Formulaire d'Analyse Vectorielle

Coordonnées cylindriques La base est $(\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_z})$.

$$\overrightarrow{\text{grad}}\ M = \frac{\partial M}{\partial r}\overrightarrow{e_r} + \frac{1}{r}\frac{\partial M}{\partial \theta}\overrightarrow{e_\theta} + \frac{\partial M}{\partial z}\overrightarrow{e_z}$$

$$\text{div}\,\overrightarrow{A} = \frac{1}{r}\frac{\partial (r.A_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\overrightarrow{\text{rot}}\ \overrightarrow{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\overrightarrow{e_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\overrightarrow{e_\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\overrightarrow{e_z}$$

$$\Delta M = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial M}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 M}{\partial \theta^2} + \frac{\partial^2 M}{\partial z^2}$$

$$\overrightarrow{\Delta}\overrightarrow{A} = \left[\Delta A_r - \frac{1}{r^2}\left(A_r + 2\frac{\partial A_\theta}{\partial \theta}\right)\right]\overrightarrow{e_r} + \left[\Delta A_\theta - \frac{1}{r^2}\left(A_\theta - 2\frac{\partial A_r}{\partial \theta}\right)\right]\overrightarrow{e_\theta} + \Delta A_z\overrightarrow{e_z}$$

Coordonnées sphériques La base est $(\overrightarrow{e_r}, \overrightarrow{e_\theta}, \overrightarrow{e_\phi})$.

$$\overrightarrow{\text{grad}}\ M = \frac{\partial M}{\partial r}\overrightarrow{e_r} + \frac{1}{r}\frac{\partial M}{\partial \theta}\overrightarrow{e_\theta} + \frac{1}{r\sin\theta}\frac{\partial M}{\partial \phi}\overrightarrow{e_\phi}$$

$$\text{div}\,\overrightarrow{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial \sin\theta A_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\overrightarrow{\text{rot}}\ \overrightarrow{A} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(\sin\theta A_\phi) - \frac{\partial A_\theta}{\partial \phi}\right)\overrightarrow{e_r} + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi)\right)\overrightarrow{e_\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\overrightarrow{e_\phi}$$

$$\Delta M = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial M}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial M}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 M}{\partial \phi^2}$$

$$\begin{aligned}\overrightarrow{\Delta}\overrightarrow{A} = &\left(\Delta A_r - \frac{2A_r}{r^2} - \frac{2A_\theta\cos\theta}{r^2\sin\theta} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial \theta} - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial \phi}\right)\vec{e}_r \\ &+ \left(\Delta A_\theta - \frac{A_\theta}{r^2\sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial \theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial \phi}\right)\vec{e}_\theta \\ &+ \left(\Delta A_\phi - \frac{A_\phi}{r^2\sin^2\theta} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial \phi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial \phi}\right)\vec{e}_\phi\end{aligned}$$

Composition des opérateurs

$$\mathrm{div}(\overrightarrow{\mathrm{rot}}\ \vec{A}) = 0$$

$$\overrightarrow{\mathrm{rot}}(\overrightarrow{\mathrm{rot}}\ \vec{A}) = \overrightarrow{\mathrm{grad}}(\mathrm{div}\ \vec{A}) - \Delta\vec{A}$$

$$\mathrm{div}\ \overrightarrow{\mathrm{grad}}\ M = \Delta M$$

$$\overrightarrow{\mathrm{rot}}\ \overrightarrow{\mathrm{grad}}\ M = \vec{0}$$

Formules pour les produits

$$\overrightarrow{\mathrm{grad}}(\vec{A}\cdot\vec{B}) = (\vec{A}\cdot\overrightarrow{\mathrm{grad}})\vec{B} + \vec{A}\wedge\overrightarrow{\mathrm{rot}}\ \vec{B} + (\vec{B}\cdot\overrightarrow{\mathrm{grad}})\vec{A} + \vec{B}\wedge\overrightarrow{\mathrm{rot}}\ \vec{A}$$

$$\overrightarrow{\mathrm{grad}}(\vec{A}^2) = 2(\vec{A}\cdot\overrightarrow{\mathrm{grad}})\vec{A} + 2\vec{A}\wedge\overrightarrow{\mathrm{rot}}\ \vec{A}$$

$$\mathrm{div}(\vec{A}\wedge\vec{B}) = -\vec{A}\cdot\overrightarrow{\mathrm{rot}}\ \vec{B} + \vec{B}\cdot\overrightarrow{\mathrm{rot}}\ \vec{A}$$

$$\overrightarrow{\mathrm{rot}}(\vec{A}\wedge\vec{B}) = \vec{A}\mathrm{div}\ \vec{B} - (\vec{A}\cdot\overrightarrow{\mathrm{grad}})\vec{B} - \vec{B}\mathrm{div}\ \vec{A} + (\vec{B}\cdot\overrightarrow{\mathrm{grad}})\vec{A}$$

$$\overrightarrow{\mathrm{grad}}(U\ V) = U\overrightarrow{\mathrm{grad}}\ V + V\overrightarrow{\mathrm{grad}}\ U$$

$$\mathrm{div}(M\vec{A}) = M\ \mathrm{div}\ \vec{A} + \vec{A}\cdot\overrightarrow{\mathrm{grad}}(M)$$

$$\overrightarrow{\mathrm{rot}}(M\vec{A}) = M\overrightarrow{\mathrm{rot}}\ \vec{A} + \overrightarrow{\mathrm{grad}}\ (M)\wedge\vec{A}$$

$$\Delta(U\cdot V) = U\Delta V + 2\ \overrightarrow{\mathrm{grad}}\ U\cdot\overrightarrow{\mathrm{grad}}\ V + V\Delta U$$

$$\mathrm{div}(U\overrightarrow{\mathrm{grad}}\ V - V\overrightarrow{\mathrm{grad}}\ U) = U\Delta V - V\Delta U$$