} Cyl\right) + J(O, G(m))$$

$$\vec{OG} \begin{vmatrix} 0 \\ y_G \\ H/2 \end{vmatrix}$$

$$J(O, G(m)) = \begin{bmatrix} m\left(y_G^2 + \frac{H^2}{4}\right) & 0 & 0 \\ 0 & m\frac{H^2}{4} & -m y_G \frac{H}{2} \\ 0 & -m y_G \frac{H}{2} & m y_G^2 \end{bmatrix}_{R_S}$$

$$J\left(0, \tfrac{1}{2}(y+1)\right) = \begin{pmatrix} A_0 & 0 & 0 \\ 0 & B_0 & -D_0 \\ 0 & -D_0 & C_0 \end{pmatrix}_{R_S}$$