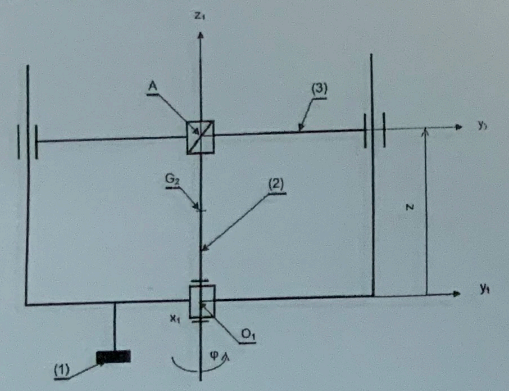
Colle: Énergétique (TEC)
ÉLÉVATEUR

1 Présentation

Un élévateur est constitué d'un socle 1, d'un coulisseau 3 et d'un mécanisme d'entraînement par la vis 2, l'écrou en A est solidaire du coulisseau. La position du coulisseau est repéré par $O_1A = z$ et la position angulaire de la vis par l'angle φ.

On suppose dans un premier temps que les liaisons sont sans frottement.



On précise que $\vec{g} = -g.\vec{z_1}$ et que la liaison hélicoïdale possède un pas à droite. Un couple moteur C_m s'exerce en O_1 sur la vis 2. Un solide, masse m_4, est lié au coulisseau.

? Problématique
L'inertie de ce rotor étant inconnue, cet appareillage permet de la déterminer expérimentalement en mesurant la période de ses petites oscillations autour de sa position d'équilibre.

2 Travail demandé

Question 1 Donner le graphe des liaisons du mécanisme. On placera l'ensemble des notions nécessaires sur celui-ci.

Question 2 Déterminer la liaison équivalente entre 3 et 1. Quel est le degré d'hyperstatisme de cette liaison ?

Question 3 Proposer la(les) matrice(s) d'inertie du(des) solide(s) « intéressante(s) ».

Question 4 Réaliser la figure de changement de base.

Question 5 Donner la relation de type entrée sortie de ce mécanisme, relation liant $\dot{z}$ et φ.

Question 6 Calculer $T_{(2/1)}$ et $T_{(3/1)}$.

Question 7 En déduire l'inertie équivalente du système ramené sur l'arbre moteur.

Question 8 Déterminer les expressions des puissances s'exerçant sur l'ensemble mobile.

Question 9 En déduire la relation liant le couple moteur à l'accélération du coulisseau.

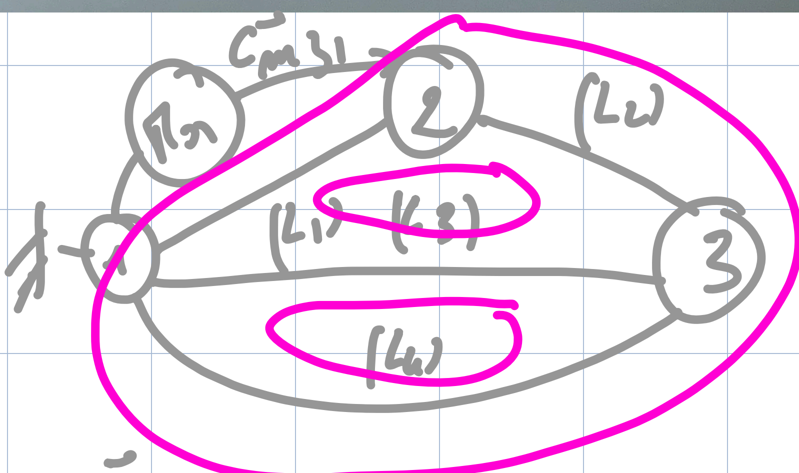
Pour aller plus loin...

On suppose que les liaisons entre 3 et 2 sont avec frottement de coefficient f.

Question 10 Déterminer le nouveau couple moteur.

De plus, la liaison hélicoïdale est avec frottement, du même coefficient.

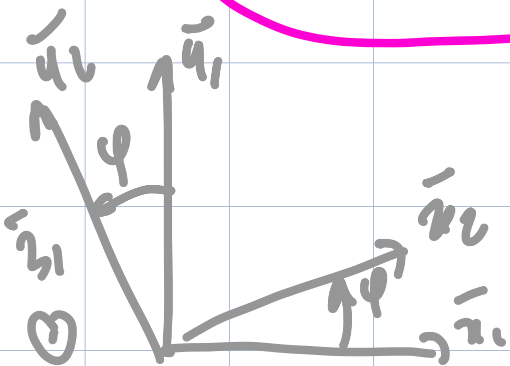
Question 11 Déterminer le nouveau couple moteur.



$\gamma = l - m = 4 - 2$

$= l - (p-1)$

$= 4 - (3-1)$



$N_{c1} = 1$

$\mathcal{V}(2/1) = \left\{ \dot{\varphi}\,\vec{z}_1 \; , \; \vec{0} \right\}_{O_1,A}$

$\mathcal{S}(1\to 2) = \begin{vmatrix} X_{12} & L_{12} \\ Y_{12} & M_{12} \\ Z_{12} & 0 \end{vmatrix}_{O_1,(A)}$

$(-,-,\vec{z_1})$

$N_{s1} = 5$

$$\mathcal{V}(3|2| = \left\{ \omega_{232}\,\vec{3}_{31},\ V_{232}\,\vec{3} \right\}_A \qquad N_{c_2} = 1$$

$$V_{232} = + \frac{P}{2\pi}\,\omega_{232}$$

$$\mathcal{F}(2\to3) = \begin{vmatrix} X_{23} & L_{23} \\ Y_{23} & M_{23} \\ Z_{23} & N_{23} \end{vmatrix}_A \qquad N_{S_2} = 5$$

$(-1-, \vec{3}_{2}|3)$

$$L_{23} = -\frac{P}{2\pi} X_{23} \quad -$$

$$\mathcal{V}_3(3|1| = \left\{ \omega^{(B)}_{231}\,\vec{3}_1,\ V_{B31}\,\vec{3}_1 \right\}_B$$

$N_{c3} = 2$

$$\mathcal{V}_4(3|1| = \left\{ \omega^{(C)}_{231}\,\vec{3}_1,\ V_{C31}\,\vec{3}_1 \right\}_C$$

$N_{c4} = 2$

$$\mathcal{F}_3(1\to3) = \begin{vmatrix} X_B & L_B \\ Y_B & M_B \\ 0 & 0 \end{vmatrix}_B \qquad N_{S_3} = 4$$

$(-1-, \vec{3}_1)$

$| X_C \quad L_C |$

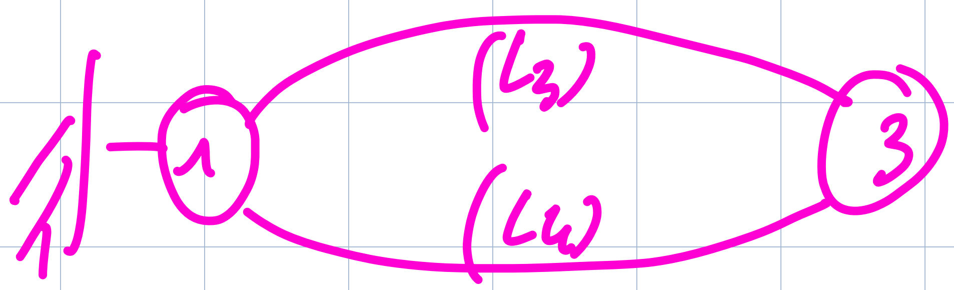
$$\mathcal{T}_4 (1 \to 3) = \begin{Bmatrix} X_C & L_C \\ Y_C & M_C \\ 0 & 0 \end{Bmatrix}_{C \, (-,-,\vec{z}_1)} \qquad N_{S_4} = 4$$

$$N_c = 2 + 2 + 1 + 1 = 6$$

$$N_s = 4 + 4 + 5 + 5 = 18$$

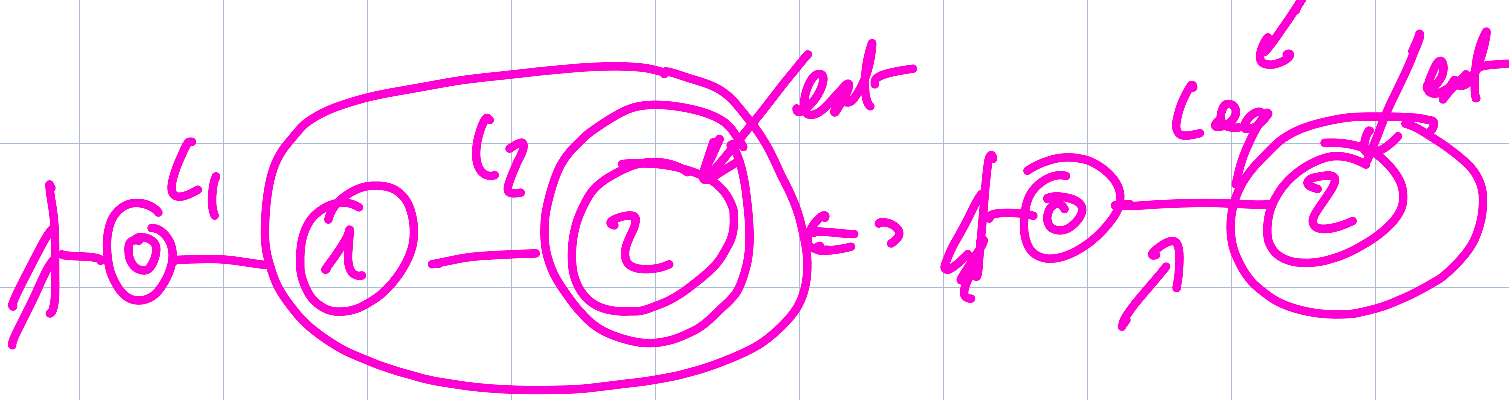
$$E_s = 6n = 12$$

hyperstatisme : $h = N_s - r_s$



$\Leftrightarrow$

Leq : glissière de direction $\vec{z}_1$



$v(L_{eq}) = v(2(s))$

$v(L_{eq}) = v(2(1)) + v(1(s))$

$v(L_{eq}) = v(L_1) + v(L_2)$ → L_{eq} ans

$F(\bar{2} \to 2) = \{\vec{0}, \vec{0}\}$
$F(ext \to 2) + F(L_{eq}) = \{\vec{0}, \vec{0}\}$

$F(\bar{2} \to 2) = \{\vec{0}, \vec{0}\}$

$F(1 \to 2) + F(ext \to 2) = \{\vec{0}, \vec{0}\}$

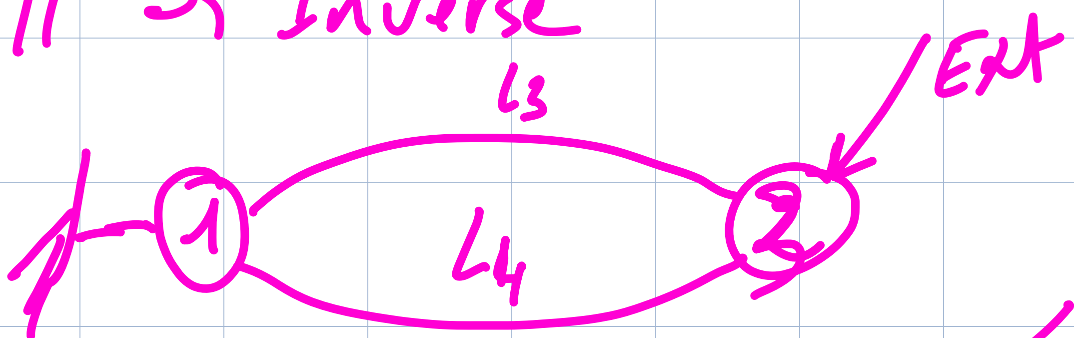
$F(\overline{1+2} \to 1+2) = \{\vec{0}, \vec{0}\}$

$\mathfrak{F}(\partial \to 1) + \mathfrak{F}(ext \to 1) = \{\vec{0} + \vec{0}\}$

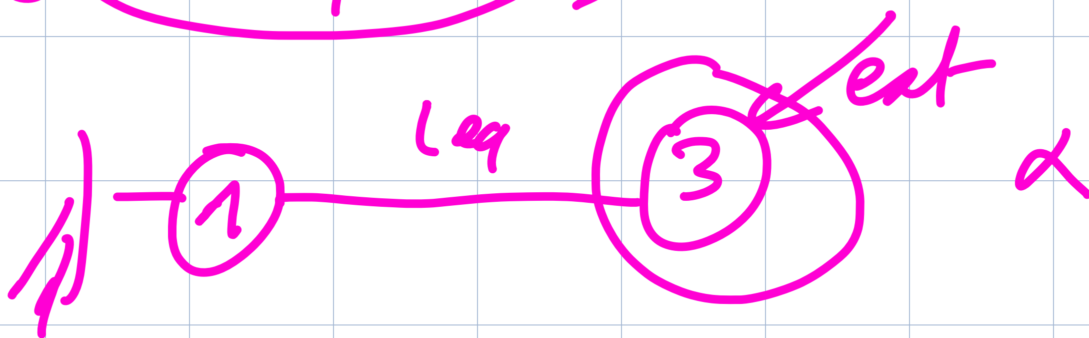
$\mathfrak{F}(L_{eq}) = \mathfrak{F}(L_1) = \mathfrak{F}(L_2)$

$6\ell \ eq^{us}$

$\|$ → Inverse



$\Leftrightarrow$



$\mathfrak{F}(\bar{3} \to 3) = \{\vec{0}, \vec{0}\}$

$\mathfrak{F}(ext \to 3) + \mathfrak{F}(L_{eq}) = \{\vec{0}, \vec{0}\}$

$\mathfrak{F}(ext \to 3) + \mathfrak{F}(L_3) + \mathfrak{F}(L_4) = \{\vec{0}, \vec{0}\}$

$$\mathcal{F}(L_{eq}) = \mathcal{F}(L_3) + \mathcal{F}(L_4)$$

6 eq^{ns}

$$\mathcal{V}(L_{eq}) = \mathcal{V}(3/1) = \mathcal{V}(L_3) = \mathcal{V}(L_4)$$

$\rightarrow$ 6 eq^{ns}

<u>Etude Statique</u> : $\mathcal{F}(L_{eq})_{0\to 3} = \begin{vmatrix} X_{eq} & L_{eq} \\ Y_{eq} & M_{eq} \\ Z_{eq} & N_{eq} \end{vmatrix}_B$

$$\mathcal{F}_3(0\to 3) = \begin{vmatrix} X_B & L_B \\ Y_B & M_B \\ 0 & 0 \end{vmatrix}_B$$

$$\mathcal{F}_4(0\to 3) = \begin{vmatrix} X_C & L_C \\ Y_C & M_C \\ 0 & 0 \end{vmatrix}_C \overset{!}{\equiv} \begin{vmatrix} X_C & L_C \\ Y_C & M_C \\ 0 & aY_C \end{vmatrix}_B$$

$$\vec{M}_4(B, 0\to 3) = M_4(C, 0\to 3) + \vec{BC} \wedge \vec{R}_4(0\to 3)$$

$$= \begin{vmatrix} L_C \\ M_C \\ \end{vmatrix} + \begin{vmatrix} a \\ 0 \\ \end{vmatrix} \wedge \begin{vmatrix} X_C \\ Y_C \\ \end{vmatrix} = \begin{vmatrix} L_C \\ M_C \\ aY_C \end{vmatrix}$$

$$\mathcal{F}(L_{eq}) = \mathcal{F}(L_3) + \mathcal{F}(L_4)$$

$$X_{eq} = X_C + X_B$$
$$Y_{eq} = Y_C + Y_B$$
$$Z_{eq} = 0$$
$$L_{eq} = L_C + L_B$$
$$M_{eq} = M_C + M_B$$
$$N_{eq} = a'/c$$

$$\left\{ \begin{matrix} X_{eq} & L_{eq} \\ Y_{eq} & M_{eq} \\ 0 & N_{eq} \end{matrix} \right\}_B$$

glissière de direction $\vec{z}_1$

$$N_S = 4 + 4 = 8$$

$X_C, X_D, Y_C, Y_D, L_C, L_D, M_C, M_D$

$$X_C + X_B = X_{eq} \quad x \longrightarrow X$$

$$Y_C + Y_B = Y_{eq} \longrightarrow Y_B$$

$$\cancel{0 = Z_{eq}}$$

$$L_C + L_B = l_{eq} \quad x \longrightarrow L$$

$$M_C + M_B = M_{eq} \quad x \longrightarrow M$$

$$a\, Y_C = N_{eq} \longrightarrow Y_C = \frac{N_{eq}}{a}$$

5 eq^{ns} à 8 inconnues

15 = 5

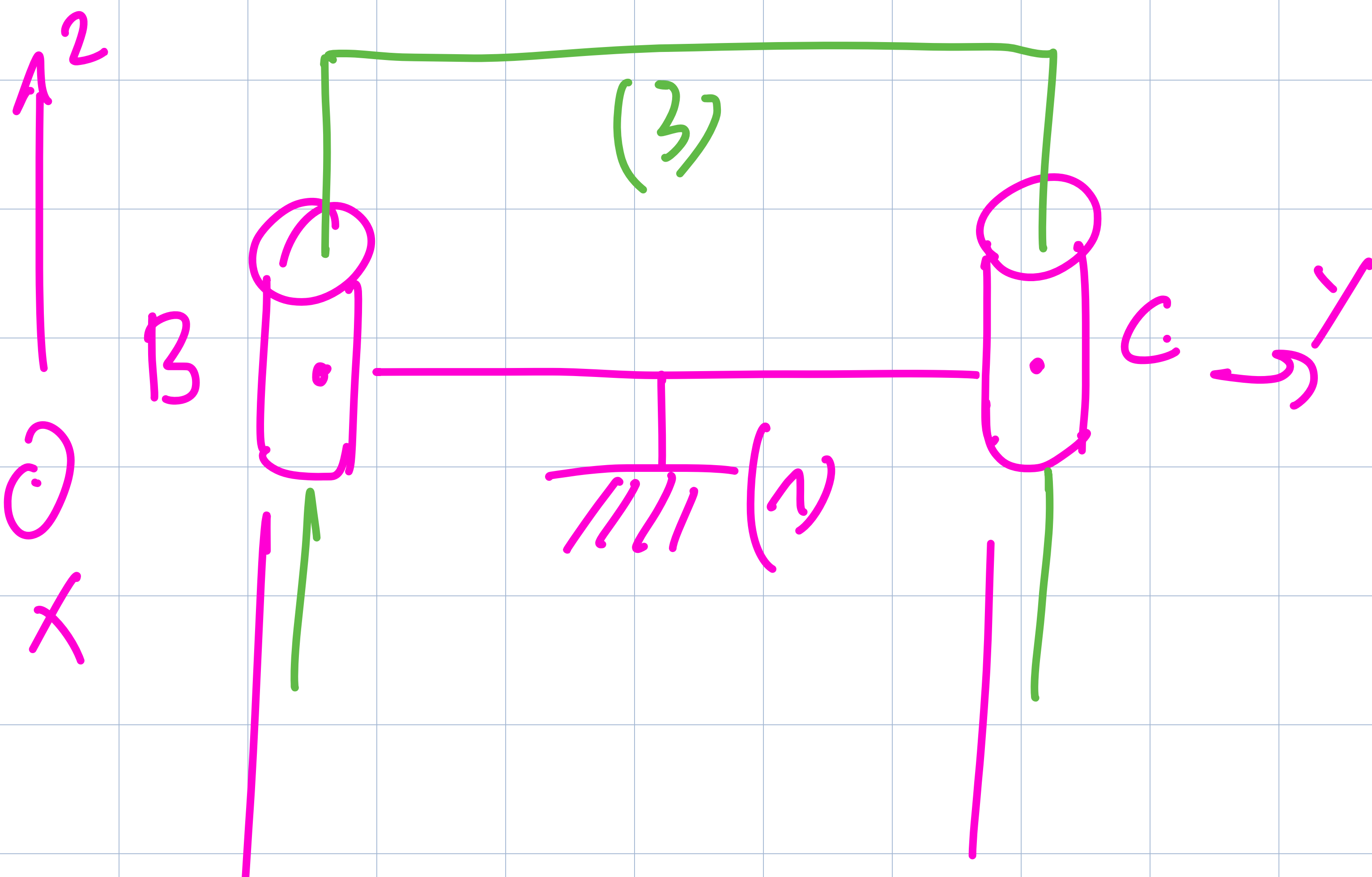
$F_x \quad M_x \quad M_y$

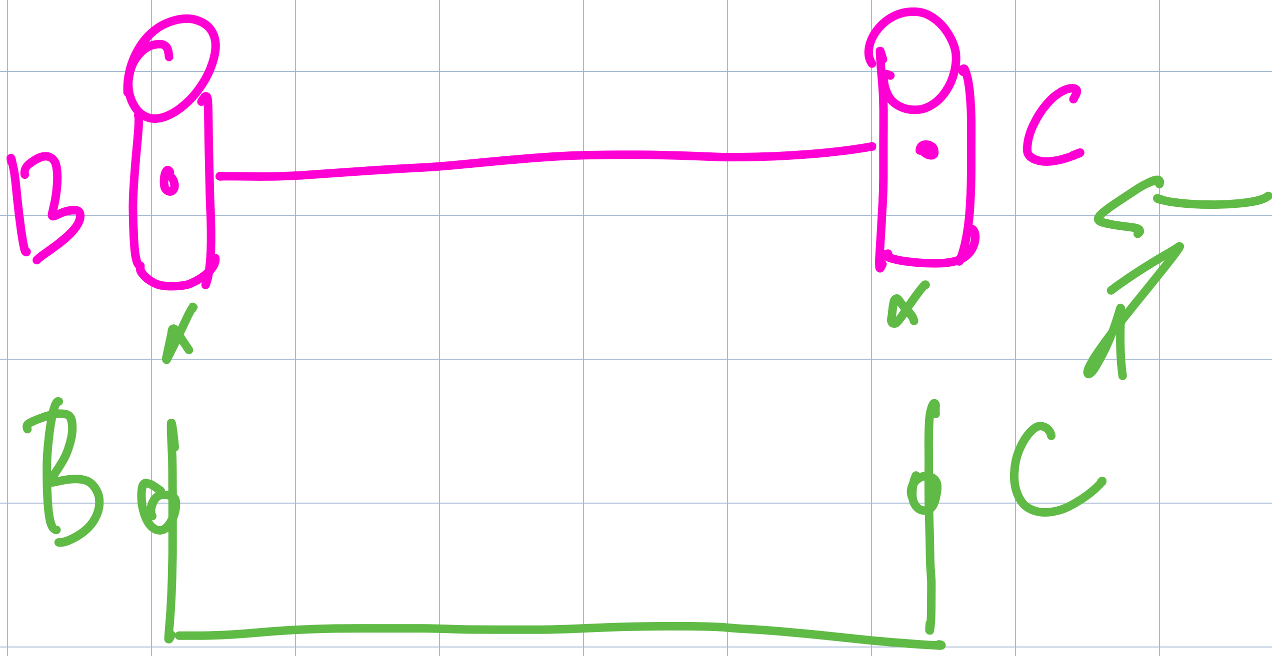
3 inconnues hyperstatiques X, L, M

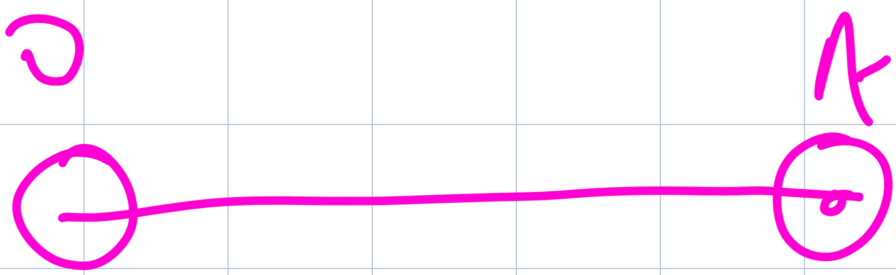
$h = 8 - 5 = 3$

↓ N_S ↓ I_S

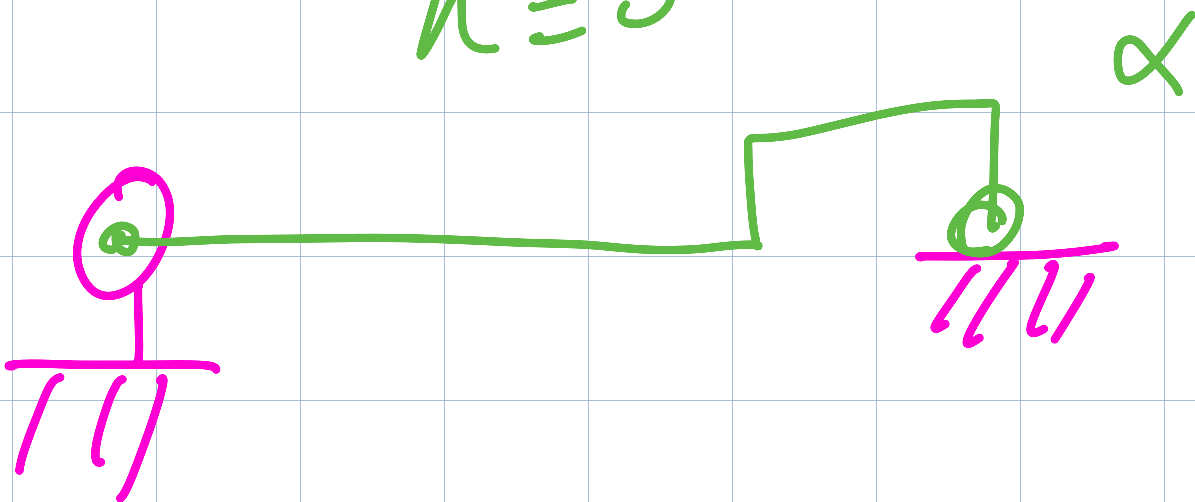
$h = 0 \rightarrow$ ISOSTATIQUE







$h = 0$



$$\mathcal{V}(L_{eq}) = \begin{vmatrix} \omega_{x\,eq} & V_{x\,eq} \\ \omega_{y\,eq} & V_{y\,eq} \\ \omega_{z\,eq} & V_{z\,eq} \end{vmatrix}_{B\,\alpha}$$

$$\mathcal{V}(L_3) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ \omega_{z_B} & V_{z_B} \end{vmatrix}_{B\,\alpha}$$

$$\mathcal{V}(L_4) = \begin{vmatrix} 0 & 0 \\ 0 & 0 \\ \omega_{z_C} & V_{z_C} \end{vmatrix}_{C} = \begin{vmatrix} 0 & 0 \\ 0 & -a\omega_{z_C} \\ \omega_{z_C} & V_{z_C} \end{vmatrix}_{B}$$

$$\mathcal{V}(L_{eq}) = \mathcal{V}(L_3) \qquad \mathcal{V}(L_{eq}) \underset{\text{en } B}{=} \mathcal{V}(L_4)$$

$$\boxed{\omega_{x\,eq} = 0} \qquad \omega_{x\,eq} = 0$$

$$\boxed{\omega_{y\,eq} = 0} \qquad \omega_{y\,eq} = 0$$

$\omega_{zeq} = \omega_{z_B}$

$\boxed{V_{xeq} = 0}$

$\boxed{V_{yeq} = 0}$

$V_{zeq} = V_{z_B}$

$\omega_{zeq} = \omega_{z_C} = 0$

$V_{xeq} = 0$

$\boxed{V_{yeq} = -a\omega_{z_C}}$

$V_{zeq} = V_{z_C}$

$$\vec{BC} \wedge \vec{\Omega} = \begin{vmatrix} a \\ 0 \\ 0 \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ \omega_{z_C} \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 \\ 0 & 0 \\ 0 & V_{zeq} \end{vmatrix}_B$$

donc glissière

$\omega_{xeq} = \boxed{0 = 0} \to R_x$

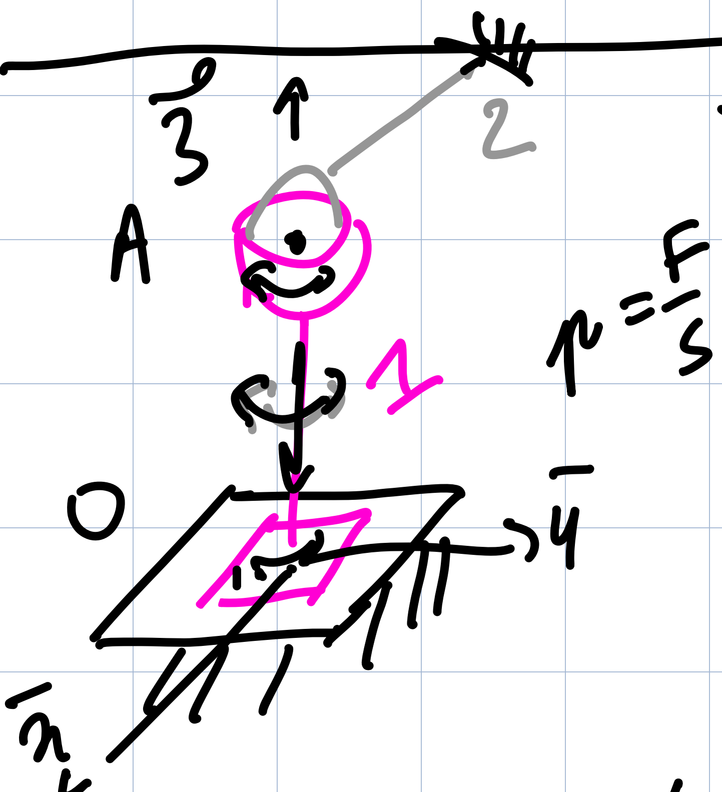
$\omega_{yeq} = \boxed{0 = 0} \to R_y$

$$\omega_{z\,eq} = \omega_{2_B} = \omega_{2_C} = 0$$

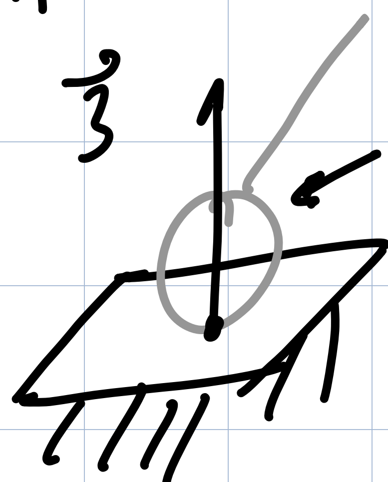
$$V_{x\,eq} = \boxed{0 = 0} \to T_x \swarrow)$$

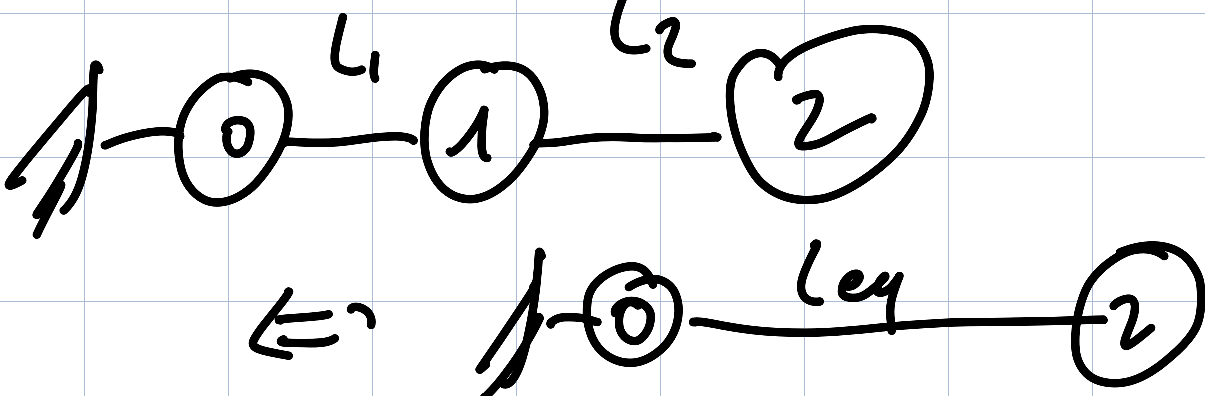
$$V_{y\,eq} = 0 = -a\,\omega_{2_C}$$

$$V_{z\,eq} = V_{2_B} = V_{2_C}$$



$OA = a$





$$\vartheta(C_{eq})_{(b)} = \begin{vmatrix} \omega_{xeq} & v_{xeq} \\ \omega_{yeq} & v_{yeq} \\ \omega_{zeq} & v_{zeq} \end{vmatrix}_A$$

$$\vartheta(L_1) = \vartheta(1/0) = \begin{vmatrix} 0 & v_{x10} \\ 0 & v_{y10} \\ \omega_{z10} & 0 \end{vmatrix}_{0,A} \qquad N_{C1} = 3$$

$$\vartheta(L_2) = \vartheta(2/1) = \begin{vmatrix} \omega_{x21} & 0 \\ \omega_{y21} & 0 \\ \omega_{z21} & 0 \end{vmatrix}_A \qquad N_{C2} = 3$$

$$\vartheta(C_{eq}) = \vartheta(L_2) + \vartheta(L_1)$$

$$\omega_{xeq} = \omega_{x21}$$
$$\omega_{yeq} = \omega_{y21}$$

$$\omega_{zeq} = \omega_{z21} + \omega_{z10} \rightarrow m_c$$

$$V_{xeq} = V_{x10}$$

$$V_{yeq} = V_{y10}$$

$$\boxed{V_{zeq} = 0}$$

$$N_{ceq} = 5$$

$$\mathcal{V}_{eq} = \left| \begin{matrix} \omega_{xeq} & V_{xeq} \\ \omega_{yeq} & V_{yeq} \\ \omega_{zeq} & 0 \end{matrix} \right|_A$$

(b)

S/P de normale $(A\vec{z})$

$$N_C = 6$$

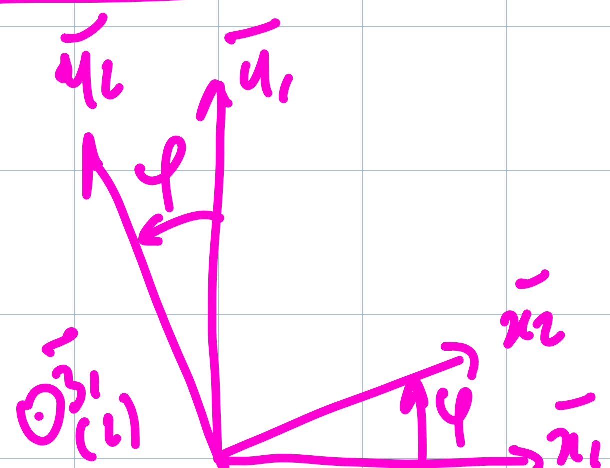
6 5 + 1

$$m = \underbrace{m_u}_{} + \underbrace{m_i}_{}$$

$$m_{cinématique} = \sum NC_i$$

$$I(2, O_1) = \begin{bmatrix} A_2 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & C_2 \end{bmatrix}_{R_2} \qquad (A_2 = B_2)$$

2 : Solide de Révolution autour de $(O_1, \vec{z}_1)$



$$\vec{\Omega}(2/1) = \dot{\varphi}\, \vec{z}_{1(1)}$$

- $\{\mathcal{V}(2/1)\} = \left\{ \dot{\varphi}\, \vec{z}_{1(1)} \,,\, \vec{0} \right\}_{O_1, A}$
- $\{\mathcal{V}(3/1)\} = \left\{ \vec{0} \,,\, \dot{z}\, \vec{z}_1 \right\}_{\forall P}$

$\propto \mathcal{V}(3/2) = \left\{ \begin{matrix} \omega_{21} \vec{z}_1 \\ \end{matrix} , \; V_{21} \vec{z}_1 \right\}_{A, \vec{z}_1} \propto$

$$V_{21} = \overset{(d)}{+} \frac{p}{2\pi} \, \omega_{21}$$

Fermeture de chaîne Cinématique

$\mathcal{V}(1/2) + \mathcal{V}(2/3) + \mathcal{V}(3/1) = \{ \vec{0} , \vec{0} \}$

$\mathcal{V}(3/1) = \mathcal{V}(3/2) + \mathcal{V}(2/1)$

$\propto \; \vec{\Omega}(3/1) = \vec{\Omega}(3/2) + \vec{\Omega}(2/1)$

$\vec{0} = \omega_{232} \vec{z}_1 + \dot{\varphi} \vec{z}_1$

$/ \vec{z}_1 \Rightarrow \omega_{232} = -\dot{\varphi}$

$\downarrow \; \vec{v}(A, 3/1) = \vec{v}(A, 3/2) + \overset{\vec{0}}{\cancel{\vec{v}(A, 2/1)}}$

$\dot{z} \vec{z}_1 = V_{232} \vec{z}_1$

$/ \vec{z}_1 \rightarrow V_{232} = \dot{z}$

$$V_{Z3_2} = + \frac{P}{2\pi} \omega_{Z3_2} \Rightarrow \dot{z} = -\frac{P}{2\pi} \dot{\varphi}$$

$$T(2/1) = \text{"}\frac{1}{2} J \omega^2\text{"} = \frac{1}{2} C_2 \dot{\varphi}^2$$

$$T(3/1) = \text{"}\frac{1}{2} m v^2\text{"} = \frac{1}{2}(m_3 + m_4)\dot{z}^2$$

$$E = (2+3)$$

$$T(E/1) = T(2/1) + T(3/1)$$

$$T(E/1) = \frac{1}{2} C_2 \dot{\varphi}^2 + \frac{1}{2}(m_3 + m_4)\dot{z}^2$$

$$= \frac{1}{2}\Big[\underbrace{C_2 + (m_3 + m_4)\left(\frac{-P}{2\pi}\right)^2}\Big]\dot{\varphi}^2$$

$$= \frac{1}{2} J_{eq}(E \to 2)\,\dot{\varphi}^2$$

$$T(E/1) = \frac{1}{2}\,\underbrace{\left[C_2\left(-\frac{2\pi}{p}\right)^2 + (m_3+m_4)\right]}\,\dot{z}^2$$

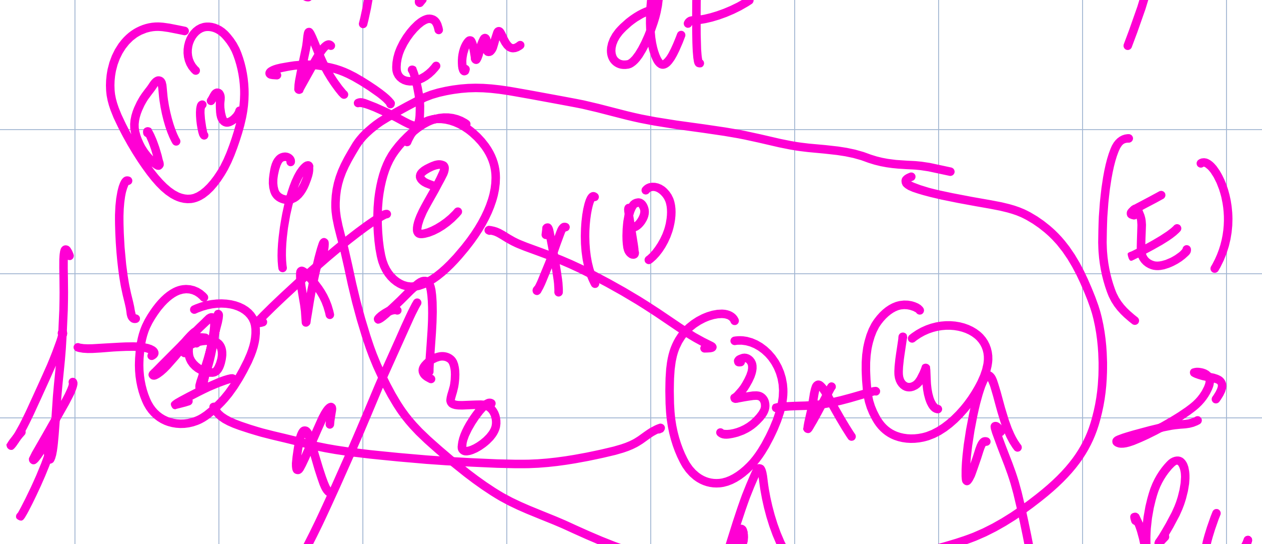
$$T(E/1) = \frac{1}{2}\, m_{eq}(E)\,\dot{z}^2 \leftarrow$$

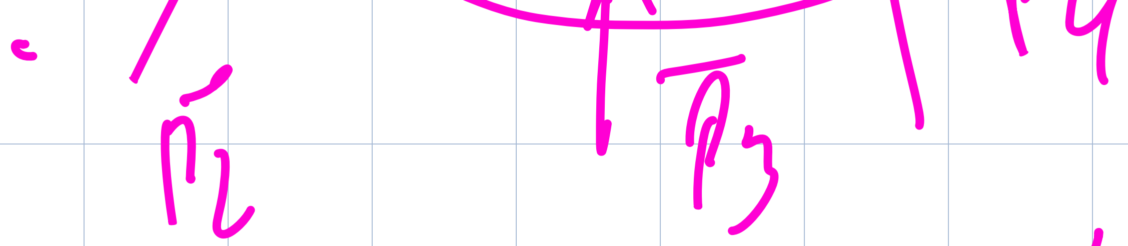
$$\frac{dT(E/1)}{dt} = J_{eq}(E\to 2)\,\dot{\varphi}\,\ddot{\varphi} \quad \alpha$$

$$= m_{eq}(E)\,\dot{z}\,\ddot{z} \quad \alpha$$

$$\dot{z} = -\frac{p}{2\pi}\dot{\varphi} \rightarrow \ddot{z} = -\frac{p}{2\pi}\ddot{\varphi}$$

T.E.C. a (E) : $\frac{dT(E/1)}{dt} = P_{ext}(E/1) + P_{int}(E)$





$$P_{ext}(E/1) = P(pes \to E/1) + \underbrace{\cancel{P(1 \leftrightarrow 2)}}_{\substack{0 \\ L.P.}} + \underbrace{\cancel{P(1 \leftrightarrow 3)}}_{\substack{0 \\ L.P.}} + \underbrace{P_m}_{P(\Pi_{12} \to 2/1)}$$

$$P_m = P(\Pi_{12} \to 2/1) = C_m \, \omega_m = C_m \, \dot{\varphi}$$

$$P(pes \to E/1) = P(pes \to 2/1) + P(pes \to 3+4/1)$$

$$P(pes \to 2/1) = \vec{P}_2 \cdot \vec{V}(G_2, 2/1) = 0$$

$$= -m_2 g \, \vec{z} \cdot \vec{0}$$

$G_2 \in (O, \vec{z}_1)$

$$P(pes \to 3+4/1) = (\vec{P}_3 + \vec{P}_4) \cdot \vec{V}(G_3, 3/1)$$

$$= -(m_3 + m_4) g \vec{z} \cdot \dot{z} \vec{z}$$

$$= -(m_3 + m_4) g \dot{z} \quad \alpha \quad \text{"PV"}$$

1) $\dot{z} > 0 \Rightarrow \quad < 0$

2) $\dot{z} < 0 \Rightarrow \quad \geq 0$

$$P_{int}(E) = P(2 \leftrightarrow 3) = 0 \text{ si L.P.}$$

T.E.C. $\bar{a}(E)$: $m_{eq} \dot{z} \ddot{z} = C_m \dot{\varphi} - (m_3 + m_4) g \dot{z}$

$$m_{eq} \cancel{\dot{z}} \ddot{z} = C_m \left(\frac{-2\pi}{p} \cancel{\dot{z}} \right) - (m_3 + m_4) g \cancel{\dot{z}}$$

$$C_m = -\left[m_{eq}\ddot{\theta} + (m_3+m_4)g\right]\frac{p}{2\pi}$$

Si $\ddot{\theta} > 0$ $\quad C_m < 0$

Si $\ddot{\theta} < 0$ $\quad C_m > 0$

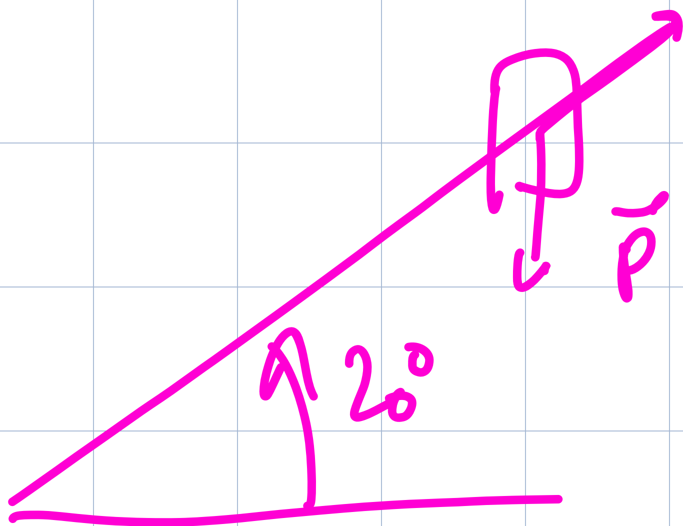
$\eta \rightarrow$ pertes $\quad (1-\eta)P_m$

$$m_{eq}\dot{\theta}\ddot{\theta} = P_m - \text{pertes} - (m_3+m_4)g\dot{\theta}$$

$$P_m - \underbrace{(1-\eta)P_m}$$

$$m_{eq}\dot{\theta}\ddot{\theta} = \underbrace{\eta P_m} - (m_3+m_4)g\dot{\theta}$$

$$C_m = -\frac{1}{\eta}\left(m_{eq}\ddot{\theta} + (m_3+m_4)g\right)\frac{p}{2\pi}$$



$$\vec{P}\cdot\vec{v} = -PV\sin\alpha$$

$$P = 40000\,N \qquad V = 0{,}5\,m/s$$

$$-PV\sin\alpha = -40000 \times 0{,}5 \times \sin 20^\circ$$

$$= -20000\,\sin 20^\circ =$$

$$= -6840\,W$$

$$\cancel{m_{eq}\, v\, \dot{v}}^{\,0} = \eta\, P_m - 6840$$

$$\simeq 0{,}6$$

$$P_m = \frac{2 \times 6840}{7000} \simeq 14000\,W$$

$$= 14\,kW$$