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Spiral path of a spinning coin

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Abstract

If a coin is spun on its edge about a vertical axis on a horizontal surface it will follow a spiral path of decreasing radius and eventually come to a stop when the coin falls onto the surface. An experimental result is described and compared with theoretical calculations.

Keywords: angular momentum, torque, precession

Supplementary material for this article is available online

1. Introduction

The last few seconds of a spinning coin or disk has been studied by many authors as an interesting example of precession [1, 2]. The first few seconds while a coin is still upright is rarely mentioned but it is just as interesting because the coin remains upright instead of falling over [3]. While the coin is upright, it spins like a top, apart from the fact that it follows a spiral path instead of being anchored to a fixed spot on the surface.

It is not immediately obvious why a spinning coin would spiral radially inwards while it is spinning. A first guess is that it might behave like a ball that spins about a vertical axis on a horizontal surface. The ball can spin on a fixed spot or it can travel along a straight line path if it is also given a gentle horizontal push. However, a coin is different since it can lean slightly away from a vertical position. If it does so, then a torque will arise which would cause it to fall over if it was not spinning. In that respect, it behaves in the same way as a spinning top. A spinning top does not fall over if it leans away from the vertical. Instead, a rapidly spinning top rotates slowly around a vertical axis through its bottom end. The effect is known as precession and arises because the torque acting on the top acts to change the direction of its angular momentum vector rather than changing its magnitude.

The same thing happens with a spinning coin, but the coin precesses about a remote vertical axis instead of precessing about a fixed vertical axis through its bottom end. The bottom end of a top is usually a sharp point and it cannot roll away from its fixed position. The bottom end of a coin is round and it can roll around the horizontal surface in a circular or spiral path. A typical result is shown in supplementary video Spinning coin.mov, filmed in slow motion at

300 frames s^{-1} on a smooth horizontal surface. Similar observations of spiral motion are obtained if a coin rolls on its edge on a horizontal surface about a horizontal axis [4, 5], but that effect was not investigated in the present paper.

The coin used in the video was an Australian 20 cent coin of mass $M = 11.3$ g, diameter $2a = 28.65$ mm and thickness 2.50 mm. It had a slightly rough, milled edge. A coin with a smooth edge, or a smooth toroidal ring, spirals radially inwards at a slower rate than the 20 cent coin used in the present experiment, and spins for a longer time before it falls, due to the decrease in the rolling friction force on the coin. A spinning top with a round bottom end also rolls along a spiral path, as shown in supplementary video Top.mov. The top continued to spiral inwards for 80 s, whereas the 20 cent coin spiralled inwards for only 4 s before falling.

2. Theory

The moment of inertia for rotation about the vertical axis is $I = Ma^2/4 = 2.32 \times 10^{-6}$ kg m^2. If it spins about that axis at angular velocity ω then its angular momentum is $I\omega$. Suppose that the coin is slightly tilted and leans at an angle θ to the vertical, as shown in figure 1. Suppose also that the centre of mass rotates in a circular orbit of radius R at angular velocity Ω about a remote vertical axis, as observed experimentally. The normal force acting at the bottom of the coin is $N = Mg$ and there is also a centripetal friction force $F = Mv^2/R$ acting at the bottom of the coin where $v = R\Omega$ is the horizontal velocity of the centre of mass.

The normal force exerts a torque $Na\sin\theta$ about the centre of mass, and the friction force exerts a torque $Fa\cos\theta$ in the opposite direction. The total torque is $\tau = Na\sin\theta - Fa\cos\theta$ in a clockwise sense. The horizontal component of the angular momentum is $L_X = I\omega\sin\theta$ and the change ΔL_X as the coin rotates about the remote axis is shown in figure 2. In a small time dt, L_X rotates through an angle Ωdt and the change in the vector L_X is $\Delta L_X = L_X\Omega \mathrm{d}t$. L_X does not change in magnitude but it rotates in the Y direction (into the page in figure 1) since that is the torque direction.

Since the net torque equals the rate of change of angular momentum,

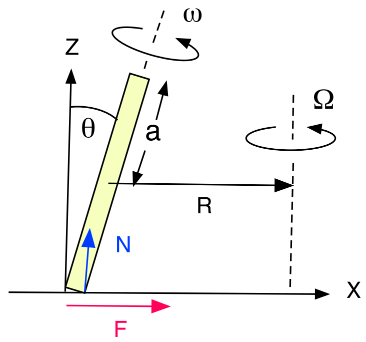

Figure 1. A spinning coin viewed edge-on in the XZ plane.

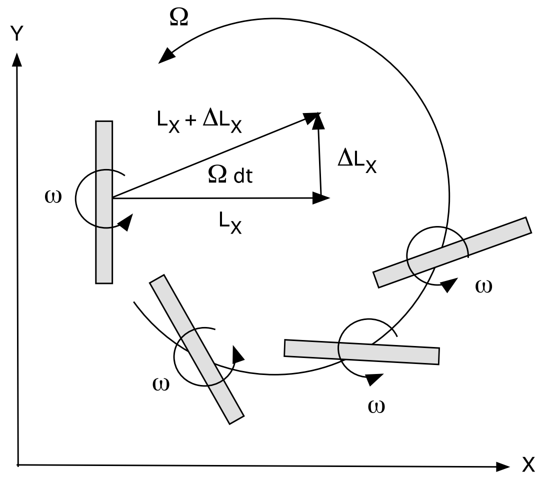

Figure 2. Rotation of L_X in the horizontal (XY) plane, viewed from above.

$$Mga\sin\theta - Fa\cos\theta = \frac{\Delta L_X}{\mathrm{d}t} = \Omega L_X = I\omega\Omega\sin\theta. \quad (1)$$

The precession frequency, Ω, is therefore given from equation (1) by

$$\left(\frac{v}{\tan\theta} + \frac{a\omega}{4}\right)\Omega = g, \quad (2)$$

where $v = R\Omega$ is the horizontal speed of the coin.

Solutions of equation (2) are shown in figure 3 as a function of θ for the 20 cent coin when $v = 0.1$ or 0.2 ms^{-1} and $\omega = 40$ or 80 rad s^{-1}. In all cases, the torque due to N is only slightly larger than the opposing torque due to F. The solutions depend primarily on the tilt angle, θ, and the horizontal launch speed of the coin rather than

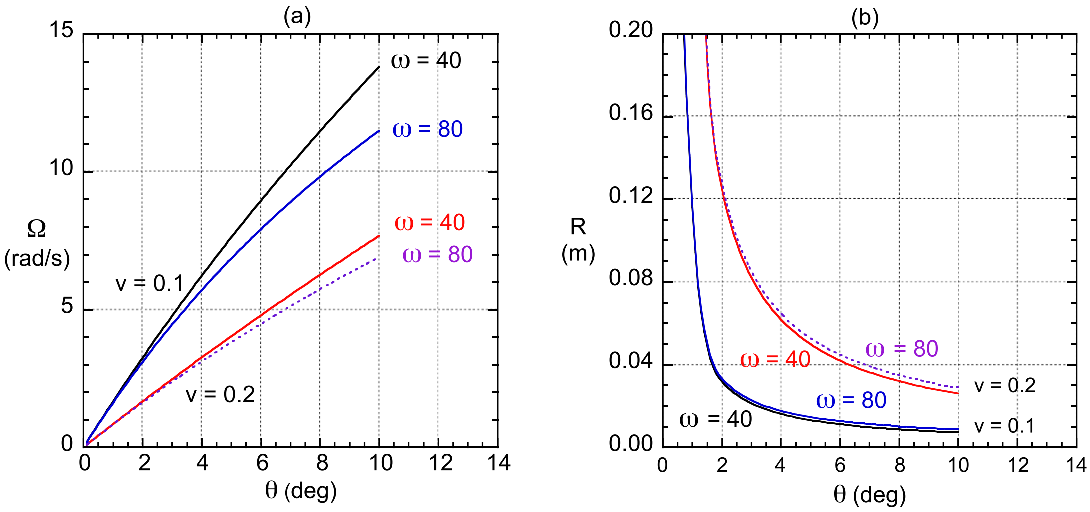

Figure 3. Solutions of equation (2) showing (a) Ω vs θ and (b) R vs θ when $v = 0.1$ or $0.2\,\mathrm{m\,s^{-1}}$ and when $\omega = 40$ or $80\,\mathrm{rad\,s^{-1}}$.

its spin frequency, ω. The radius, R, decreases as θ increases and as v decreases, which is why the coin spirals inwards as it tilts and as it slows down. If $v > 0.15\,\mathrm{m s^{-1}}$ and $\theta < 1^\circ$ then the coin will travel along an almost straight line path, as shown in supplementary video Straight.mov. If $\theta > 20^\circ$ then R approaches zero and the coin then precesses about a vertical axis through the centre of mass, which remains at rest. In that case, $\Omega^2 = 4g/a\cos\theta$ which increases rapidly as $\theta \to 90^\circ$ [2].

3. Experimental results

A number of coin spins were filmed, all with similar results, so only one video result was analysed in detail. The x, y coordinates of the centre of mass of the coin were digitised from the video using Tracker motion analysis software and the results are shown over a 4 s period in figure 4(a). That data was used to calculate the spiral radius, R, vs time using an assumed coordinate origin near the end of the spin. The origin of the spiral track drifted slowly with time and the values of R shown in figure 4(b) after $t = 3$ s were unreliable.

The y and x vs time data were used to calculate the velocity, v, of the centre of mass, as shown in figure 5(a), indicating that v decreased slowly from about 0.2 to $0.1\,\mathrm{m s^{-1}}$ while R decreased from 0.07 m to 0.03 m over the first three seconds. The precession frequency was calculated from the relation $\Omega = v/R$, as shown in figure 5(b). Ω remained approximately constant at about $3.0\,\mathrm{rad\,s^{-1}}$, while ω decreased smoothly with time from $110\,\mathrm{rad\,s^{-1}}$ at $t = 0$ to 48 rad $\mathrm{s^{-1}}$ at $t = 3$ s.

Figure 3(a) indicates that θ is between 2 and 4 degrees for these parameters, in which case figure 3(b) indicates that R is about 0.04 to 0.06 m in theory. During the first 0.5 s, figure 5 shows that v was about $0.2\,\mathrm{m\,s^{-1}}$ and Ω was about $3.5\,\mathrm{rad\,s^{-1}}$. Figure 3(a) indicates that θ is about 4 degrees for these parameters. θ then decreased to about 3° during the next three seconds, but θ could not be measured accurately from the overhead video. Nevertheless, it was clear from the video that θ was about 5° at the start, decreased quickly to about 3° and then slowly increased over the next three seconds. Equation (2) was used to calculate a theoretical estimate of θ vs time, shown in

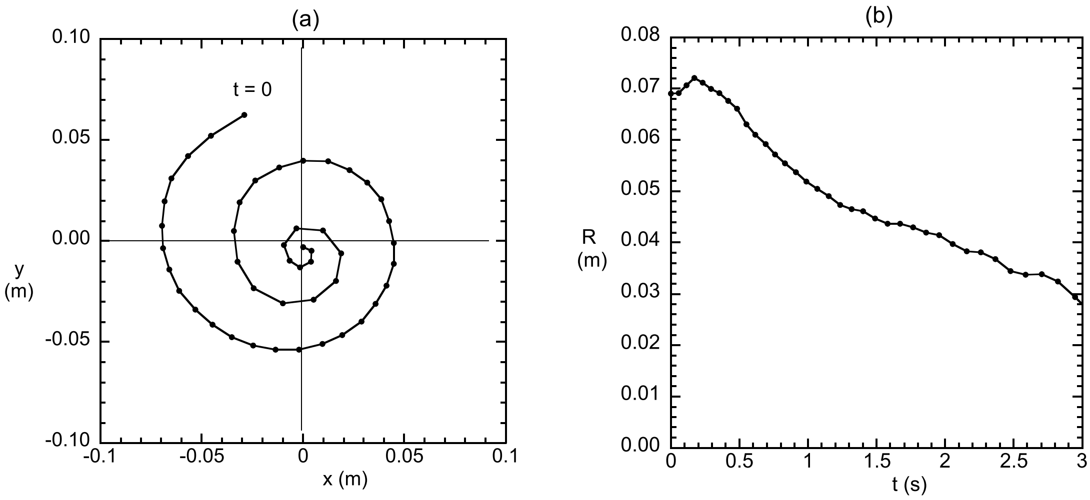

Figure 4. Experimental values of (a) y vs x and (b) R vs t.

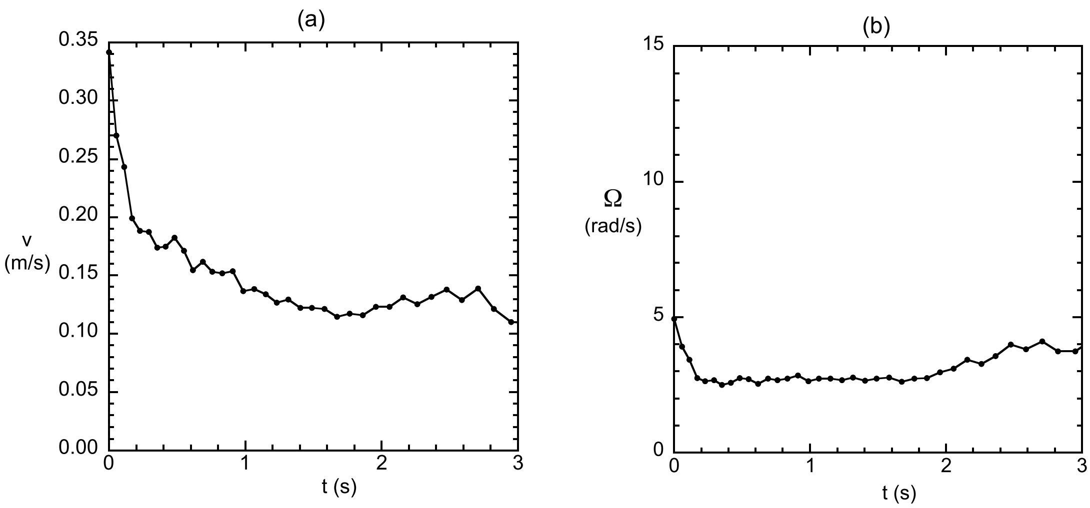

Figure 5. Experimental values of (a) v vs t and (b) Ω vs t.

figure 6, based on the measured values of v, ω and Ω vs time. The theoretical estimates of θ are consistent with the observed inclination angle of the coin observed in the video. Eventually, the coin fell onto the table but the last few seconds of the fall were not analysed. The initial decrease in θ

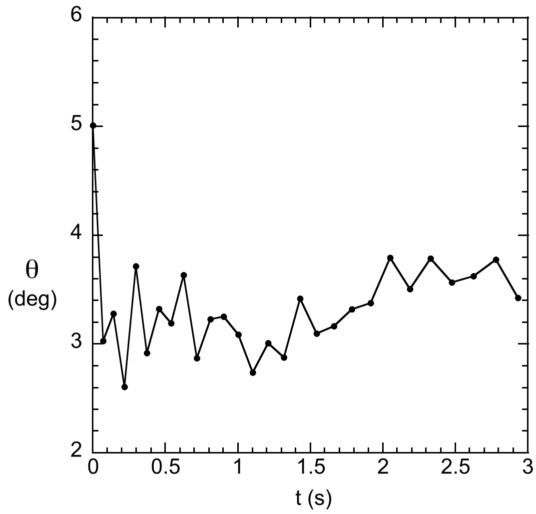

Figure 6. Estimate of θ vs t from equation (2), based on the measured values of v, ω and Ω vs time.

corresponds to a rise in the centre of mass, and is likely due to initial sliding motion, as it is for a spinning egg or tippe top.

4. Conclusion

Spiral motion of a spinning coin is easily observed, is quite reproducible, and has probably been observed many times by students. As such, it makes for an interesting experiment that can be analysed by students using a smartphone camera, especially if they are already familiar with the fact that the rate of change of angular momentum is equal to the applied torque.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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References

[1] Olssen M G 1972 Coin spinning on a table *Am. J. Phys.* **40** 1543–5
[2] Cross R 2015 Experimenting with a spinning disk *Phys. Educ.* **50** 197–202
[3] Cross R 2024 Experiments with a spinning coin *Phys. Educ.* **59** 013002
[4] Poonyawatpornkul J and Wattanakasiwich P 2015 High-speed video analysis of a rolling disc in three dimensions *Eur. J. Phys.* **36** 065027
[5] Humble S 2001 Rolling and spinning coin: a level gyroscopic processional motion *Teach. Math. Appl.* **20** 18–24