

```
> with(plots) :
```

```
Warning, the name changecoords has been redefined
```

```
> alpha:=n->cos(ln(n)) ;
```

$$\alpha := n \rightarrow \cos(\ln(n))$$

```
> seq(evalf(alpha(n),2),n=1..300) ;
```

```
1., .77, .45, .17, -.029, -.23, -.32, -.50, -.59, -.67, -.74, -.80, -.86, -.86, -.90, -.94, -.94,
-.97, -.97, -.99, -.99, -1.0, -1.0, -1.0, -1.0, -.99, -.99, -.99, -.97, -.97, -.97, -.94, -.94,
-.94, -.90, -.90, -.90, -.90, -.85, -.85, -.85, -.85, -.79, -.79, -.79, -.79, -.73, -.73, -.73,
-.73, -.73, -.65, -.65, -.65, -.65, -.65, -.65, -.57, -.57, -.57, -.57, -.57, -.57, -.49, -.49,
-.49, -.49, -.49, -.49, -.40, -.40, -.40, -.40, -.40, -.40, -.40, -.31, -.31, -.31, -.31,
-.31, -.31, -.31, -.31, -.21, -.21, -.21, -.21, -.21, -.21, -.21, -.21, -.21, -.11, -.11, -.11,
-.11, -.11, -.11, -.11, -.11, -.11, -.11, -.11, -.012, -.012, -.012, -.012, -.012, -.012,
-.012, -.012, -.012, .087, .087, .087, .087, .087, .087, .087, .087, .087, .087, .087, .087,
.19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19, .19,
.19, .28, .28, .28, .28, .28, .28, .28, .28, .28, .28, .38, .38, .38, .38, .38, .38, .38, .38,
.38, .38, .38, .38, .38, .38, .38, .38, .38, .38, .47, .47, .47, .47, .47, .47, .47, .47,
.47, .47, .47, .47, .47, .47, .47, .47, .47, .47, .55, .55, .55, .55, .55, .55, .55, .55,
.55, .55, .55, .55, .55, .55, .55, .55, .55, .55, .63, .63, .63, .63, .63, .63, .63, .63,
.63, .63, .63, .63, .63, .63, .63, .63, .63, .63, .71, .71, .71, .71, .71, .71, .71, .71,
.71, .71, .71, .71, .71, .71, .71, .71, .71, .71, .71, .71, .71, .71, .78, .78, .78, .78,
.78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78, .78,
.78, .78, .78, .78, .78, .78, .78, .83, .83, .83, .83, .83, .83, .83, .83, .83, .83, .83,
.83, .83, .83, .83
```

```
> a:=n->min(evalf(alpha(n)),evalf(alpha(n+1))) ;
```

```
b:=n->max(evalf(alpha(n)),evalf(alpha(n+1))) ;
```

$$a := n \rightarrow \min(\text{evalf}(\alpha(n)), \text{evalf}(\alpha(n+1)))$$
$$b := n \rightarrow \max(\text{evalf}(\alpha(n)), \text{evalf}(\alpha(n+1)))$$

```
>
```

```
> f:=proc(n,x)
```

```
  if x<a(n) then 0
```

```
  elif x<b(n) then 1
```

```
  else 0;
```

```
fi;
```

```
end:
```

```
> a(5);b(5) ;
```

```
-.2191694497
```

```
-.03863196950
```

```
> f(5,0.3);f(5,-0.1);
```

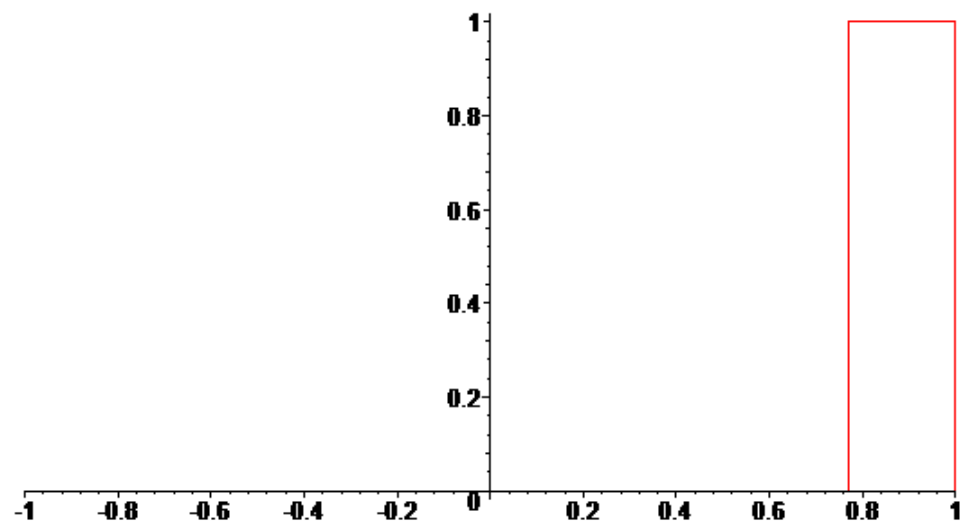
0

1

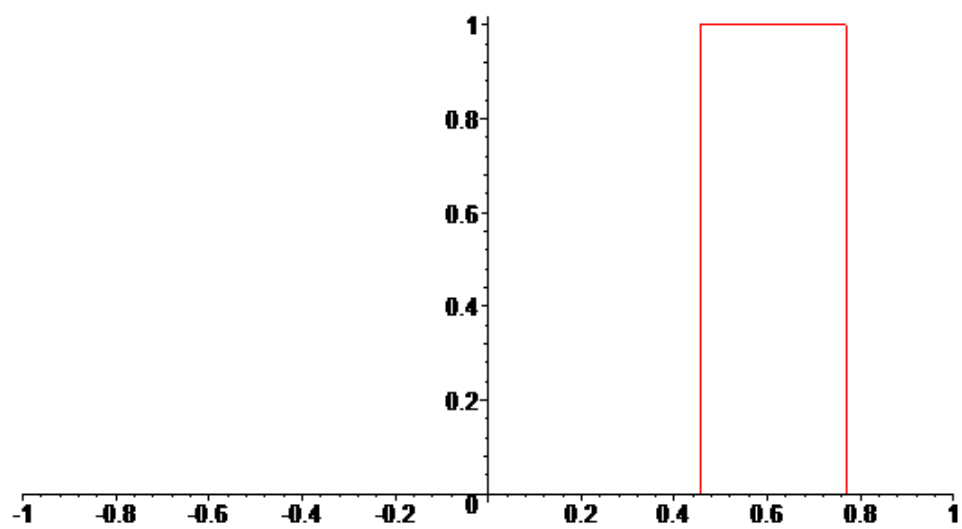
```
> p:=proc(n)
  g:=x->f(n,x);
  plot(g,-1..1,scaling=constrained,numpoints=10000);
end;
```

Warning, `g` is implicitly declared local to procedure `p`

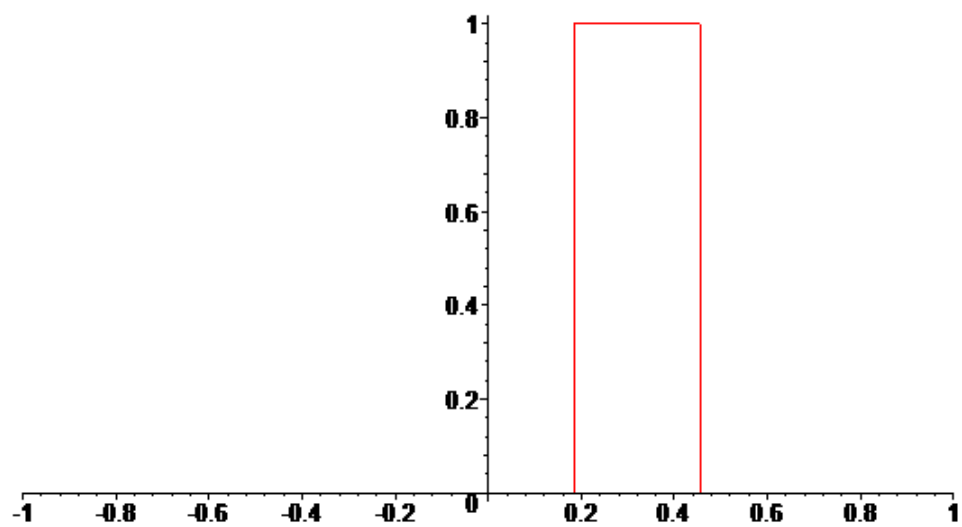
```
> p(1);
```



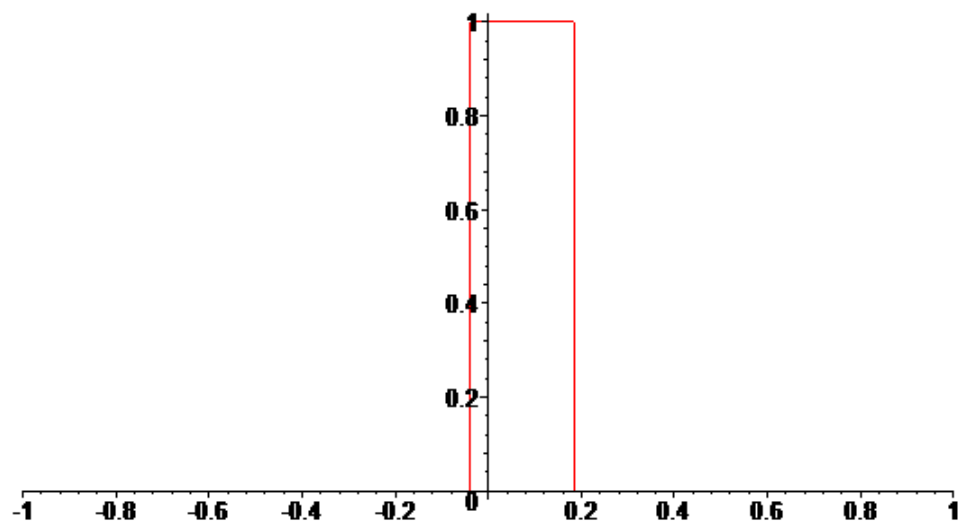
```
> p(2) ;
```



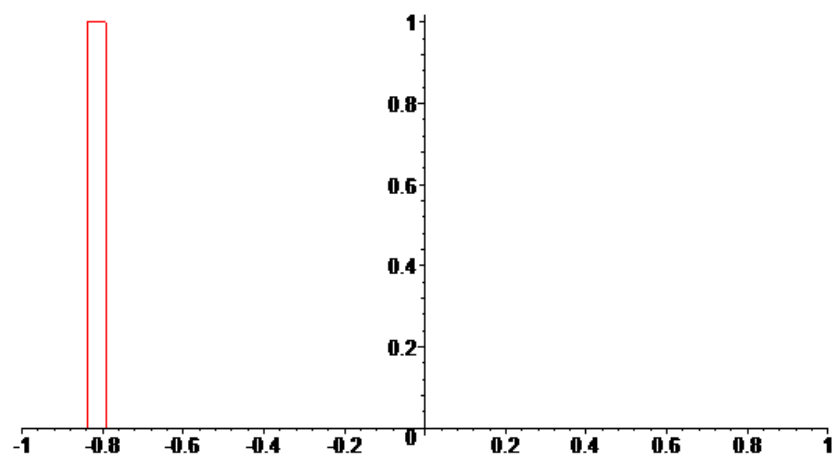
```
> p(3) ;
```



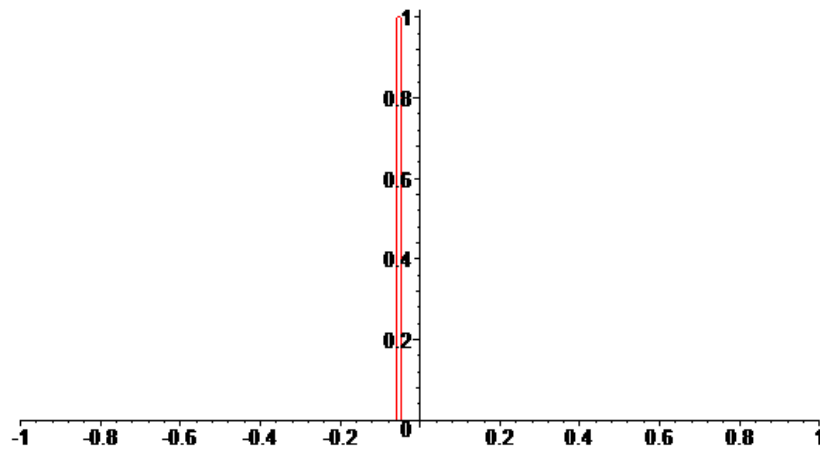
```
> p(4) ;
```



```
> p(12) ;
```



```
> p(105);
```



```
> seq(f(n,0.4),n=1..100);
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
```

```
> seq(f(n,-0.2022),n=1..200);
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
```

```
> trouve:=proc(x,N)
  for n from 1 to N do
    if f(n,x)=1 then print(n);
    fi;
  od;
```

```
end;
```

```
Warning, `n` is implicitly declared local to procedure `trouve`
```

```
> trouve(-0.2022,10^6);
```

```
5
```

```
90
```

```
3157
```

```
48628
```

```

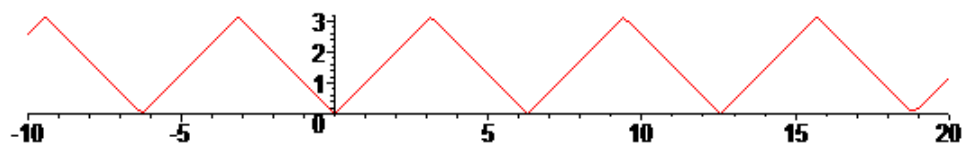
>
>
>
>
> F:=proc(x)
  if x>evalf(Pi) then F(x-2*Pi);
  elif x<-evalf(Pi) then F(x+2*Pi);
  else evalf(abs(x));
fi;
end:

> F(3);

> plot(F,-10..20,scaling=constrained);

```

3.



```

> S:=(n,t)->Pi/2-sum(4/Pi/(2*p+1)^2*cos((2*p+1)*t),p=0..n);

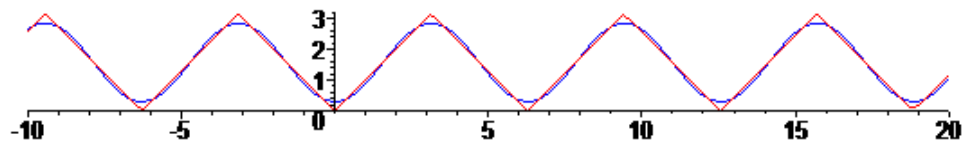
```

$$S := (n, t) \rightarrow \frac{1}{2} \pi - \left(\sum_{p=0}^n \left(4 \frac{\cos((2p+1)t)}{\pi(2p+1)^2} \right) \right)$$

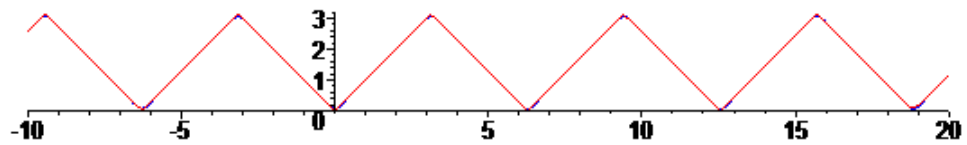
```

>
> p1:=plot(F,-10..20,scaling=constrained,color=red):
p0:=plot(S(0,t),t=-10..20,scaling=constrained,color=blue):
p3:=plot(S(3,t),t=-10..20,scaling=constrained,color=blue):
> display(p1,p0);

```

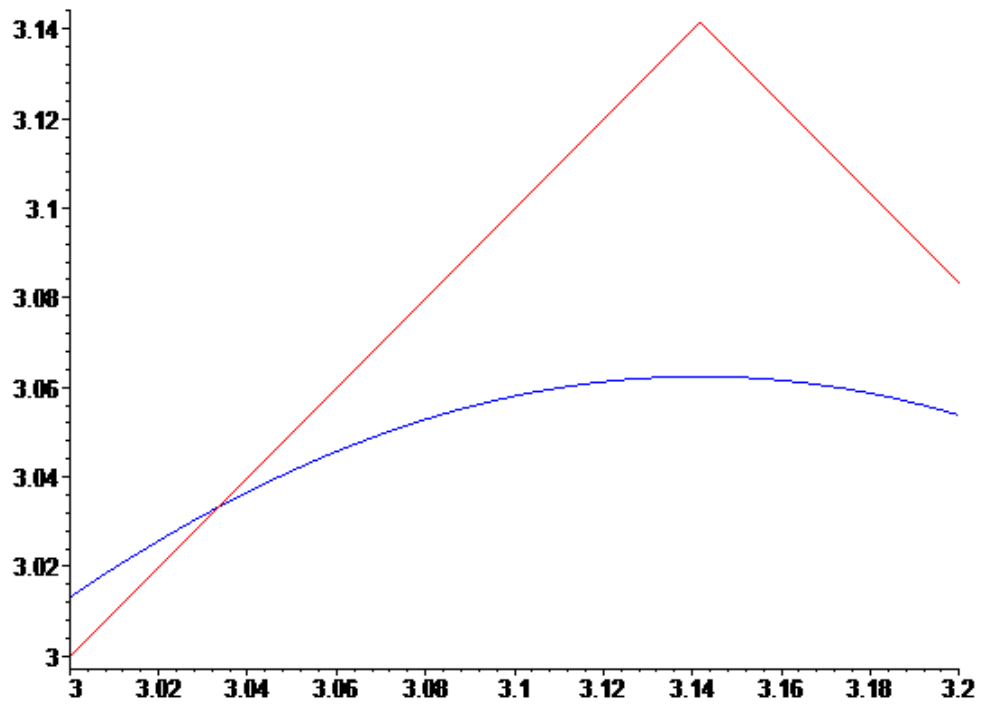


```
> display(p1,p3);
```

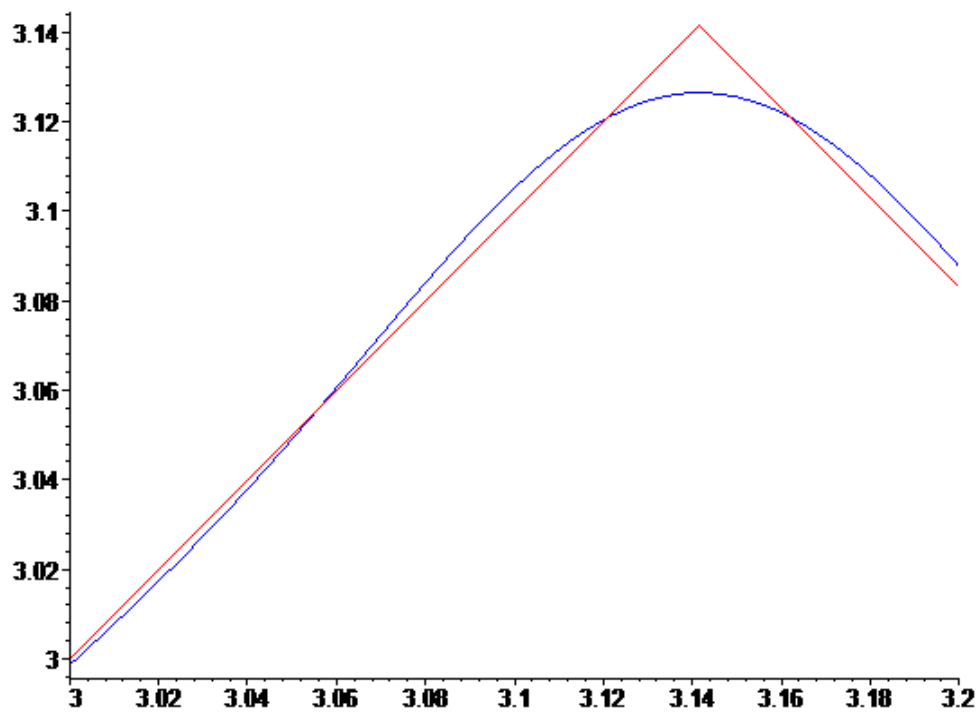


```
> q1:=plot(F,3..3.2,scaling=constrained,color=red):
q0:=plot(S(0,t),t=3..3.2,scaling=constrained,color=blue):
```

```
q3:=plot(S(3,t),t=3..3.2,scaling=constrained,color=blue):  
q20:=plot(S(20,t),t=3..3.2,scaling=constrained,color=blue):  
> display(q1,q3);
```



```
> display(q1,q20);
```

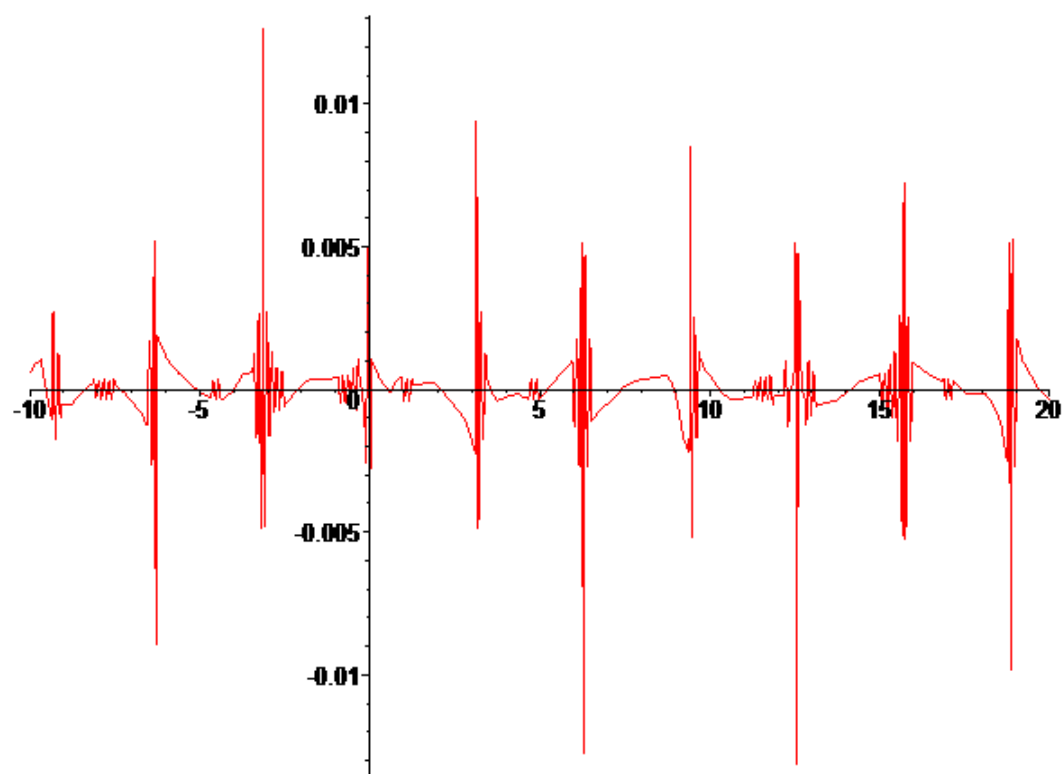
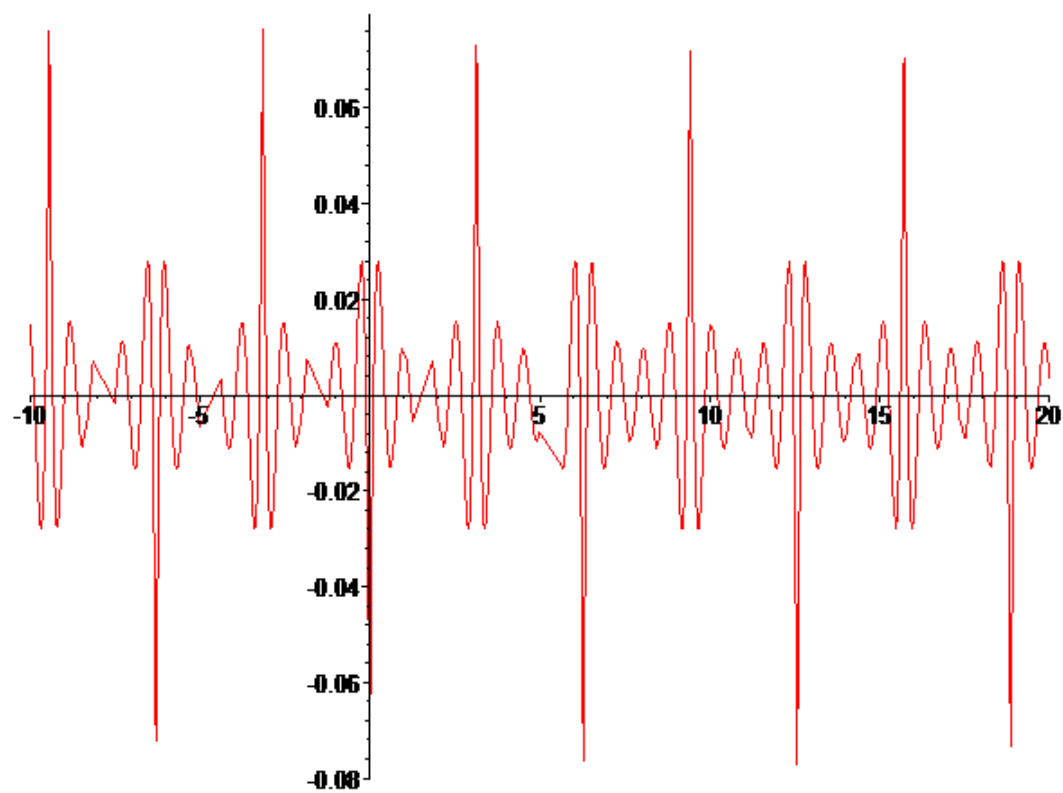
```
> G3:=x->F(x)-S(3,x);
G20:=x->F(x)-S(20,x);
G200:=x->F(x)-S(200,x);
```

$$G3 := x \rightarrow F(x) - S(3, x)$$

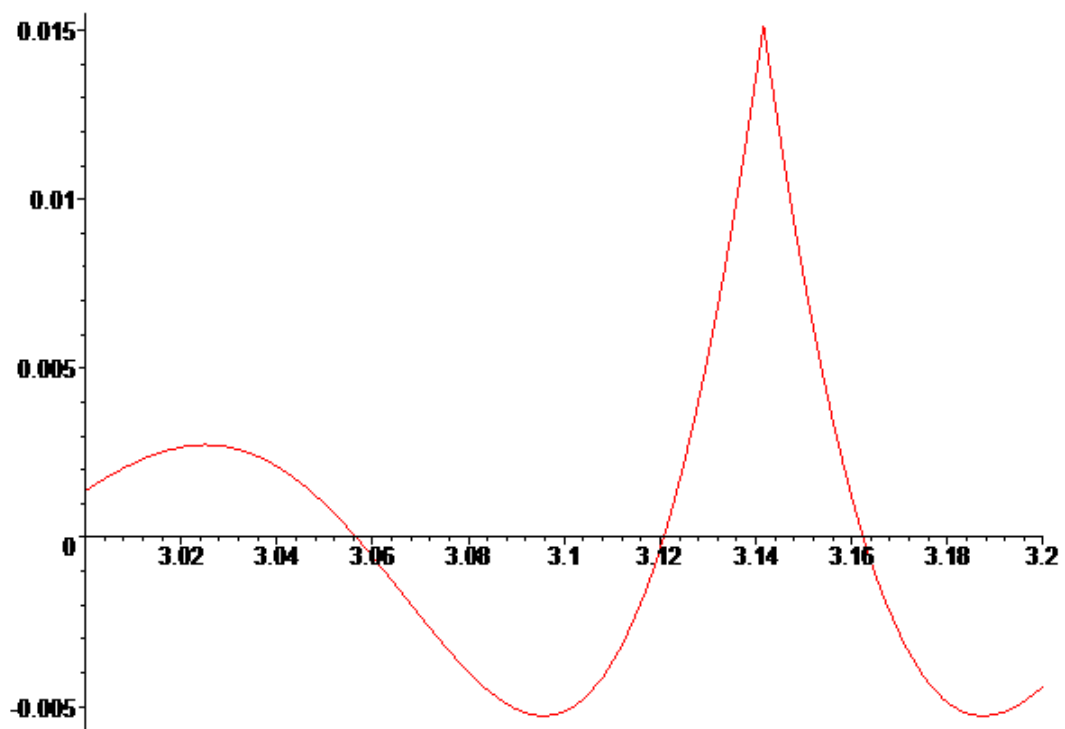
$$G20 := x \rightarrow F(x) - S(20, x)$$

$$G200 := x \rightarrow F(x) - S(200, x)$$

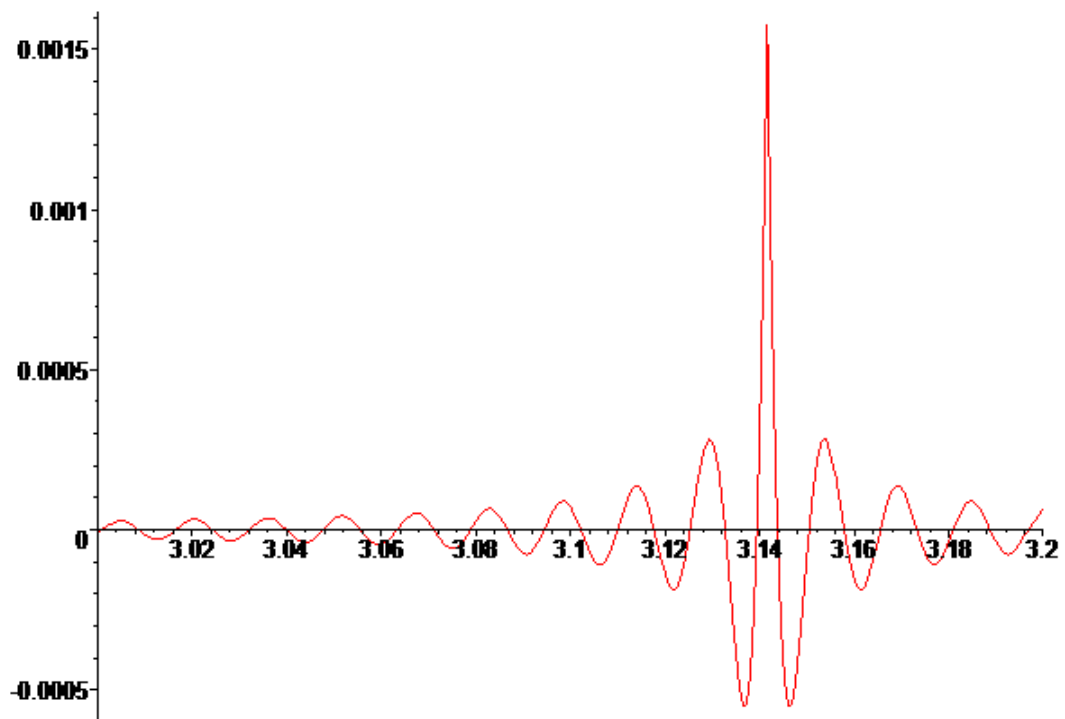
```
> plot(G3,-10..20);
plot(G20,-10..20);
```



```
> plot(G20,3..3.2);
```



```
> plot(G200,3..3.2);
```



```
>
```