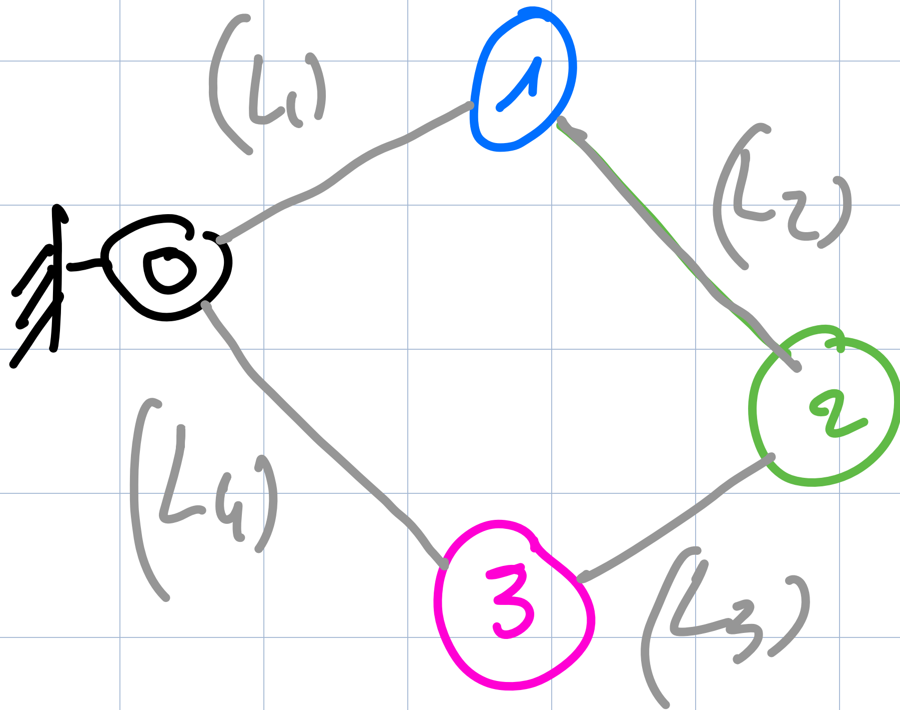
Système Bielle Manivelle

(0) Bati

(1) Manivelle

(2) bielle

(3) Piston



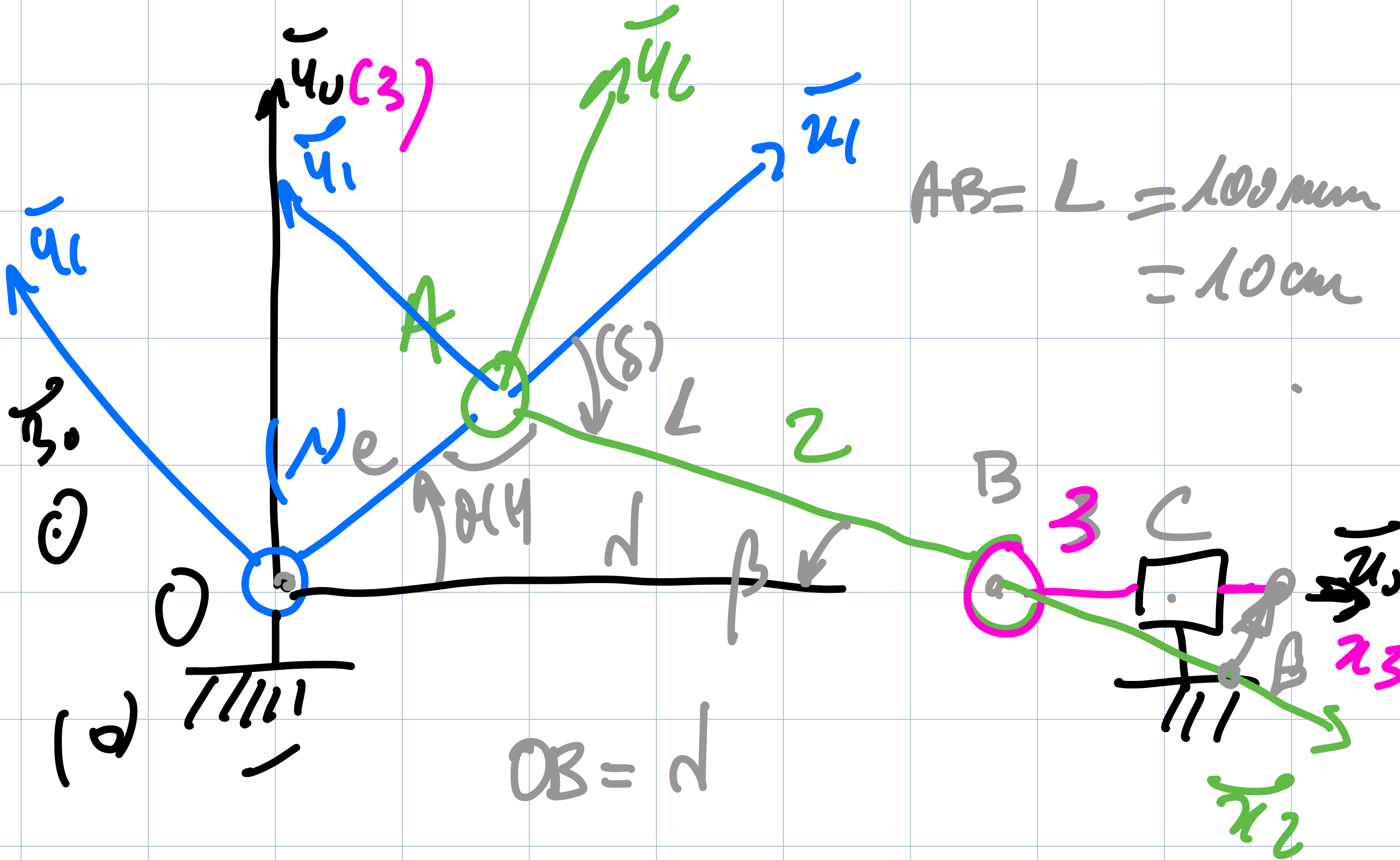
Mvt 1/0 : Rotation d'axe $(O, \vec{z}_{0(1)})$

(L1) : Pivot d'axe $(O, \vec{z}_{0(1)})$

Mvt 2/1 : Rotation d'axe $(A, \vec{z}_{1(2)})$

(L2) : Pivot d'axe $(A, \vec{z}_{1(2)})$

$OA = e = 20\,mm = 2\,cm$ excentricité



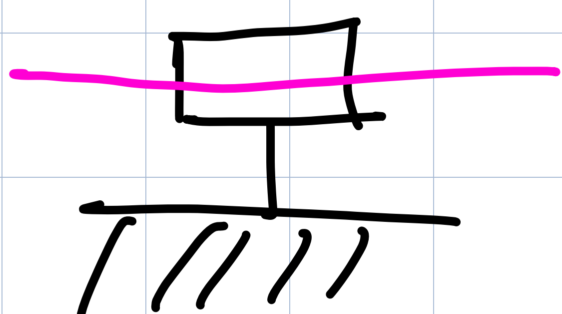
AB = L = 100 mm
= 10 cm

OB = d

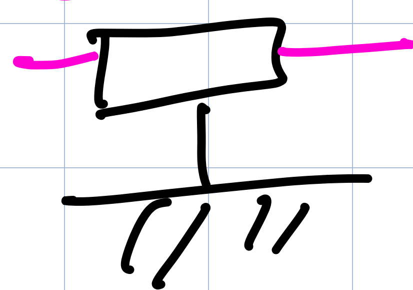
Piston/Bâti : mvt Translation
de direction $\vec{x}_0$

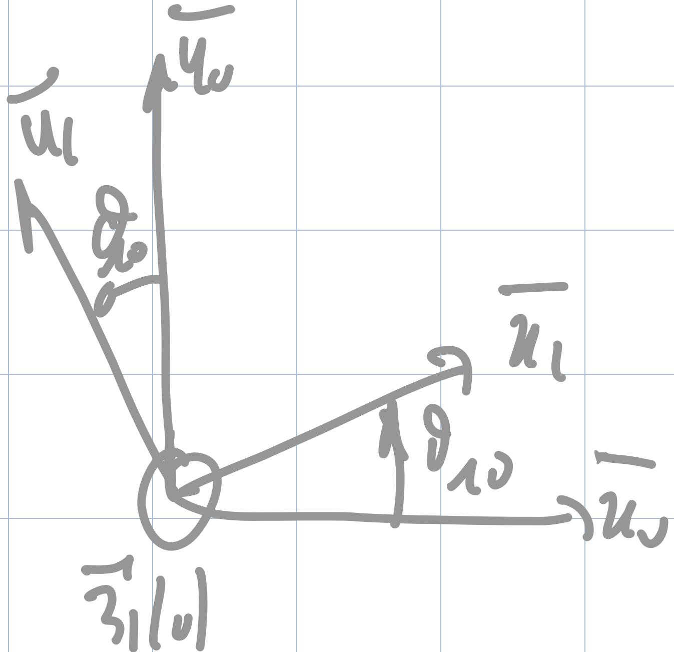
Rotation autour de $\vec{x}_0$

(L4) : Pivot glissant d'axe $(C, \vec{x}_0)$



Glissière





$$\vec{\Omega}(1/0) = \dot{\theta}_{10}\,\vec{z}_{0(1)}$$
$$= \omega_{z_{10}}\,\vec{z}_{0(1)}$$

$$\mathcal{V}(1/0) = \left\{ \dot{\theta}\,\vec{z}_{1(0)}\,,\ \vec{0} \right\}_{0,\ \forall P(0,\vec{z}_{0(1)})}$$

Mvt 3/2 : Rotation d'axe $(B, \vec{z}_{2(3)})$

$\rightarrow$ Liaison Pivot d'axe $(B, \vec{z}_{2(3)})$

3/0 $\rightarrow$ glissière $\vec{\Omega}(3/0) = \vec{0}$

$$b_3 = b_2$$

3/2 $\rightarrow$ Pivot

Relation d'Entrée / Sortie

Pompe : $(E) \rightarrow (1) \rightarrow (\theta)$
$(S) \rightarrow (2) \rightarrow (\alpha)$

$\rightarrow \alpha = f(\theta)$ → E/S geometrique

$\downarrow d/dt$ $\uparrow \int dt$ à C^{ste}

$\dot{\alpha} = g(\dot{\theta})$ → E/S cinematique

Moteur $(E) \rightarrow (3)$ (α)
(S) (1) θ

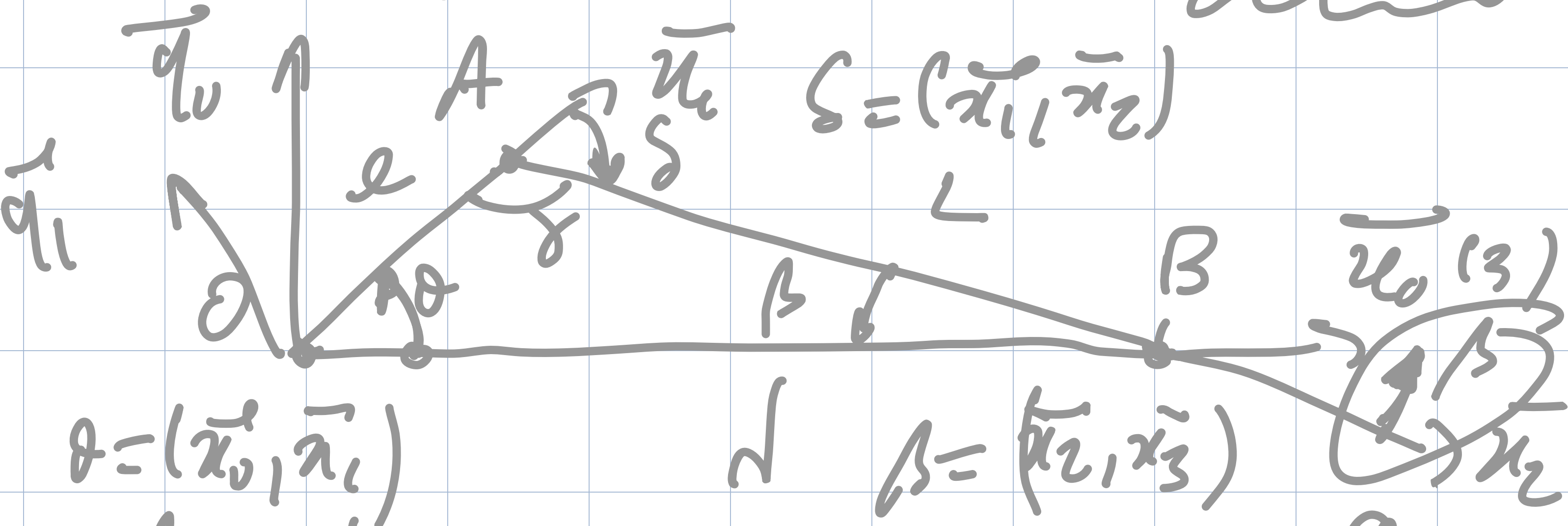
$\theta = f^{-1}(\alpha)$

d/dt $\uparrow \int dt$

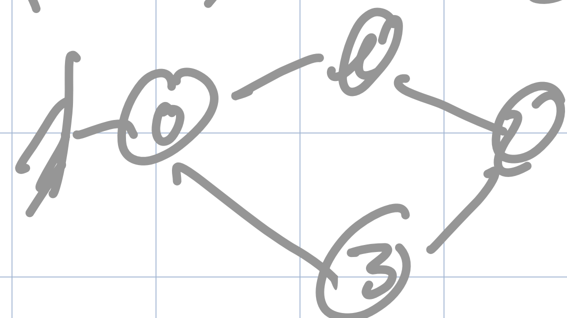
$$\theta^0 = g^{-1}(d^0)$$

Approche géométrique

$\theta = f(d)$



fermeture angulaire



$$(\vec{x}_0, \vec{x}_1) + (\vec{x}_1, \vec{x}_2) + (\vec{x}_2, \vec{x}_3) + (\vec{x}_3, \vec{x}_0) = 0$$

$$\theta + \delta + \beta + 0 = 0_{\alpha}$$

$$\boxed{\delta = -\theta - \beta}$$

$$\frac{d}{dt} \longrightarrow \dot{\delta} = -\dot{\theta} - \dot{\beta}$$

fermeture dimensionnelle

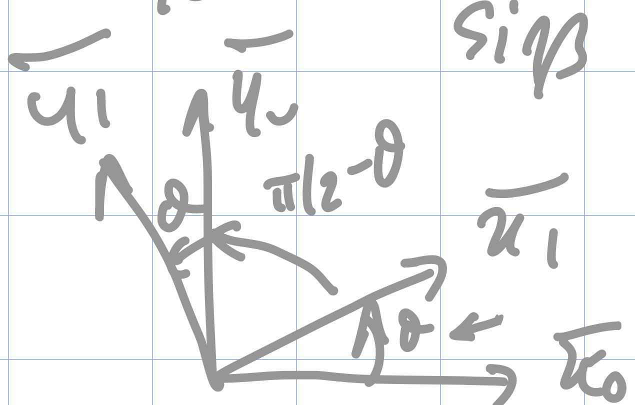
$$\vec{OA} + \vec{AB} + \vec{BO} = \vec{0}$$

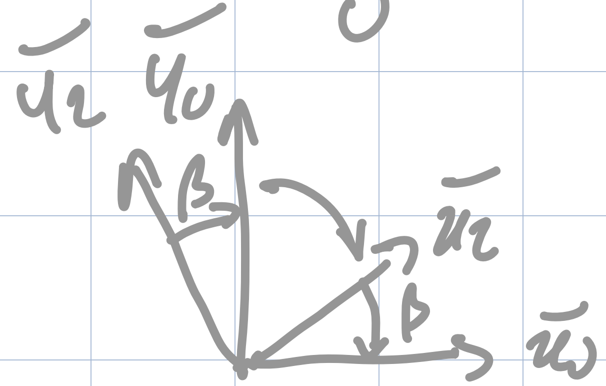
$$e\vec{x_1} + L\vec{x_2} - d\vec{x_0} = \vec{0}$$

$$L\vec{x_2} = d\vec{x_0} - e\vec{x_1} \leftarrow$$

$$\underline{|\vec{x_0}} : \quad L\underbrace{\vec{x_2}\cdot\vec{x_0}}^{\cos\beta} = d\,\overbrace{\cancel{\vec{x_0}\cdot\vec{x_0}}}^{1} - e\,\overbrace{\vec{x_1}\cdot\vec{x_0}}^{\cos\theta}$$

$$|\vec{y_0} : \quad L\underbrace{\vec{x_2}\cdot\vec{y_0}}_{\sin\beta} = d\,\underbrace{\cancel{\vec{x_0}\cdot\vec{y_0}}}_{0} - e\,\underbrace{\vec{x_1}\cdot\vec{y_0}}_{\cos(\frac{\pi}{2}-\theta) = \sin\theta}$$





(1) $L\cos\beta = d - e\cos\theta$

(2) $L\sin\beta = -e\sin\theta$

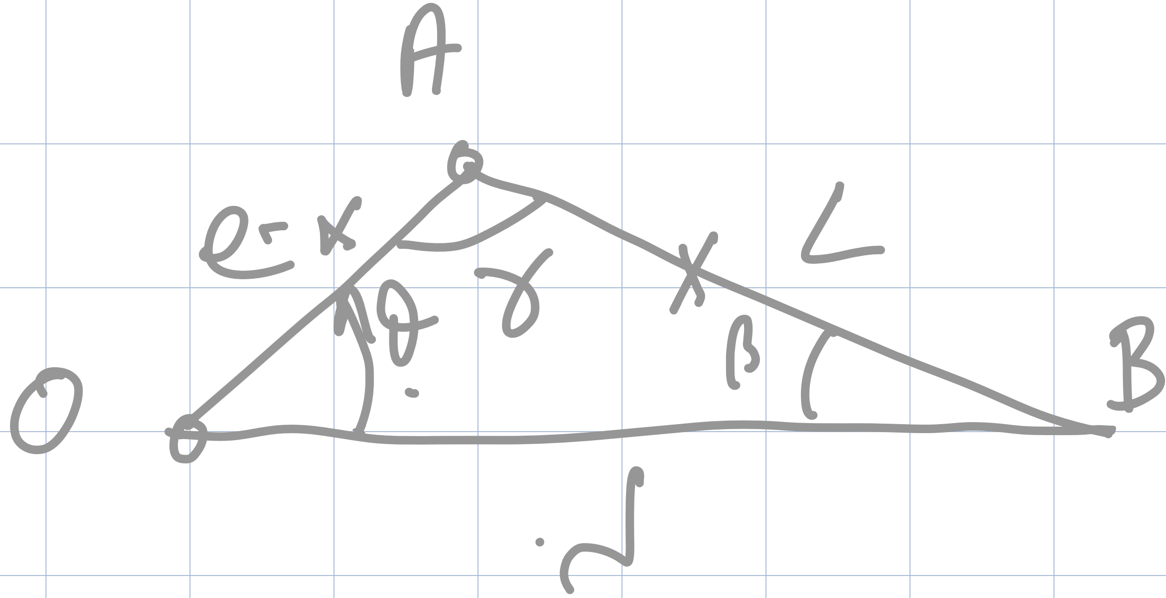
$$d = f(\theta) \quad ?$$

$(1)^2 + (2)^2$

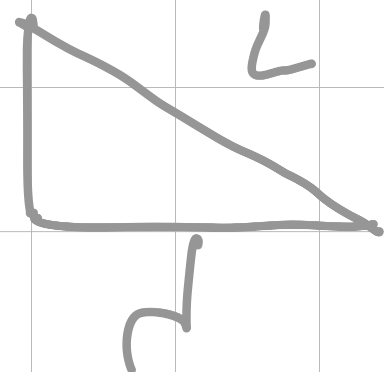
$$L^2(\underbrace{\cancel{\sin^2\beta + \cos^2\beta}}_{1}) = (d - e\cos\theta)^2 + (-e\sin\theta)^2$$

$$L^2 = d^2 - 2ed\cos\theta + e^2(\underbrace{\cancel{\cos^2\theta + \sin^2\theta}}_{1})$$

$$\boxed{L^2 = d^2 + e^2 - 2ed\cos\theta}$$



$\theta = \frac{\pi}{2}$



$$e^2 = L^2 + d^2 - 2Ld\cos\beta$$

$$d^2 = e^2 + L^2 - 2eL\cos\gamma$$

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$AB^2 = AO^2 + OB^2 + 2\,\vec{AO}\cdot\vec{OB}$$

$$L^2 = e^2 + d^2 - 2\,\vec{OA}\cdot\vec{OB}$$

$$- 2\, OA . OB . \cos\underbrace{(\vec{OA},\vec{OB})}_{=\theta}$$

$$L^2 = e^2 + d^2 - 2ed\cos\theta$$

$$d = f(\theta)$$

$$d^2 - (2e\cos\theta)d + e^2 - L^2 = 0$$

$$ad^2 + bd + c = 0$$

$$\dots \quad \begin{matrix} d_1 \\ d_2 \end{matrix} \quad \frac{-b \pm \sqrt{\Delta}}{2a} \quad < \frac{d}{d_r}$$

$$\theta = f^{-1}(d) \qquad \cos\theta = \frac{e^2 + d^2 - L^2}{2ed}$$

$$\theta = \operatorname{Arccos}\left(\frac{e^2+d^2-L^2}{2ed}\right) \quad d/dt$$

Loi cinématique

$$L^2 = e^2 + d^2 - 2ed\cos\theta$$

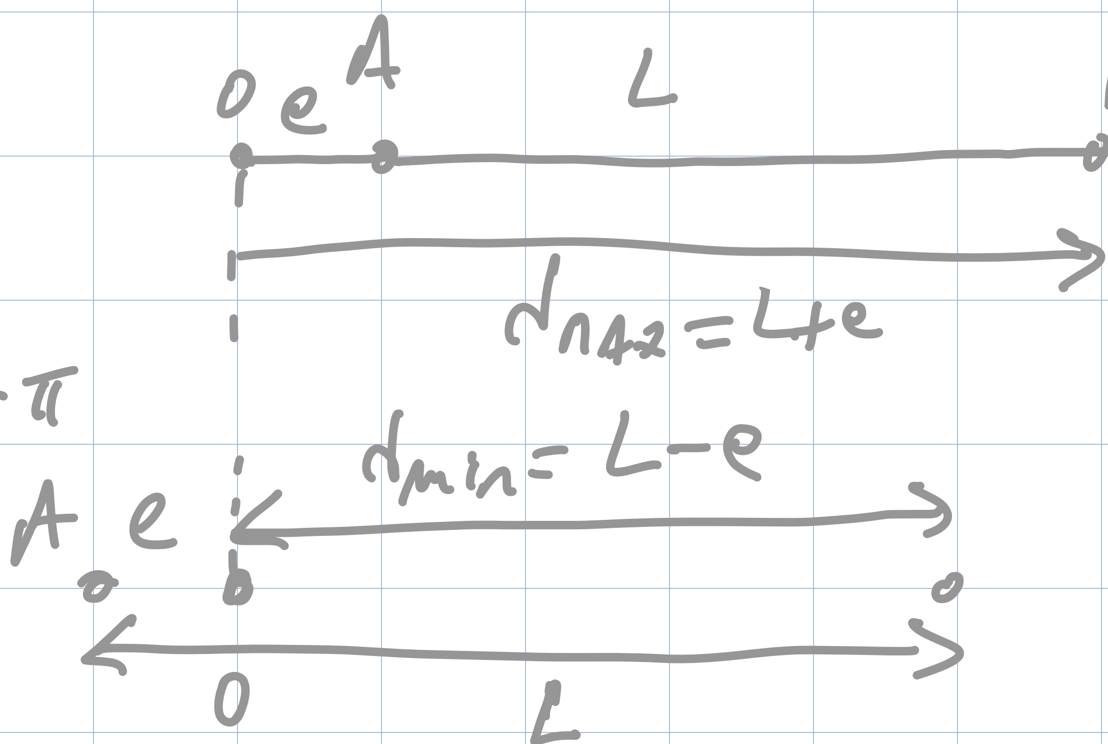
d/dt

$$0 = 0 + 2d\dot{d} - 2e\dot{d}\cos\theta + 2ed\dot{\theta}\sin\theta$$

$$\boxed{d\dot{d} - e\dot{d}\cos\theta + ed\dot{\theta}\sin\theta = 0}$$

Course du piston

$\theta = 0$

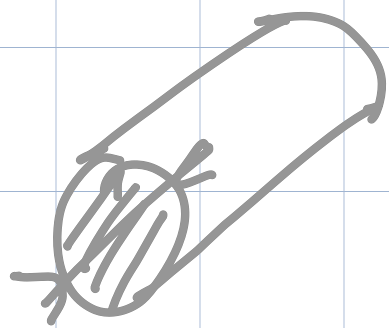


$\theta = \pi$

$Course = C = L+e-(L-e) = 2e$

Cylindrée : $cyl = C \times S$ ←

S : surface du piston



ϕD

$$S = \pi R^2 = \pi \frac{D^2}{4}$$

$$R = \frac{D}{2}$$

$$Cyl = 2e \times \frac{\pi D^2}{4} \quad mm^3/tr$$

<u>Debit</u> Q (m^3/s) (l/s) (l/h)

$$Q = \overbrace{\underset{cm^3/tr}{Cyl} \times \underset{tr/min}{N_{10}}} \quad cm^3/min$$

N_{10} tr/min $\qquad \omega_{10} = \frac{2\pi N_{10}}{60}$

$$N_{10} = 2000\ \text{tr}/\text{min}$$

$$\omega_{10} = \frac{2\pi}{60} 2000 \simeq 200\ \text{rad/s}$$

$$Q = 2 \times 2 \times \frac{\pi 2^2}{4} \times 2000$$

$$e = 20\ \text{mm} = 2\ \text{cm}$$

$$D = 2\ \text{cm}$$

$$= 8000\pi \simeq 24000\ \text{cm}^3/\text{min}$$

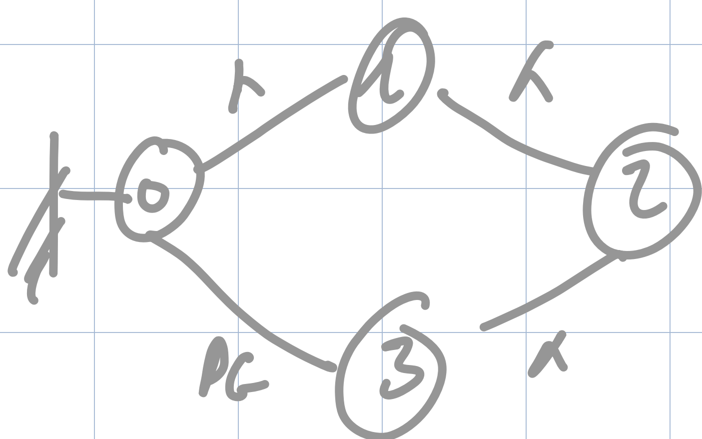
$= 60 \times 24000 \ cm^3/h$

$= 1440000 \ cm^3/h$

$= 1440 \ l/h$

$1 m^3 = 1000 \ l$

Approche cinematique



fermeture de chaîne cinématique

$V(0/1) + V(1/2) + V(2/3) + V(3/0) = \{\vec{0}, \vec{0}\}$

$\Updownarrow \qquad \Updownarrow \qquad \Updownarrow$

$V(3/0)_\alpha = V(3/2)_\alpha + V(2/1)_\alpha + V(1/0)$

$V(1/0) = \{\theta \vec{z}_{1(0)}, \vec{0}\}_{O}$ $\quad \vec{v}(O,1/0) = \vec{0}$

$V(2/1) = \{\omega_{21} \vec{z}_{1(1)}, \vec{0}\}_{A}$ $\quad \vec{v}(A, 2/1) = \vec{0}$

$$\begin{array}{c} \\ b_{2/1} \end{array} \left| \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ \omega_{21} & 0 \end{array} \right|$$

$V(3/2) = \{\dot{\beta} \vec{z}_{2(3)}, \vec{0}\}_{B}$ $\quad \alpha$

$V(3/0) = \{\vec{0}, V_{x_{30}} \vec{x}_0\}_{\forall P}$ $\quad \alpha$

Si P.G. $\quad V(3/0) = \{\underbrace{\omega_{x_{30}} \vec{x}_0}, \dot{d} \vec{x}_0\}_{B,O,C}$

$\vec{\Omega}(3/0) = \vec{\Omega}(3/2) + \vec{\Omega}(2/1) + \vec{\Omega}(1/0)$

$\omega_{x_{30}} \vec{x}_0 \qquad \dot{\beta}$

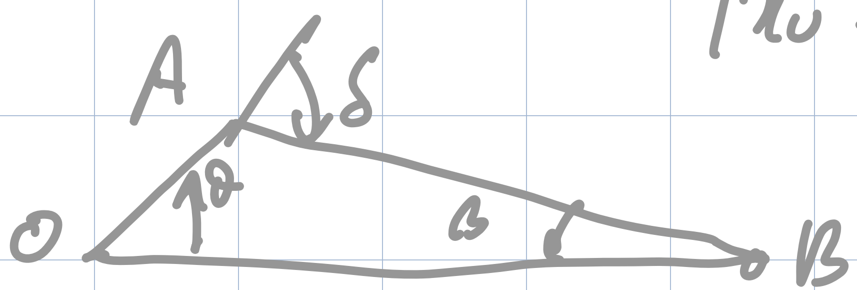
$$\vec{0} = \dot{\beta}\,\vec{z} + \dot{\theta}\,\vec{z} + \omega_{z\,2/3}\,\vec{z}$$

$$/\vec{z} : \quad \dot{\beta} + \dot{\theta} + \dot{\delta} = 0$$

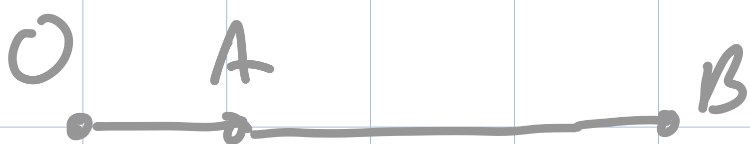
$\int dt \longrightarrow$

$$\underline{\beta + \theta + \delta = K}$$

$$/\vec{x_0} : \quad \omega_{x_0} = 0$$



$\theta = 0$



$\beta = 0$ et $\delta = 0 \implies K = 0$

$$\beta + \theta + \delta = 0$$

$$\vec{V}(B, 3/0) = \cancel{\vec{V}(B, 3/2)}^{\,0} + \underline{\vec{V}(B, 2/1)} + \underline{\vec{V}(B, 1/0)}$$

$$\dot{d}\,\vec{x}_0 = \cancel{\vec{V}(A,2/1)}_{0} + \vec{BA}\wedge\vec{\Omega}(2/1)$$

$$+\ \cancel{\vec{V}(O\in 1/0)}_{0} + \vec{BO}\wedge\vec{\Omega}(1/0)$$

$$\dot{d}\,\vec{x}_0 = \left(\vec{BA}\wedge\dot{s}\,\vec{3}\right) + \left(\vec{BO}\wedge\dot{\theta}\,\vec{3}\right)$$

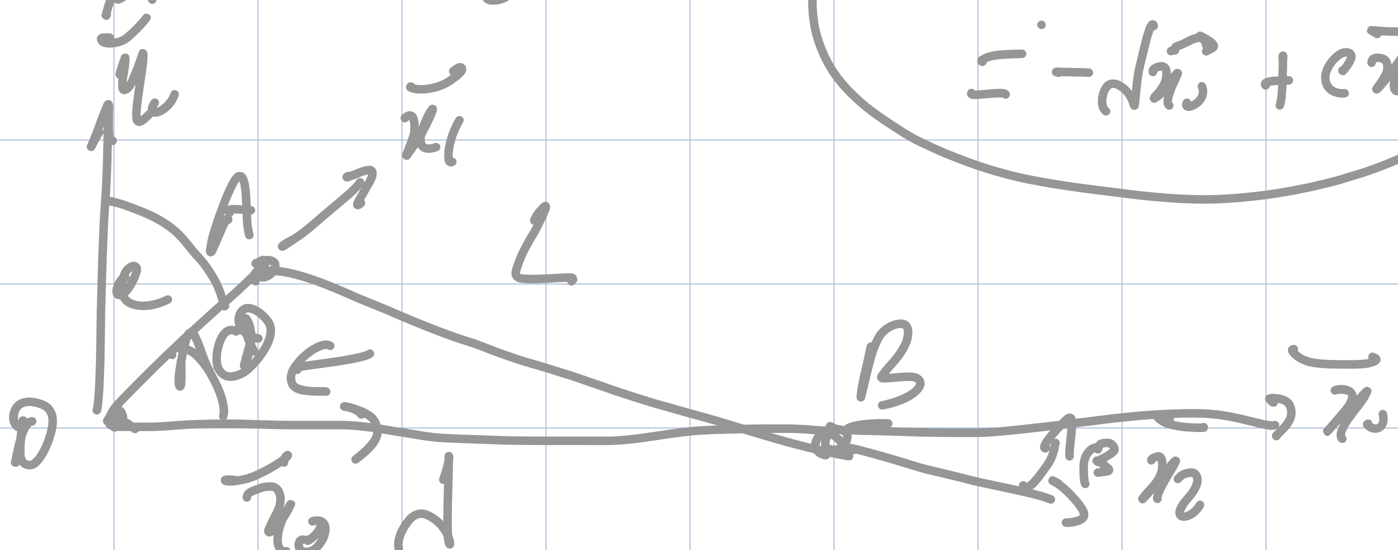
$$\dot{d} = f(\dot{\theta})$$

$$\dot{d}\,\vec{x}_0\cdot\vec{u} = \cancel{\left(\vec{BA}\wedge\dot{s}\,\vec{3}\right)\vec{u}} + \left(\vec{BO}\wedge\dot{\theta}\,\vec{3}\right)\vec{u}$$

$$\dot{d}\,\vec{x}_0\cdot\underbrace{\vec{BA}}_{-L\vec{x}_2} = \left(\vec{BO}\wedge\dot{\theta}\,\vec{3}\right)\cdot\underbrace{\vec{BA}}_{(\cancel{\vec{BO}}+\vec{OA})}$$

$$\vec{BA} = -L\,\vec{x}_2$$

$$\vec{BA} = \vec{BO} + \vec{OA} = -d\,\vec{x}_0 + e\,\vec{x}_1$$



$$\dot{d}\,\vec{x}_0 \cdot \left[-d\vec{x}_0 + e\vec{x}_1\right] = \left(-\dot{d}\vec{x}_0,\ \dot{\theta}\vec{y}_0\right)\cdot\left(-d\vec{x}_0 + e\vec{x}_1\right)$$

$$-d\dot{d}\,\underbrace{\cancel{\vec{x}_0\cdot\vec{x}_0}}_{1} + e\dot{d}\,\underbrace{\vec{x}_0\cdot\vec{x}_1}_{\cos\theta} = d\dot{\theta}\,\vec{y}_0\cdot\left(-\cancel{d\vec{x}_0} + e\vec{x}_1\right)$$

$$= de\dot{\theta}\,\underbrace{\vec{x}_1\cdot\vec{y}_0}_{\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta}$$

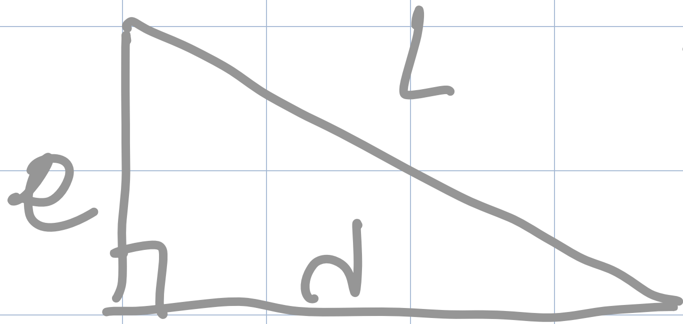
$$\boxed{d\dot{d}\ \underbrace{-\,e\dot{d}\cos\theta + ed\dot{\theta}\sin\theta}\ = 0}$$

$\int dt$

$$\hookrightarrow\ \frac{d^2}{2} - ed\cos\theta = K$$

$$d^2 - 2ed\cos\theta = K' = 2K$$

$\theta = \pi/2 \quad K' = d^2 = L^2 - e^2$



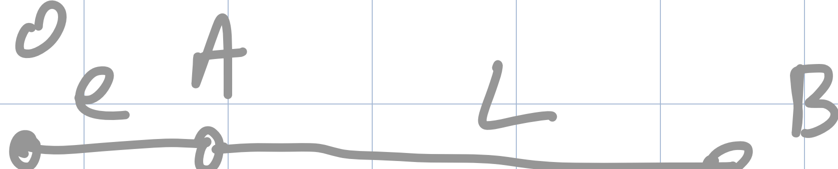
$d^2 = L^2 - e^2$

$\theta = \frac{\pi}{2}$

$d^2 - 2ed\cos\theta = L^2 - e^2$

$$L^2 = d^2 + e^2 - 2ed\cos\theta$$

$\theta = 0 \quad d^2 - 2ed\,\overset{1}{\cancel{\cos\theta}} = K'$



$$d = e + L$$

$$(e+L)^2 - 2e(e+L) = K'$$

$$e^2 + L^2 + \cancel{2eL} - 2e^2 - \cancel{2eL} = K'$$

$$K' = L^2 - e^2$$

$$\vec{V}(O, 3/0) = \vec{V}(O, 3/2) + \vec{V}(O, 2/1) + \underbrace{\cancel{\vec{V}(O, 1/0)}}_{\vec{0}}$$

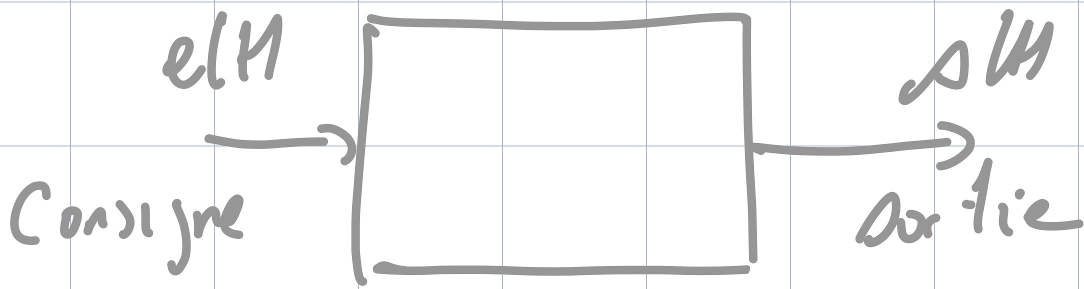
$$\overset{o}{d}\vec{x}_0 = \cancel{\vec{V}(B, 3/2)} + \vec{OB} \wedge \vec{\Omega}(3/2) + \cancel{\vec{V}(A, 2/1)} + \vec{OA} \wedge \vec{\Omega}(2/1)$$

$$\overset{o}{d}\vec{x}_0 = (\vec{OB} \wedge \dot{\beta}\,\vec{z}_3) + (\vec{OA} \wedge \dot{\delta}\,\vec{z}_3)$$

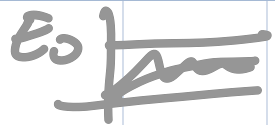
$\rightarrow A$?

SLCi

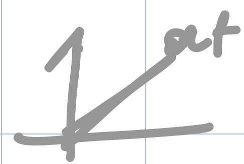
Analyse fréquentielle



analyse temporelle : $e(t) = E_0$

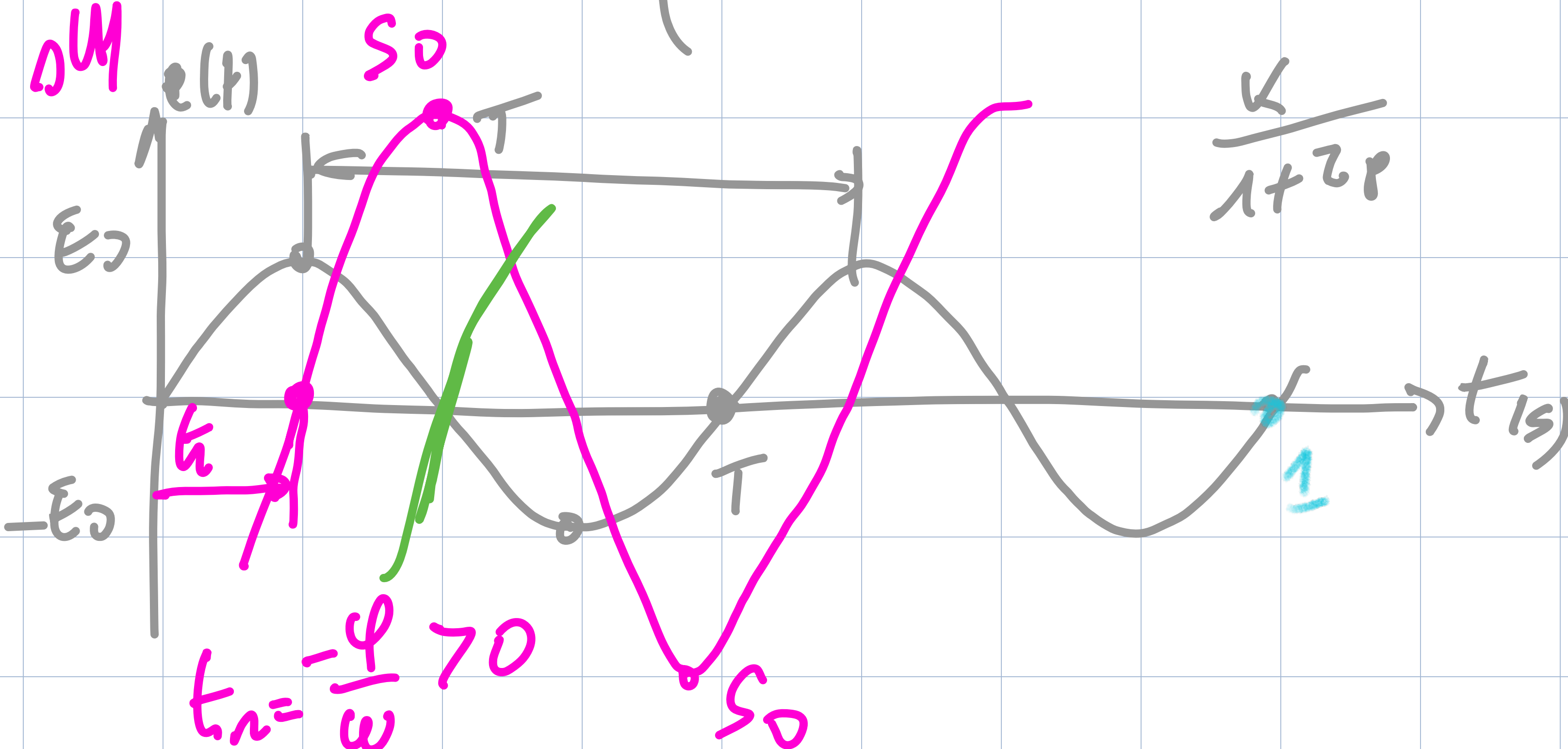


$e(t) = at$



$e(t) = E_0 \sin(\omega t)$ $\qquad E_0 \; [e]$

$\frac{K}{1+\tau p}$

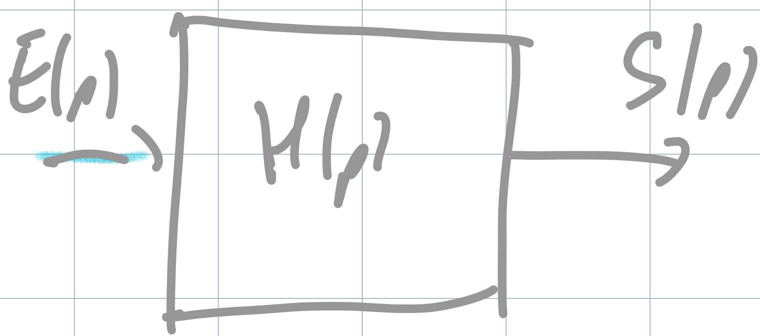


$t_1 = \frac{-\varphi}{\omega} > 0$

E_0 : amplitude

ω : pulsation (rad/s)

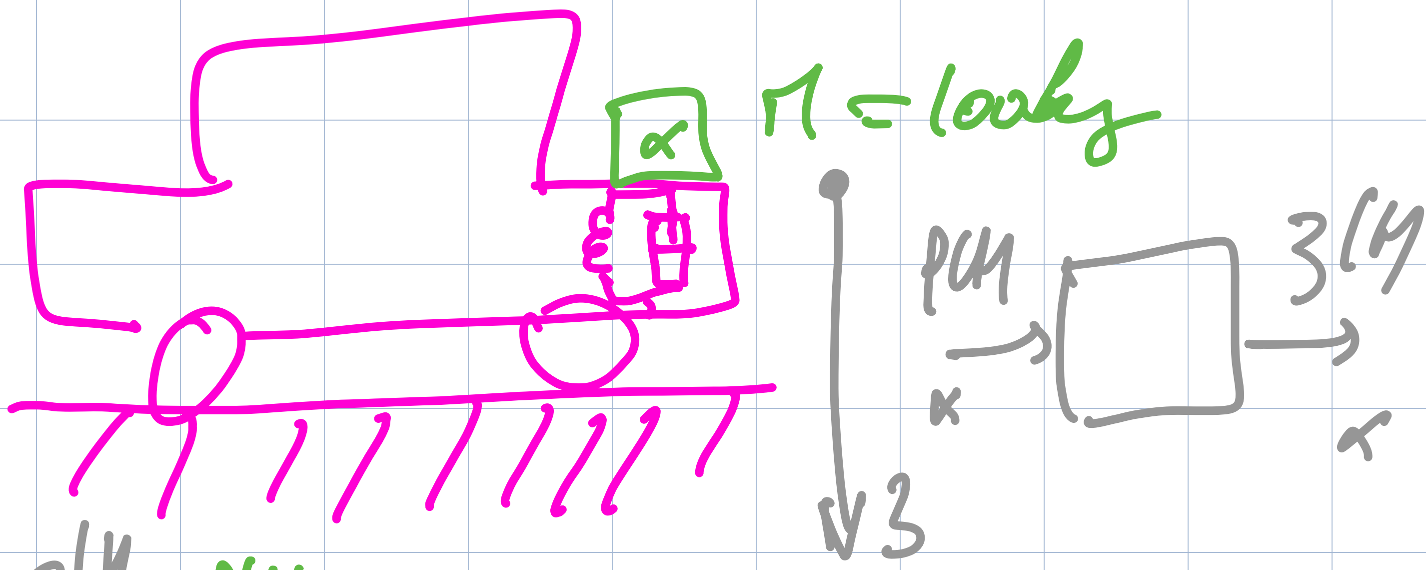
$f = \frac{1}{T}$ s^{-1} ou Hz $f = 2$ $T = 0{,}5$ s

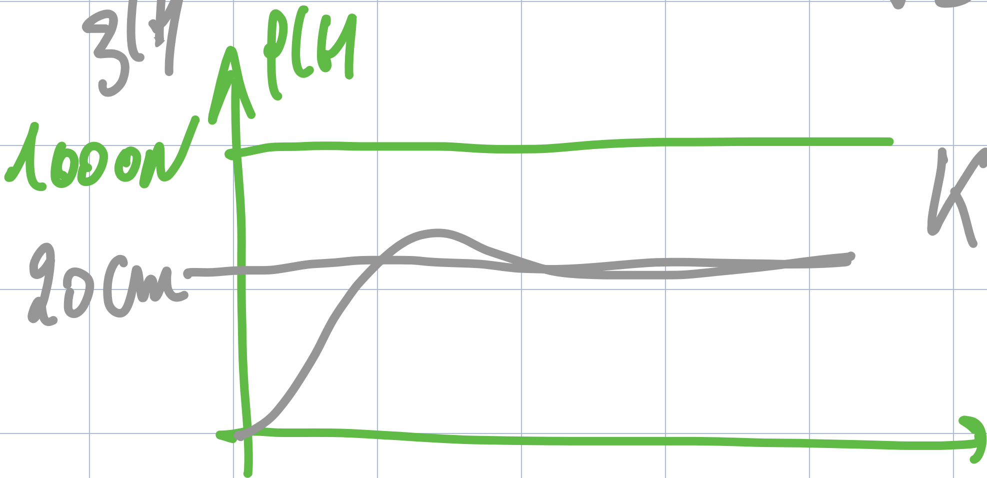


Si S.L.C.I : $s(t) = S_0 \sin(\omega t + \varphi)$

$\varphi < 0$

Gain: $G = \frac{S_0}{E_0}$





$$K = \frac{20}{1000} = 0{,}02 \text{ cm/N}$$

Gain : $G(w) = \frac{S_0}{E_0} = |H(jw)|$

Phase : $\varphi(w) = Arg(H(jw))$

Dephasage

$$E(p) \rightarrow \boxed{H(p)} \rightarrow S(p)$$

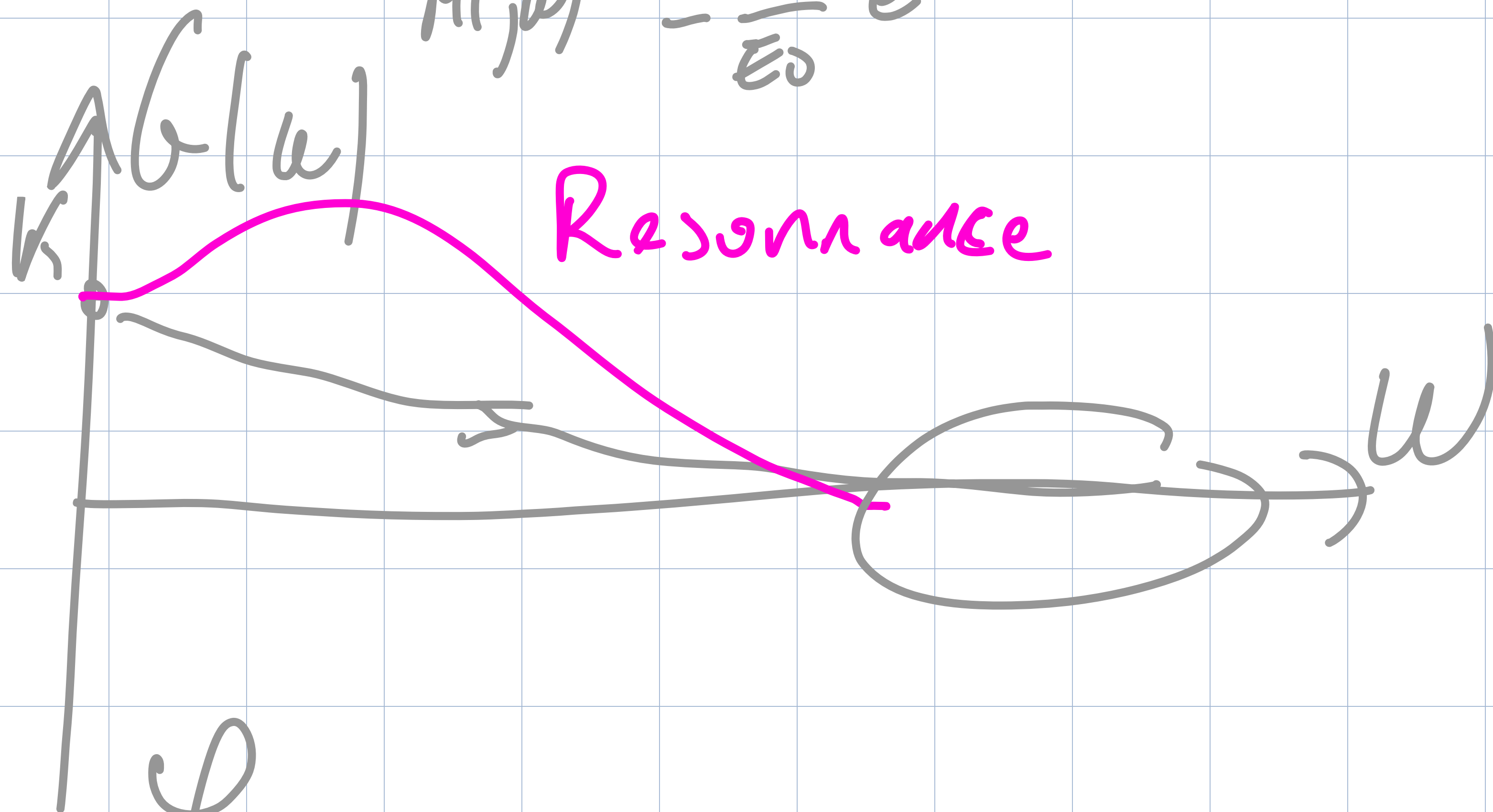
$$S(p) = H(p)\,E(p)$$

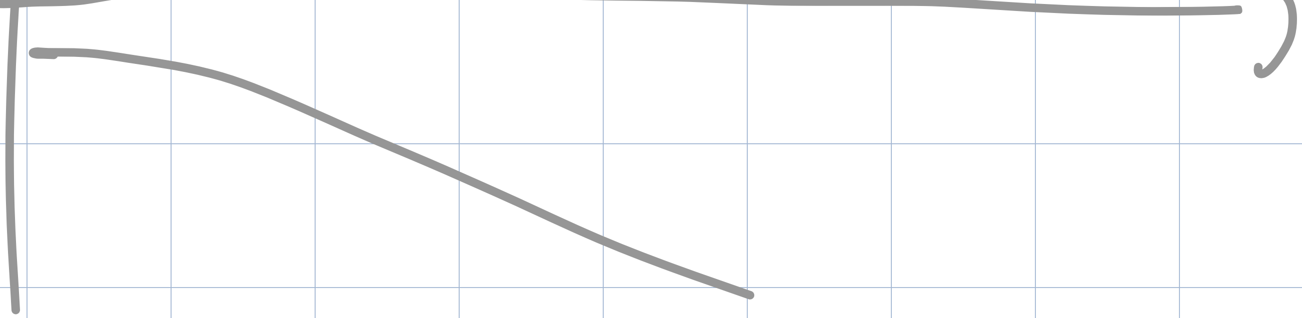
$$H(p) = \frac{S(p)}{E(p)}$$

$$p = j\omega$$

$$H(j\omega) = \frac{S(j\omega)}{E(j\omega)} = \frac{S_0 e^{j(\omega t + \varphi)}}{E_0 e^{j\omega t}}$$

$$H(j\omega) = \frac{S_0}{E_0} e^{j\varphi}$$





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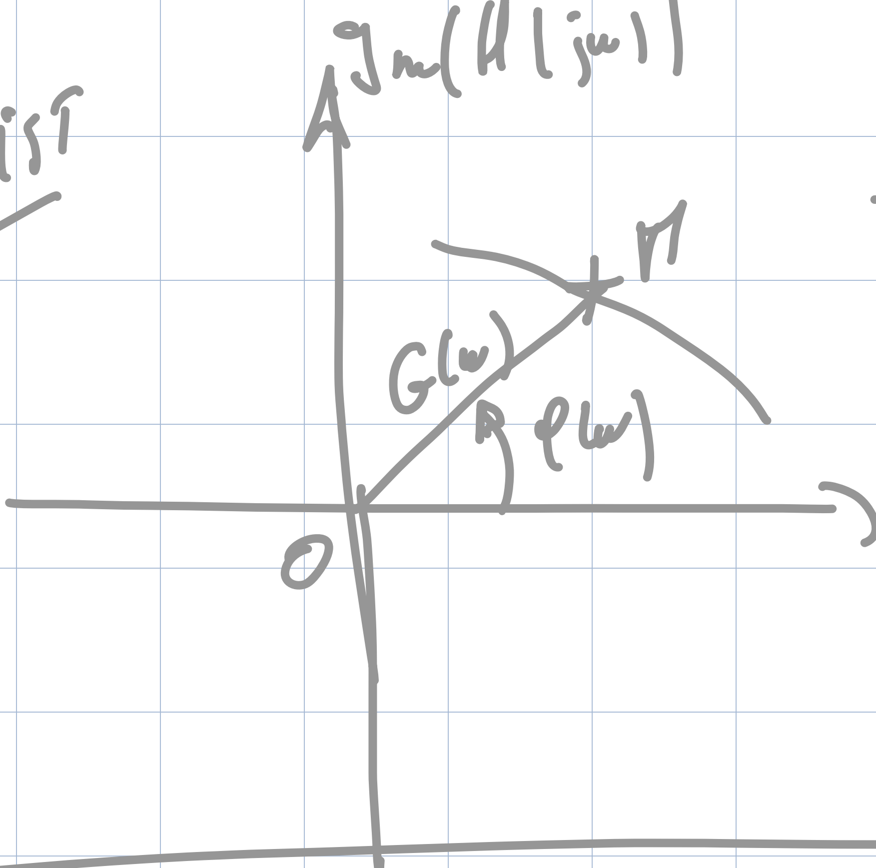

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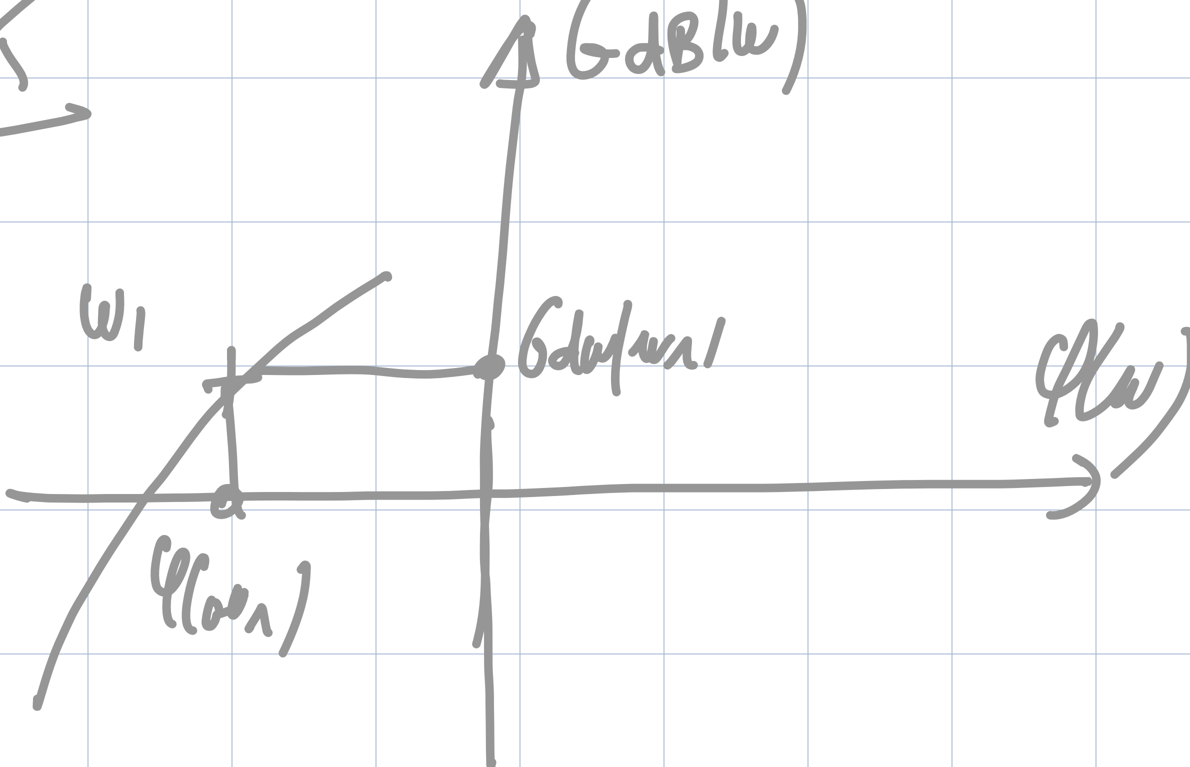
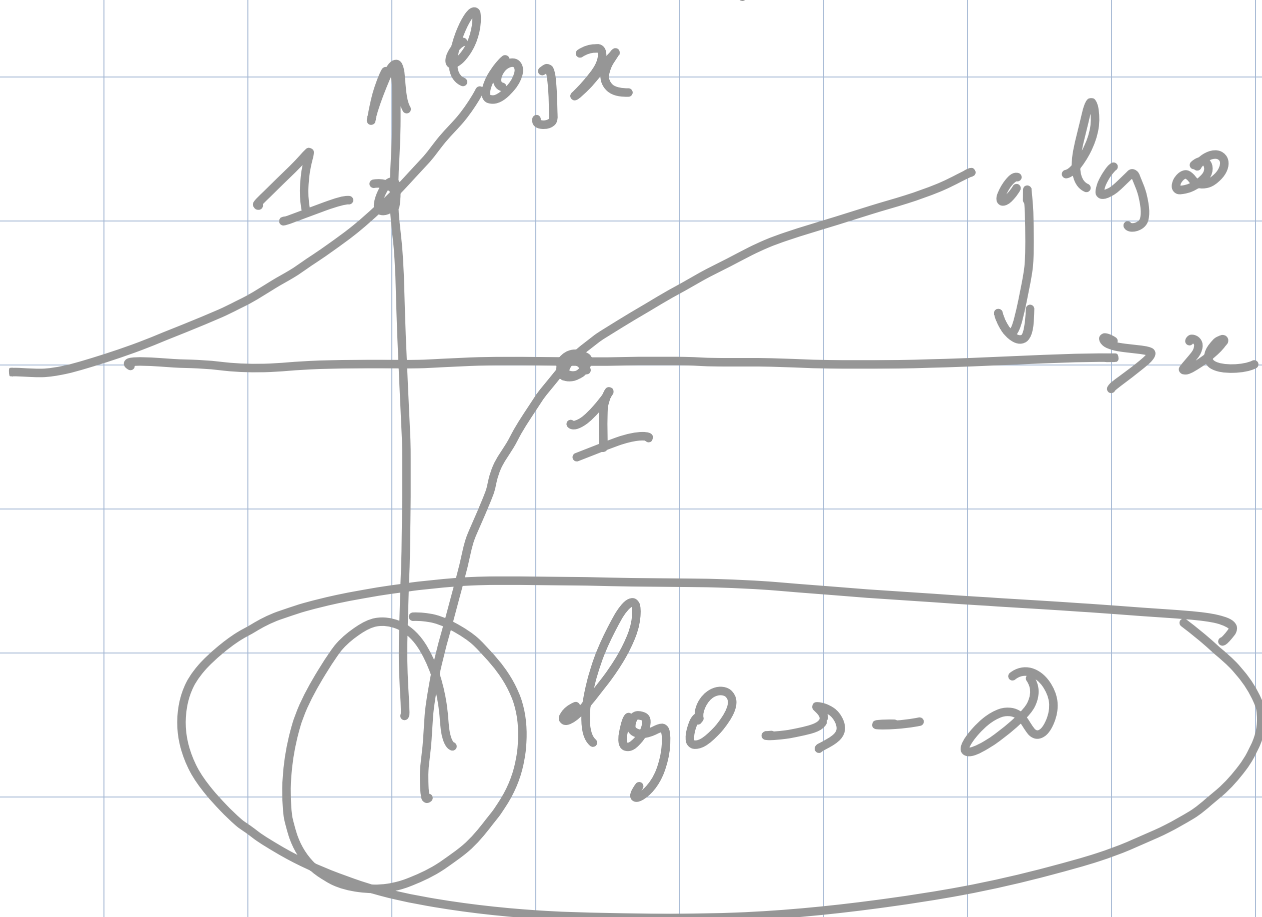


$x_{dB} = 20 \log_{(10)} x \qquad (\ln)$

$\frac{x_{dB}}{20} = \log x \Rightarrow x = 10^{x_{dB}/20}$

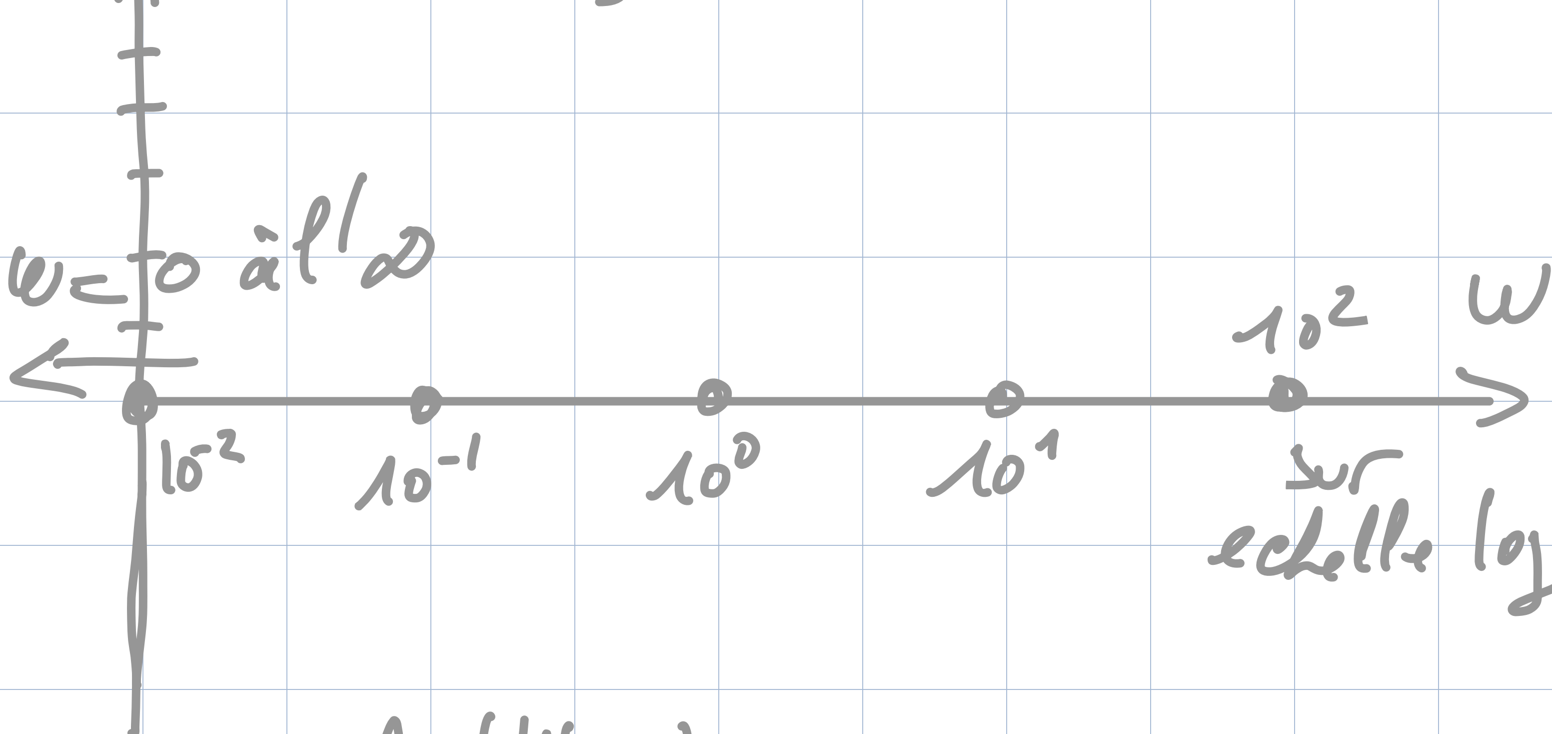
$G_{dB}(\omega) = 20 \log(G(\omega)) = 20 \log |H(j\omega)|$



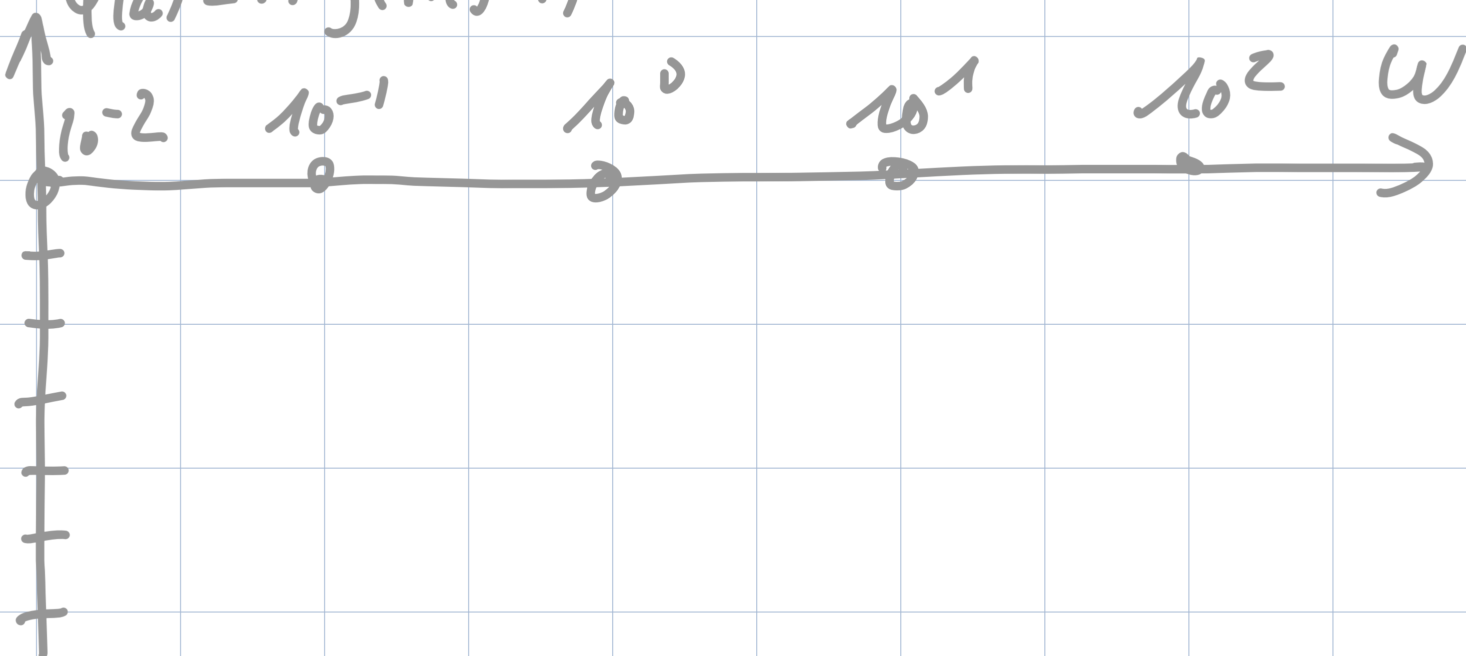
$G(\omega) \to 0 \Rightarrow G_{dB}(\omega) \to -\infty$

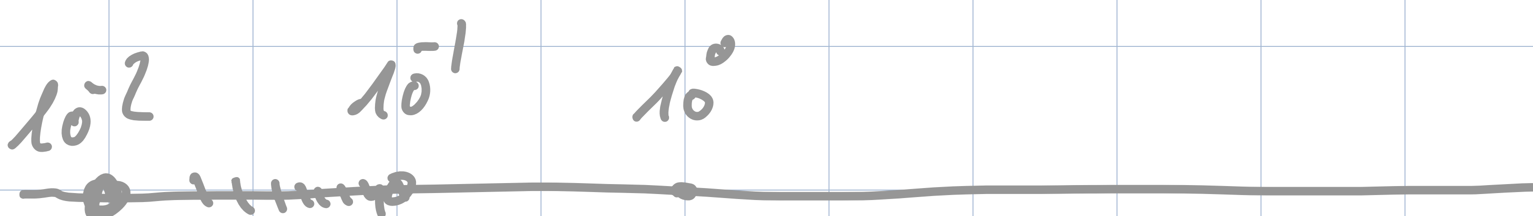
BODE

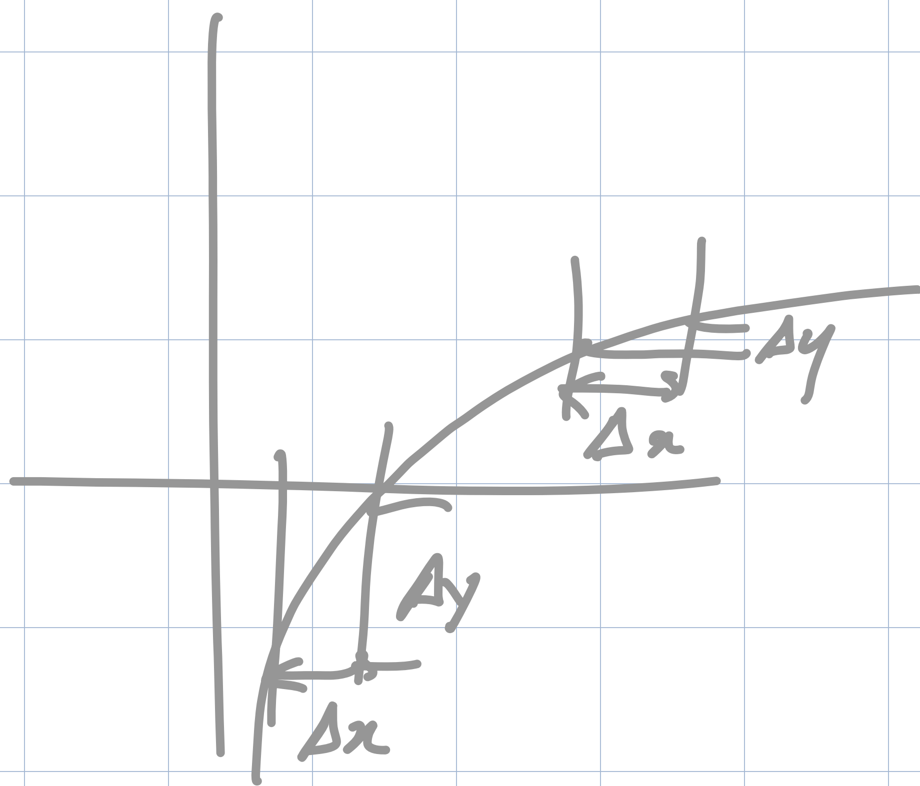
$G_{dB}(\omega) = 20\log|H(j\omega)|$

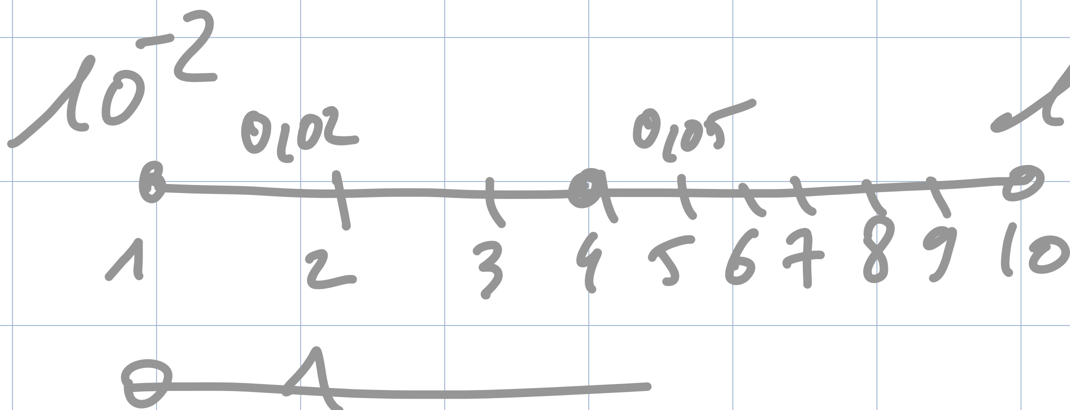


$\varphi(\omega) = Arg(H(j\omega))$






Δy
Δx
Δy
Δx


10^-2
0,02
0,05
10^-1
1
2
3
4
5
6
7
8
9
10