$\boxed{n° 58}$

$u_n = e^{-t} \cos nt \left(\frac{1}{n^2} + \frac{1}{n^4}\right) = O\left(\frac{1}{n^2}\right)$ TC ...

<ins>CS</ins>

$u_n(t) =$ ______ $\boxed{D = \mathbb{R}}$

u_n C^0/D / TG

$\forall t \in D \quad |u_n(t)| \leq \frac{1}{n^2} + \frac{1}{n^4} = \alpha_n$

... $(\sum u_n)$ $CN/\mathbb{R}$

d $\boxed{f \ C^0 \text{ sur } \mathbb{R}}$

<ins>CD</ins> u_n et f $C^0/I = \mathbb{R}^+$

$(\sum u_n)$ CS/I $(I \subset D)$

<ins>HD</ins> $\int_0^{+\infty} |u_n|$ existe car $u_n(t) = O(e^{-t})$ $_{t \to +\infty}$

et $\int_0^{+\infty} |u_n| \leq \int_0^{+\infty} e^{-t}\left(\frac{1}{n^2} + \frac{1}{n^4}\right) dt = \frac{1}{n^2} + \frac{1}{n^4} = O\left(\frac{1}{n^4}\right)$

donc $\left(\sum \int_0^{+\infty} |u_n|\right)$ CV

comme $\int_0^{\infty} e^{-t} \cos nt \, dt = \mathrm{Re}\left(\int_0^{+\infty} e^{-t} e^{int} dt\right) = \frac{1}{n^2+1}$

d $\int_0^{+\infty} f = \sum_{n=1}^{\infty} \frac{1}{n^4}$ $(= \pi^4/90)$