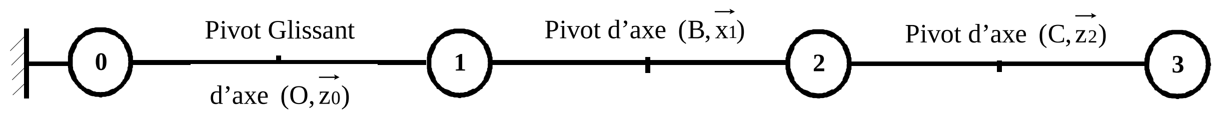
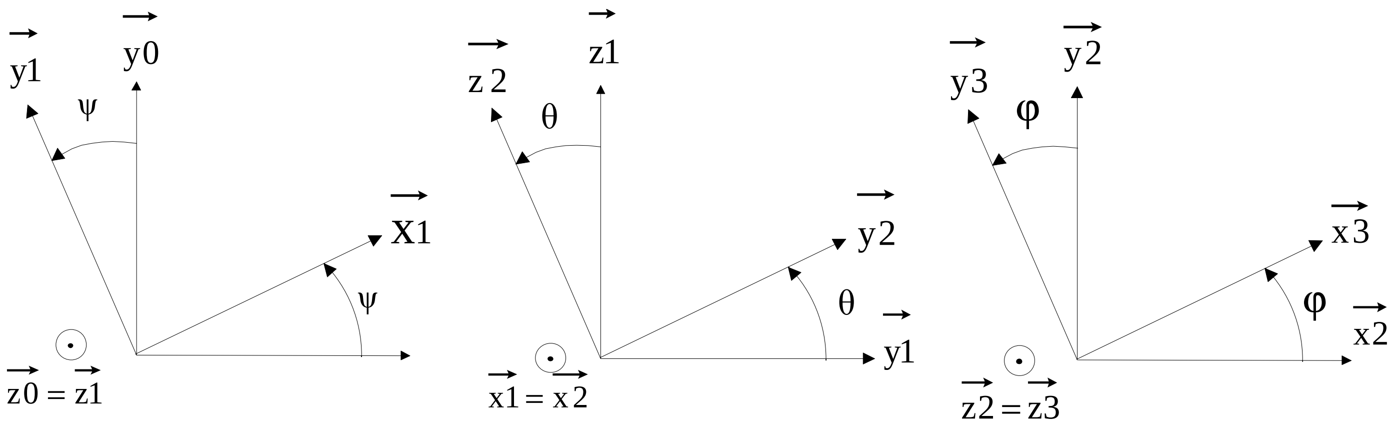
MANEGE : CORRIGE

Q.1. Graphe des liaisons



Q.2. Figures illustrant les différents changements de bases :



$\overrightarrow{\Omega(1/0)} = \dot{\psi}\,\overrightarrow{z0(1)}$ $\qquad$ $\overrightarrow{\Omega(2/1)} = \dot{\theta}\,\overrightarrow{x1(2)}$ $\qquad$ $\overrightarrow{\Omega(3/2)} = \dot{\varphi}\,\overrightarrow{z2(3)}$

Q.3. Liaison 1/0

$$\overrightarrow{V(A,1/0)} = \left[\frac{d\overrightarrow{OA}}{dt}\right]_0 = \left[\frac{d\lambda\overrightarrow{z_{0(1)}}}{dt}\right]_0 = \dot{\lambda}\,\overrightarrow{z_{0(1)}}$$

$$\mathcal{V}(1/0) = \left\{ \dot{\psi}\,\overrightarrow{z_{0(1)}} \;,\; \dot{\lambda}\,\overrightarrow{z_{0(1)}} \right\}_{O,A}$$

Q.4. Liaison 2/1

$$\mathcal{V}(2/1) = \left\{ \dot{\theta}\,\overrightarrow{x_{1(2)}} \;,\; \vec{0} \right\}_{B}$$

Q.5. Liaison 3/2

$$\mathcal{V}(3/2) = \left\{ \dot{\varphi}\,\overrightarrow{z_{2(3)}} \;,\; \vec{0} \right\}_{C,B}$$

Q.6. Calculer :

- ♦ $\overrightarrow{V(B,1/0)} = \overrightarrow{V(A,1/0)} + \overrightarrow{BA} \wedge \overrightarrow{\Omega(1/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - a\,\overrightarrow{y1} \wedge \dot{\psi}\,\overrightarrow{z1} = \dot{\lambda}\,\overrightarrow{z0(1)} - a\,\dot{\psi}\,\overrightarrow{x1}$

- ♦ $\overrightarrow{V(B,2/0)} = \overrightarrow{V(B,2/1)} + \overrightarrow{V(B,1/0)} = \overrightarrow{V(B,1/0)}$ car $\overrightarrow{V(B,2/1)} = \vec{0}$ (B centre de la pivot 2/1)

- ♦ $\overrightarrow{V(C,2/0)} = \overrightarrow{V(B,2/0)} + \overrightarrow{CB} \wedge \overrightarrow{\Omega(2/0)} = \dot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\dot{\psi}\,\overrightarrow{x_1} + b\overrightarrow{z_2} \wedge (\dot{\psi}\,\overrightarrow{z_1} + \dot{\theta}\,\overrightarrow{x_1})$

  $\overrightarrow{V(C,2/0)} = \dot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\dot{\psi}\,\overrightarrow{x_1} - b\,\dot{\psi}\sin\theta\,\overrightarrow{x_1} + b\,\dot{\theta}\,\overrightarrow{y_2} = -\dot{\psi}(a + b\sin\theta)\overrightarrow{x_1} + b\,\dot{\theta}\,\overrightarrow{y_2} + \dot{\lambda}\,\overrightarrow{z_{0(1)}}$

- ♦ $\overrightarrow{V(C,3/0)} = \overrightarrow{V(C,3/2)} + \overrightarrow{V(C,2/0)} = \overrightarrow{V(C,2/0)}$ car $\overrightarrow{V(C,3/2)} = \vec{0}$ (C centre de la pivot 3/2)

$$\blacklozenge\ \overrightarrow{V(D,3/0)} = \overrightarrow{V(C,3/0)} + \overrightarrow{DC} \wedge \overrightarrow{\Omega(3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - \dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y2} - c\vec{x_3} \wedge (\overrightarrow{\Omega(3/2)} + \overrightarrow{\Omega(2/1)} + \overrightarrow{\Omega(1/0)})$$

$$\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - \dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y2} - c\vec{x_3} \wedge (\dot{\psi}\vec{z_1} + \dot{\theta}\vec{x_2} + \dot{\varphi}\vec{z_2})$$

$$\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - \dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y2} - c\vec{x_3} \wedge (\dot{\psi}\vec{z_1} + \dot{\theta}\vec{x_2} + \dot{\varphi}\vec{z_2})$$

$$\vec{x_3} \wedge \vec{z_1} = \vec{x_3} \wedge (\cos\theta\ \vec{z_2} + \sin\theta\ \vec{y_2}) = -\cos\theta\,\vec{y_3} + \sin\theta\,\cos\varphi\,\vec{z_3}$$

$$\text{OU}\ \ \vec{x_3} \wedge \vec{z_1} = (\cos\varphi\ \vec{x_2} + \sin\varphi\ \vec{y_2}) \wedge \vec{z_1} = -\cos\varphi\,\vec{y_1} + \sin\varphi\,\cos\theta\,\vec{x_1}$$

$$\vec{x_3} \wedge \vec{x_2} = -\sin\varphi\,\vec{z_3}$$

$$\vec{x_3} \wedge \vec{z_2} = -\vec{y_3}$$

$$\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - \dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y_2} - c\vec{x_3} \wedge (\dot{\psi}\vec{z_1} + \dot{\theta}\vec{x_2} + \dot{\varphi}\vec{z_2})$$

$$\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} - \dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y_2} - c\,\dot{\psi}(-\cos\theta\,\vec{y_3} + \sin\theta\,\cos\varphi\,\vec{z_3}) + c\,\dot{\theta}\sin\varphi\,\vec{z_3} + c\,\dot{\varphi}\,\vec{y_3})$$

$$\boxed{\overrightarrow{V(D,3/0)} = -\dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y_2} + \dot{\lambda}\,\overrightarrow{z0(1)} + c\,(\dot{\varphi} + \dot{\psi}\cos\theta)\vec{y_3} + c\,(\dot{\theta}\sin\varphi - \dot{\psi}\sin\theta\,\cos\varphi)\vec{z_3}}$$

$$\text{OU}\ \ \overrightarrow{V(D,3/0)} = -\dot{\psi}(a + b\sin\theta)\vec{x1} + b\,\dot{\theta}\,\vec{y_2} + \dot{\lambda}\,\overrightarrow{z0(1)} + c\,\dot{\varphi}\,\vec{y_3} + c\,\dot{\theta}\sin\varphi\,\vec{z_3} + c\,\dot{\psi}\cos\varphi\,\vec{y_1} - c\,\dot{\psi}\sin\varphi\cos\theta\,\vec{x_1}$$

$$\text{OU}\ \ \overrightarrow{V(D,3/0)} = -\dot{\psi}(a + b\sin\theta + c\sin\varphi\cos\theta)\vec{x1} + c\,\dot{\psi}\cos\varphi\,\vec{y_1} + \dot{\lambda}\,\overrightarrow{z0(1)} + b\,\dot{\theta}\,\vec{y_2} + c\,\dot{\varphi}\,\vec{y_3} + c\,\dot{\theta}\sin\varphi\,\vec{z_3}$$

$$\blacklozenge\ \overrightarrow{A(B,1/0)} = \left[\frac{d\overrightarrow{V(B,1/0)}}{dt}\right]_0 = \left[\frac{d(\dot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\dot{\psi}\,\vec{x_1})}{dt}\right]_0 = \ddot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\ddot{\psi}\,\vec{x_1} - a\,\dot{\psi}\left[\frac{d\vec{x_1}}{dt}\right]_0 = \ddot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\ddot{\psi}\,\vec{x_1} - a\,\dot{\psi}\,\overrightarrow{\Omega(1/0)} \wedge \vec{x_1}$$

$$\overrightarrow{A(B,1/0)} = \ddot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\ddot{\psi}\,\vec{x_1} - a\,\dot{\psi}\dot{\psi}\,\vec{z_1} \wedge \vec{x_1} = \ddot{\lambda}\,\overrightarrow{z_{0(1)}} - a\,\ddot{\psi}\,\vec{x_1} - a\,\dot{\psi}^2\,\vec{y_1}$$

$$\blacklozenge\ \overrightarrow{A(C,3/0)} = \left[\frac{d\overrightarrow{V(C,3/0)}}{dt}\right]_0 = \left[\frac{d\overrightarrow{V(C,2/0)}}{dt}\right]_0 = \left[\frac{d(-\dot{\psi}(a + b\sin\theta)\vec{x_1} + b\dot{\theta}\,\vec{y_2} + \dot{\lambda}\,\overrightarrow{z_{0(1)}})}{dt}\right]_0$$

$$\overrightarrow{A(C,3/0)} = \ddot{\lambda}\,\overrightarrow{z0(1)}\ - (b\dot{\theta}\dot{\psi}\cos\theta + (a + b\sin\theta)\ddot{\psi})\vec{x1} - \dot{\psi}(a + b\sin\theta)\,\overrightarrow{\Omega(1/0)} \wedge \vec{x_1} + b\,\ddot{\theta}\,\vec{y_2} + b\dot{\theta}\ \ \overrightarrow{\Omega(2/0)} \wedge \vec{y_2}$$

$$\overrightarrow{A(C,3/0)} = \ddot{\lambda}\,\overrightarrow{z0(1)}\ - (b\dot{\theta}\dot{\psi}\cos\theta + (a + b\sin\theta)\ddot{\psi})\vec{x1} - \dot{\psi}(a + b\sin\theta)\,(\dot{\psi}\vec{z_1}) \wedge \vec{x_1} + b\,\ddot{\theta}\,\vec{y_2} + b\dot{\theta}\ \ (\dot{\psi}\vec{z_1} + \dot{\theta}\vec{x_2}) \wedge \vec{y_2}$$

$$\overrightarrow{A(C,3/0)} = \ddot{\lambda}\,\overrightarrow{z0(1)}\ - (b\dot{\theta}\dot{\psi}\cos\theta + (a + b\sin\theta)\ddot{\psi})\vec{x1} - \dot{\psi}^2\ (a + b\sin\theta)\,\vec{y_1} + b\,\ddot{\theta}\,\vec{y_2} + b\dot{\theta}\ \ (-\dot{\psi}\cos\theta\,\overrightarrow{x_{(1)2}} + \dot{\theta}\,\vec{z_2})$$

Q.7. Mouvement particuliers

$$\boxed{\begin{array}{l}\psi = Cste \Rightarrow \dot{\psi} = 0 \text{ et } \ddot{\psi} = 0 \\ \theta = Cste \Rightarrow \dot{\theta} = 0 \Rightarrow \ddot{\theta} = 0 \\ \dot{\varphi} = Cste \Rightarrow \ddot{\varphi} = 0 \\ \dot{\lambda} = Cste \Rightarrow \ddot{\lambda} = 0\end{array}}$$

➔ $$\boxed{\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} + c\dot{\varphi}\,\overrightarrow{y3}}$$

$$\overrightarrow{V(D,3/0)} = \dot{\lambda}\,\overrightarrow{z0(1)} + c\dot{\varphi}\,\overrightarrow{y3} = \dot{\lambda}(\cos\theta\ \overrightarrow{z2} + \sin\theta\ \overrightarrow{y2}) + c\dot{\varphi}(\cos\varphi\ \overrightarrow{y2} - \sin\varphi\ \overrightarrow{x2})$$

$$\overrightarrow{V(D,3/0)} = -c\dot{\varphi}\sin\varphi\ \overrightarrow{x2} + (\dot{\lambda}\sin\theta + c\dot{\varphi}\cos\varphi)\ \overrightarrow{y2} + \dot{\lambda}\cos\theta\ \overrightarrow{z2}$$

$$\left\|\overrightarrow{V(D,3/0)}\right\| = \sqrt{(c\dot{\varphi})^2 + (\dot{\lambda})^2}$$

$$\overrightarrow{A(D,3/0)} = \left[\frac{d\overrightarrow{V(D,3/0)}}{dt}\right]_0 = \left[\frac{d(\dot{\lambda}\,\overrightarrow{z0(1)} + c\dot{\varphi}\,\overrightarrow{y3})}{dt}\right]_0 = c\dot{\varphi}\left[\frac{d\overrightarrow{y3}}{dt}\right]_0 = c\dot{\varphi}\,\overrightarrow{\Omega(3/0)} \wedge \overrightarrow{y3} = c\dot{\varphi}(\dot{\varphi}\overrightarrow{z2}) \wedge \overrightarrow{y3} = -c\dot{\varphi}^2\ \overrightarrow{x3}$$

$$\left\|\overrightarrow{A(D,3/0)}\right\| = c\dot{\varphi}^2$$

Q.8. Application numérique : $\psi = 30°,\ \theta = 45°,\ \dot{\varphi} = 30\ {}^{tr}\!/_{min},\ \dot{\lambda} = 3\ {}^{m}\!/_{s}$ et $c = 2m$

$$\dot{\varphi} = 30\ {}^{tr}\!/_{min} = \frac{2\pi * 30}{60}\ {}^{rd}\!/_{s} = \pi\ {}^{rd}\!/_{s} = 3{,}14\ {}^{rd}\!/_{s}$$

$$\left\|\overrightarrow{V(D,3/0)}\right\| = \sqrt{(2 * \frac{2\pi * 30}{60})^2 + (3)^2} = \sqrt{(2\pi)^2 + (3)^2} = 6{,}96\ {}^{m}\!/_{s} = 25\ {}^{km}\!/_{h}$$

$$\left\|\overrightarrow{A(D,3/0)}\right\| = 2 * (\frac{2\pi * 30}{60})^2 = 2 * (\pi)^2 = 19{,}74\ {}^{m}\!/_{s^2} = 2g$$