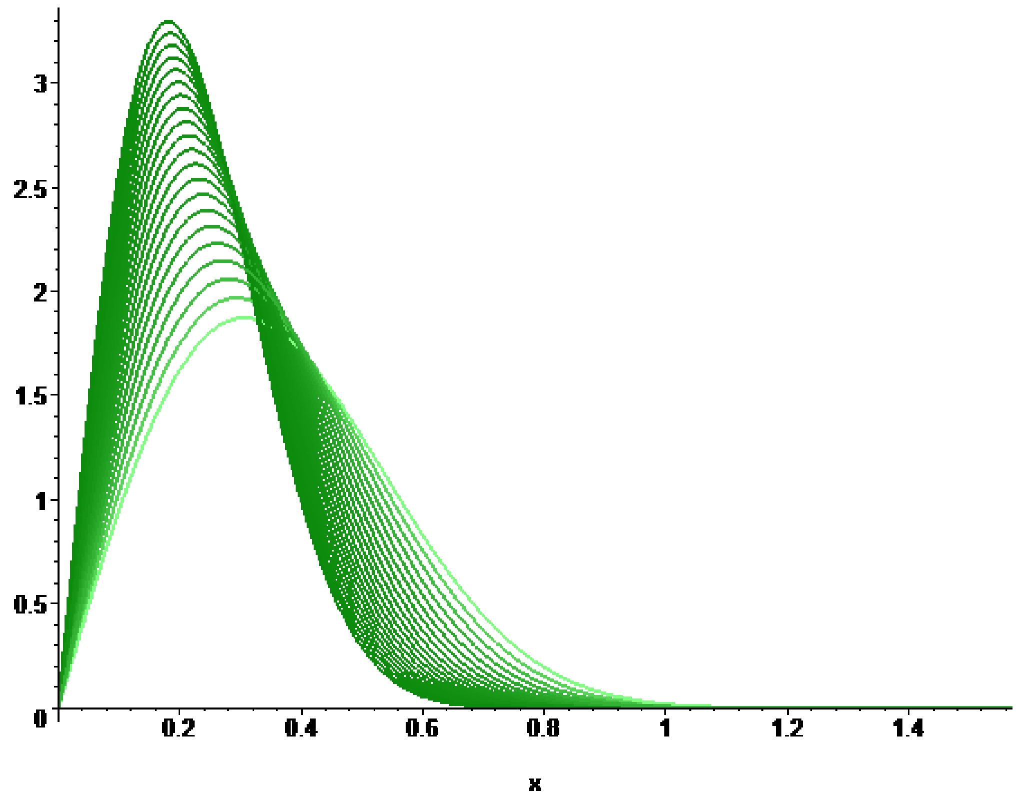
```
> # ILLUSTRATION DES SUITES ET SERIES DE FONCTIONS
> with(plots):
Warning, the name changecoords has been redefined

> col: =i->(1/(i+1),0.5+1/(i+1),1/(i+1)):
Tra: =proc(f,n1,n2,opt)
  local n,liste,p;
  for n from n1 to n2 do
    liste:=col(n-n1+1); # paramètres de la couleur

p[n]:=plot(f(n,x),op(opt),thickness=2,color=COLOR(RGB,liste));
  od;
  plots[display]({seq(p[n],n=n1..n2)});
end:
Tra2:=proc(f,n1,n2,opt)
  local p;
    p[1]:=plot(f(n1,x),op(opt),thickness=2,color=blue);
    p[2]:=plot(f(n2,x),op(opt),thickness=2,color=red);
  plots[display](p[1],p[2]);
end:
> # On entre ici les suites ou séries de fonctions
f:=(n,x)->n*cos(x)^n*sin(x);
g:=(n,x)->(1/n)*cos(x)^n*sin(x);
h:=(n,x)->sqrt(x^2+1/n);
```

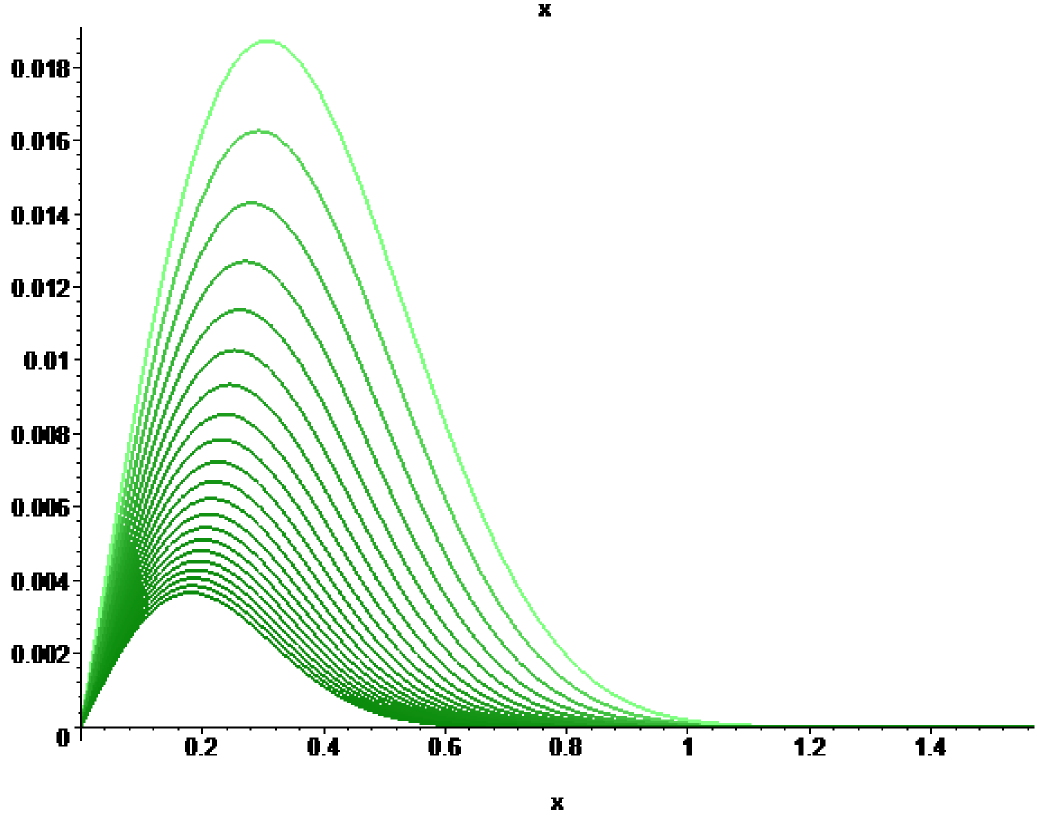
$$f := (n, x) \to n \cos(x)^n \sin(x)$$

$$g := (n, x) \to \frac{\cos(x)^n \sin(x)}{n}$$

$$h := (n, x) \to \sqrt{x^2 + \frac{1}{n}}$$

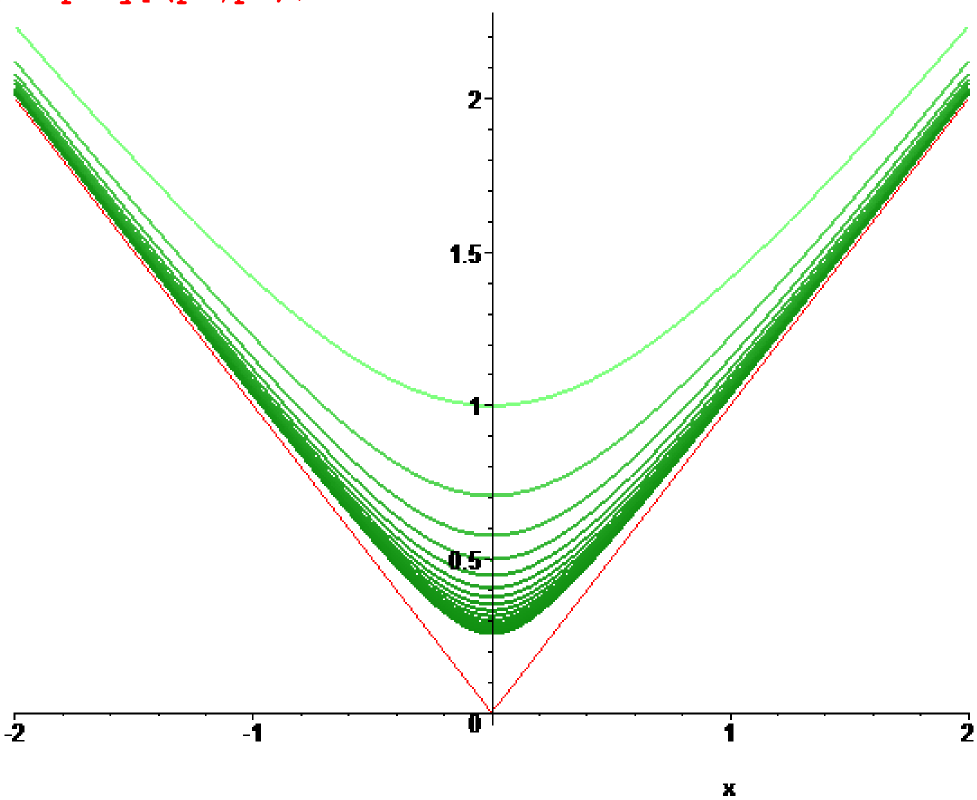
```
> Tra(f,10,30,[x=0..Pi/2]);
 Tra(g,10,30,[x=0..Pi/2]);
```






```
>
> p1:=Tra(h,1,15,[x=-2..2]):
> p2:=plot(abs(x),x=-2..2,color=red):
```

```
> plots[display](p1,p2);
```




```
>
>
```

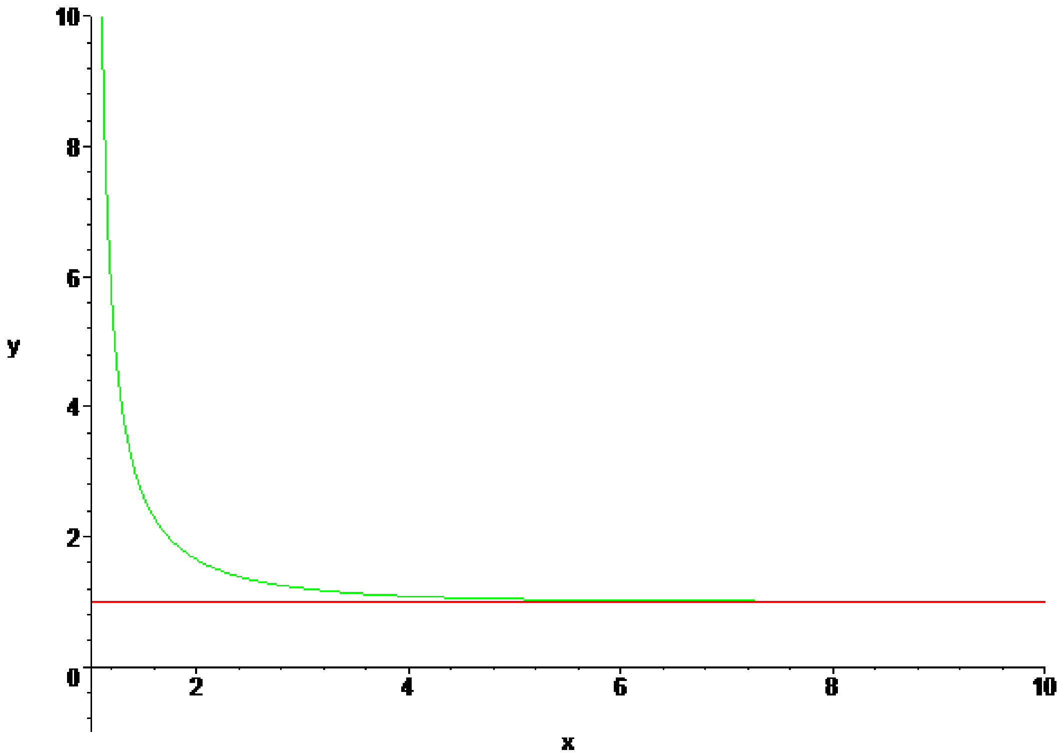
```
> S:=x->sum(1/n^x,n=1..infinity);
```

$$S := x \rightarrow \sum_{n=1}^{\infty} \frac{1}{n^x}$$

```
> 'S(2)'=S(2),'S(4)'=S(4),'S(6)'=S(6),'S(3)'=S(3);
```

$$S(2) = \frac{1}{6}\pi^2,\ S(4) = \frac{1}{90}\pi^4,\ S(6) = \frac{1}{945}\pi^6,\ S(3) = \zeta(3)$$

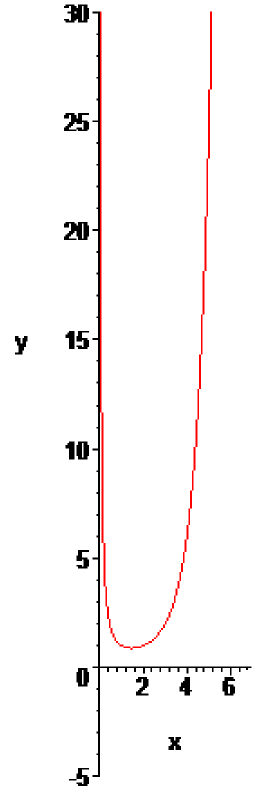
```
> plot({1,S(x)},x=1..10,y=-1..10,numpoints=1000);
```



```
> f:=x->int(t^(x-1)*exp(-t),t=0..infinity);
> plot(f(x),x=0..7,y=-5..30,scaling=constrained);
```

$$f := x \rightarrow \int_{0}^{\infty} t^{(x-1)}\, e^{(-t)}\, dt$$

```
Definite integration: Can't determine if the integral is convergent.
Need to know the sign of --> x
Will now try indefinite integration and then take limits.
```



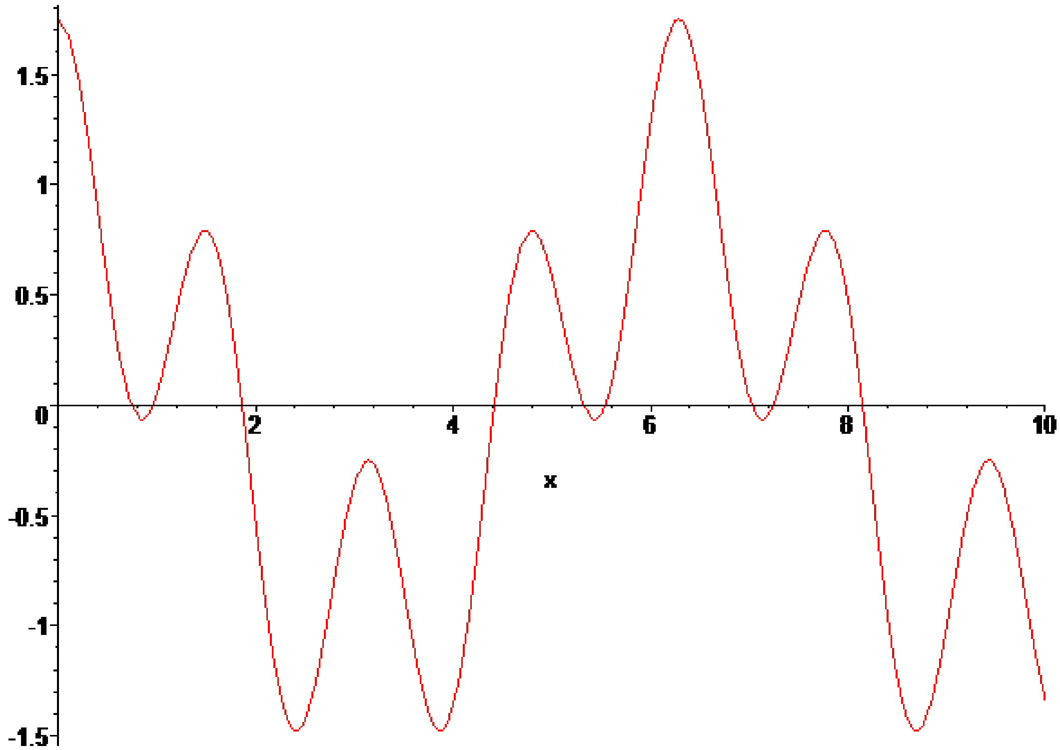
>
>
>
>
>
>

FONCTION DE WEIERSTRASS

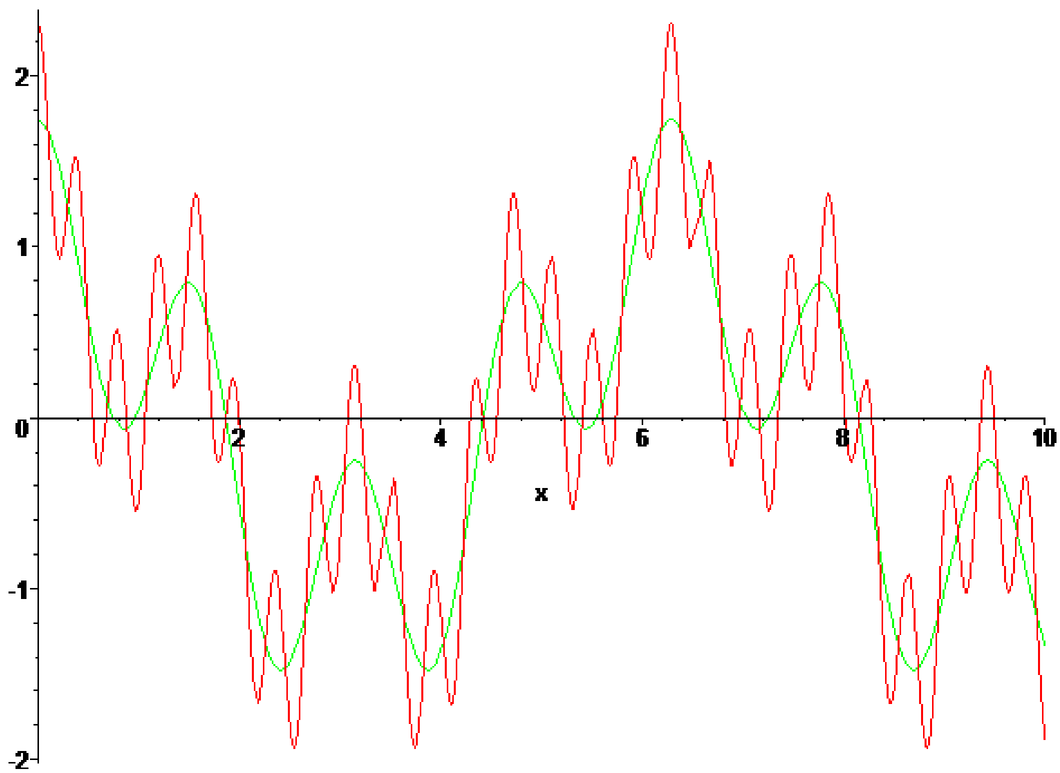
```
> W:=(n,x)->sum((3/4)^p*cos(4^p*x),p=0..n);
```

$$W := (n, x) \rightarrow \sum_{p=0}^{n} \left(\frac{3}{4}\right)^p \cos(4^p x)$$

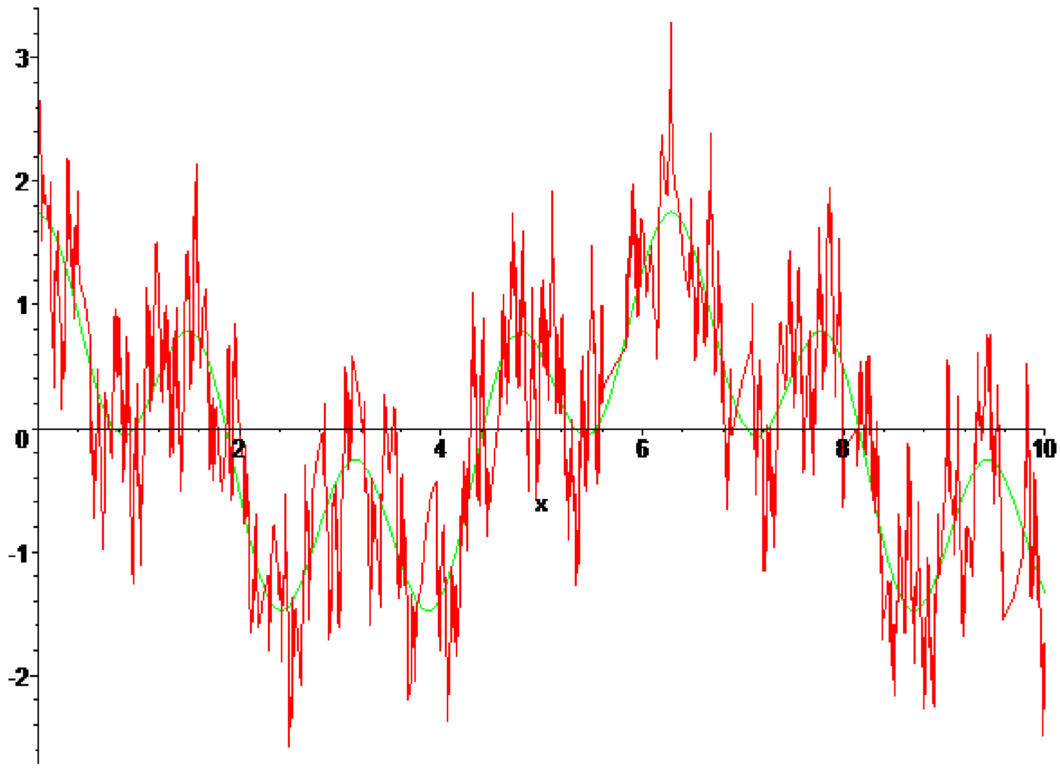
```
> plot(W(1,x),x=0..10);
```



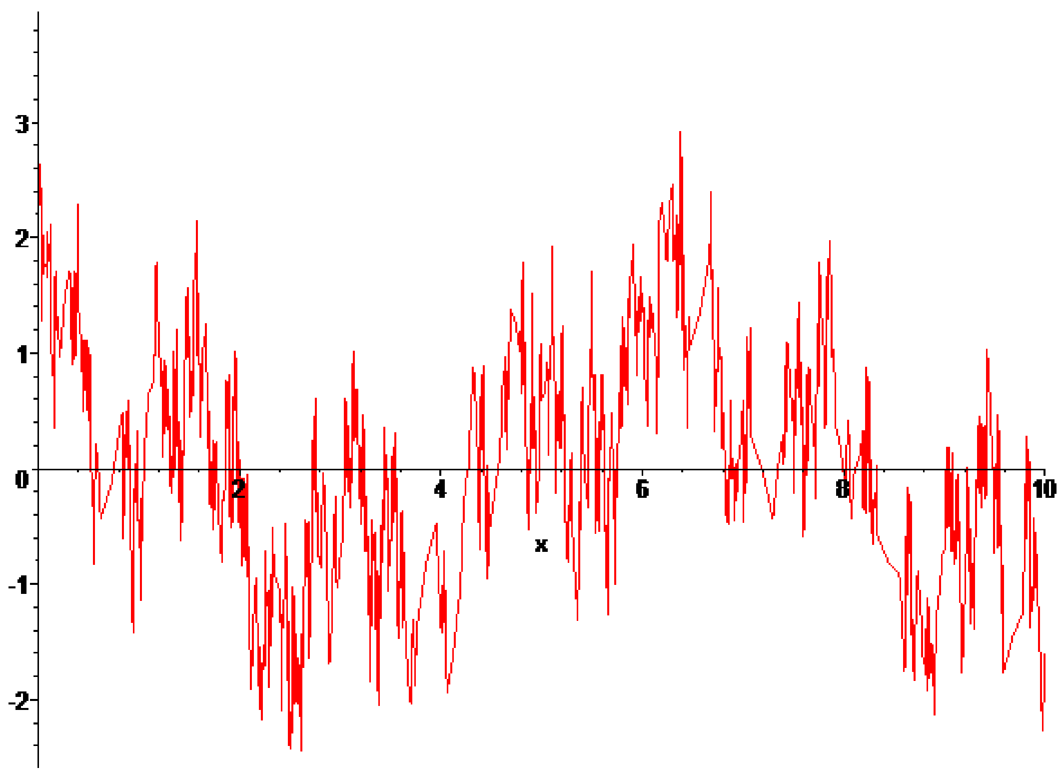
> **plot({W(1,x),W(2,x)},x=0..10);**



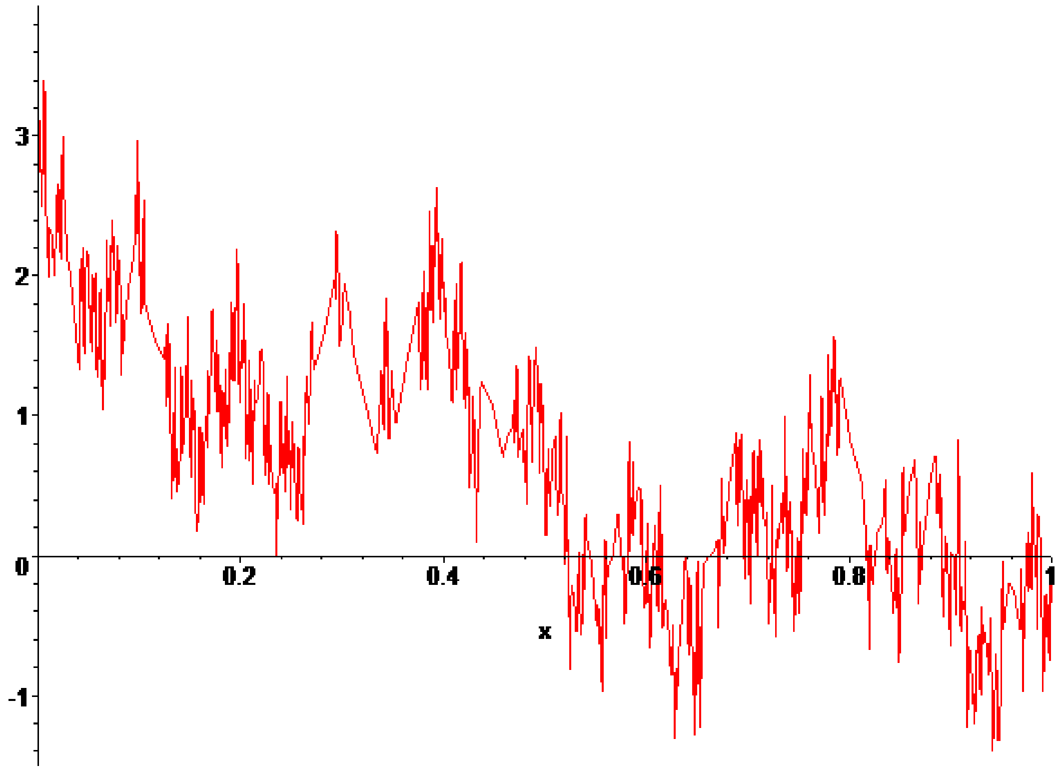
> **plot({W(1,x),W(5,x)},x=0..10);**



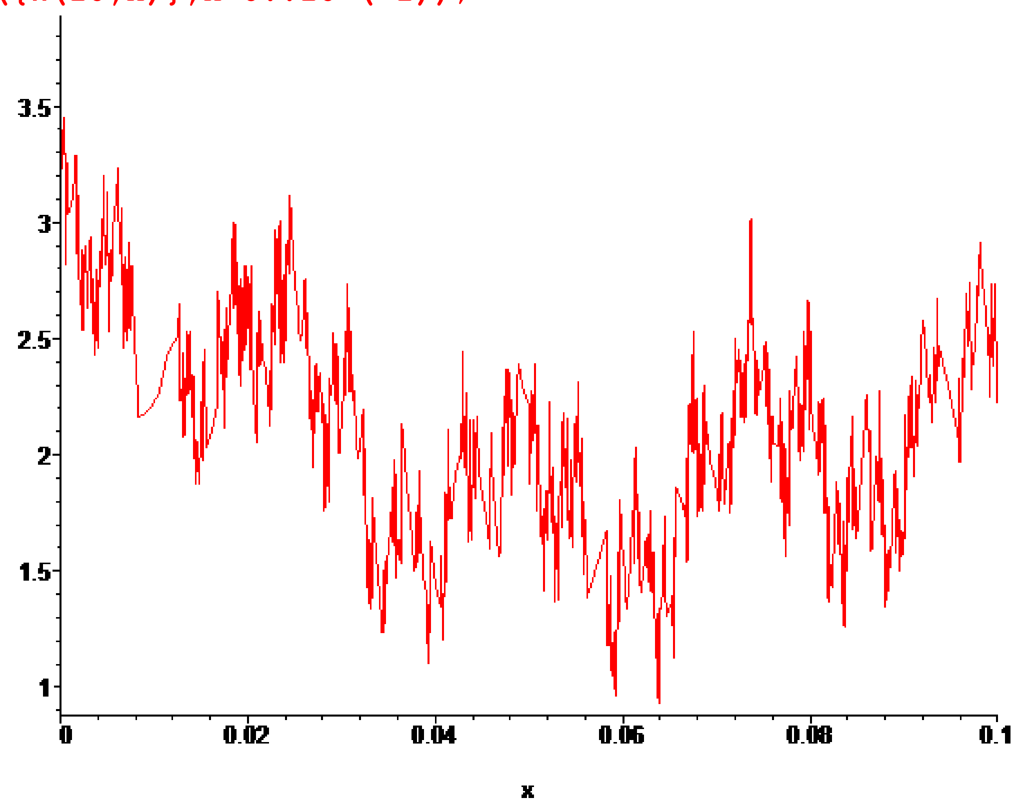
> **plot({W(10,x)},x=0..10);**



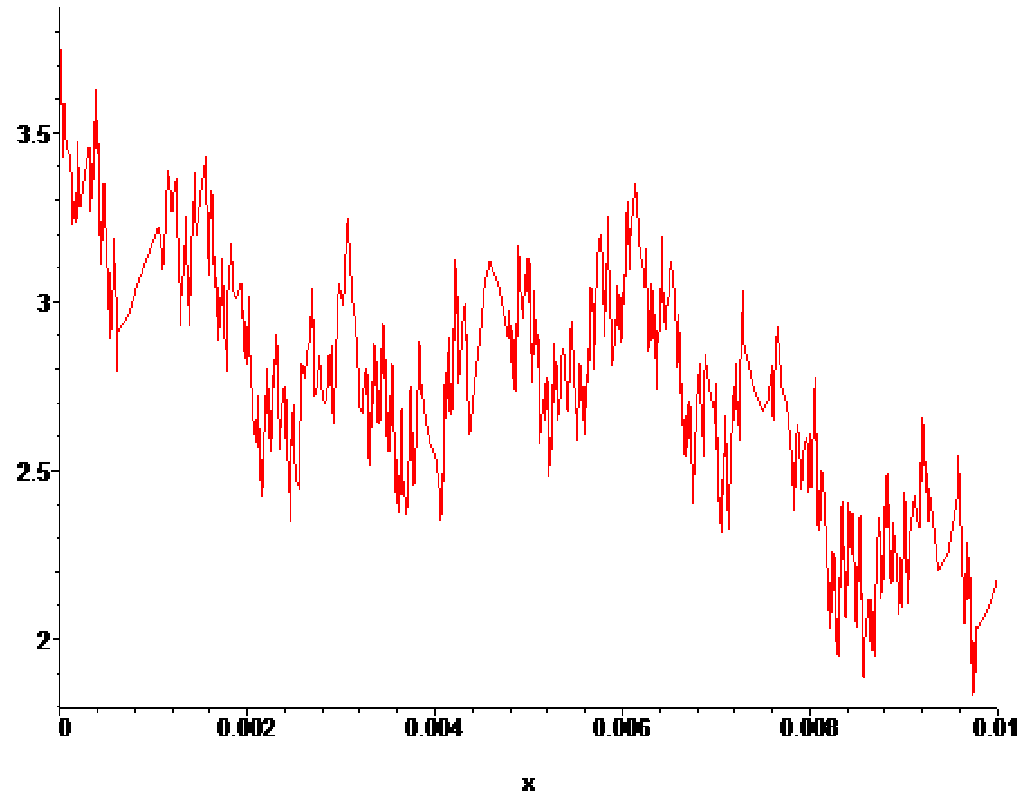
> **plot({W(10,x)},x=0..1);**



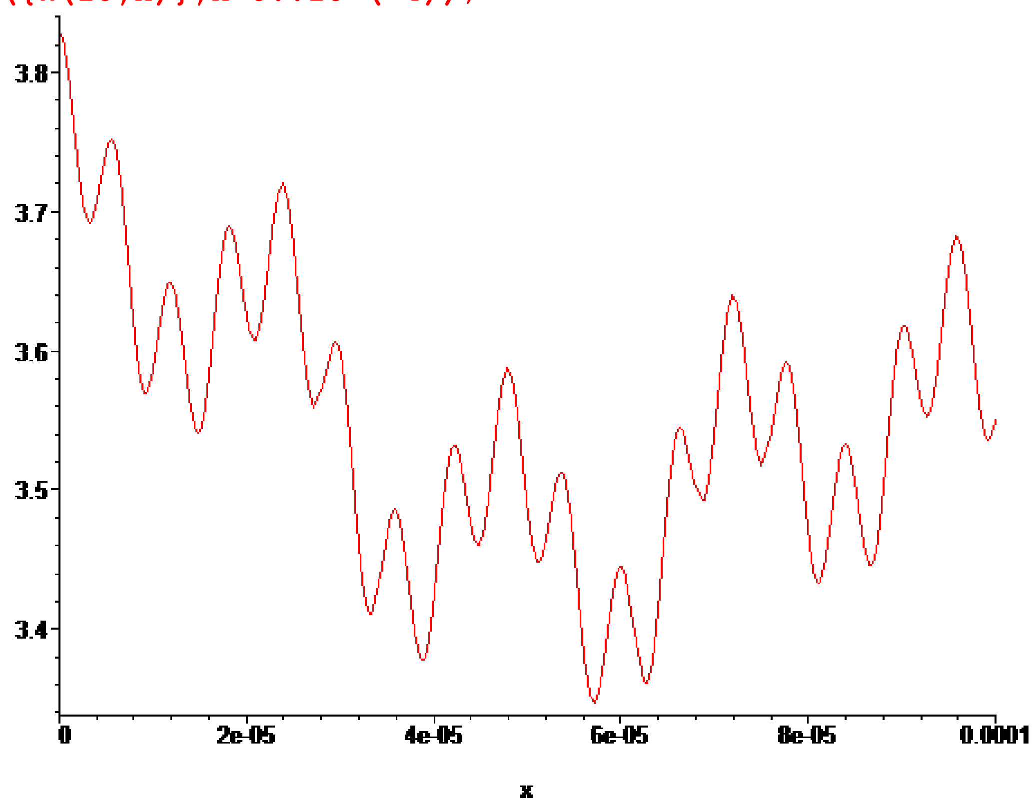

```
> plot({W(10,x)},x=0..10^(-1));
```



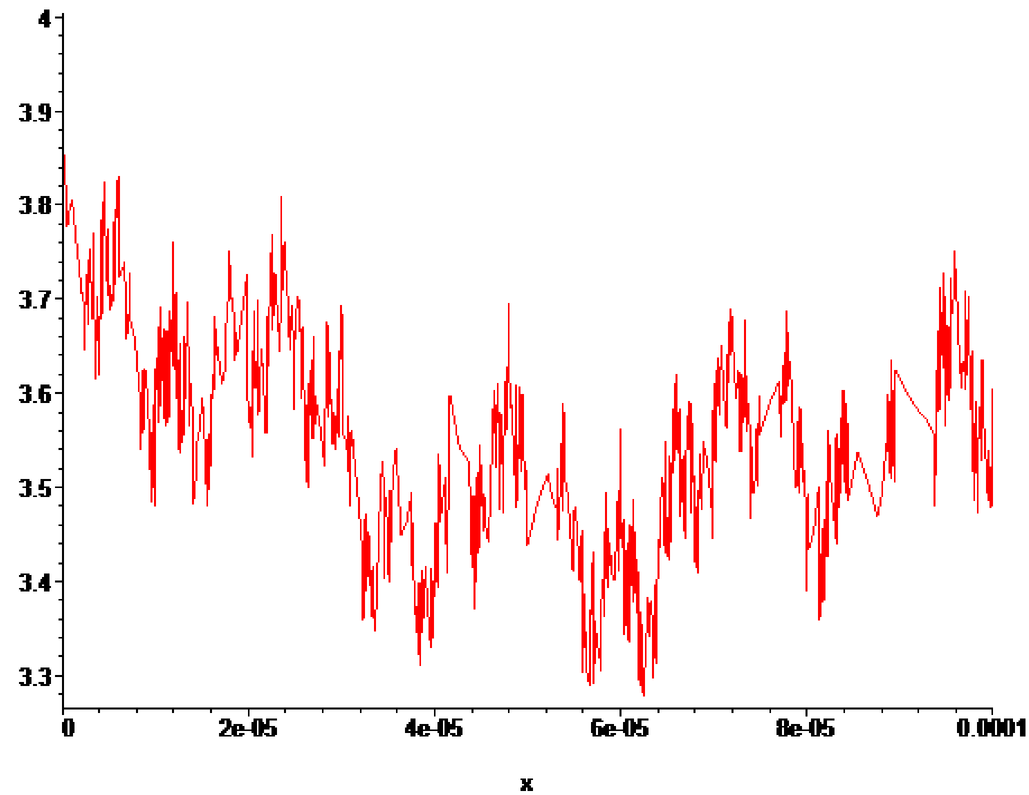

```
> plot({W(10,x)},x=0..10^(-2));
```



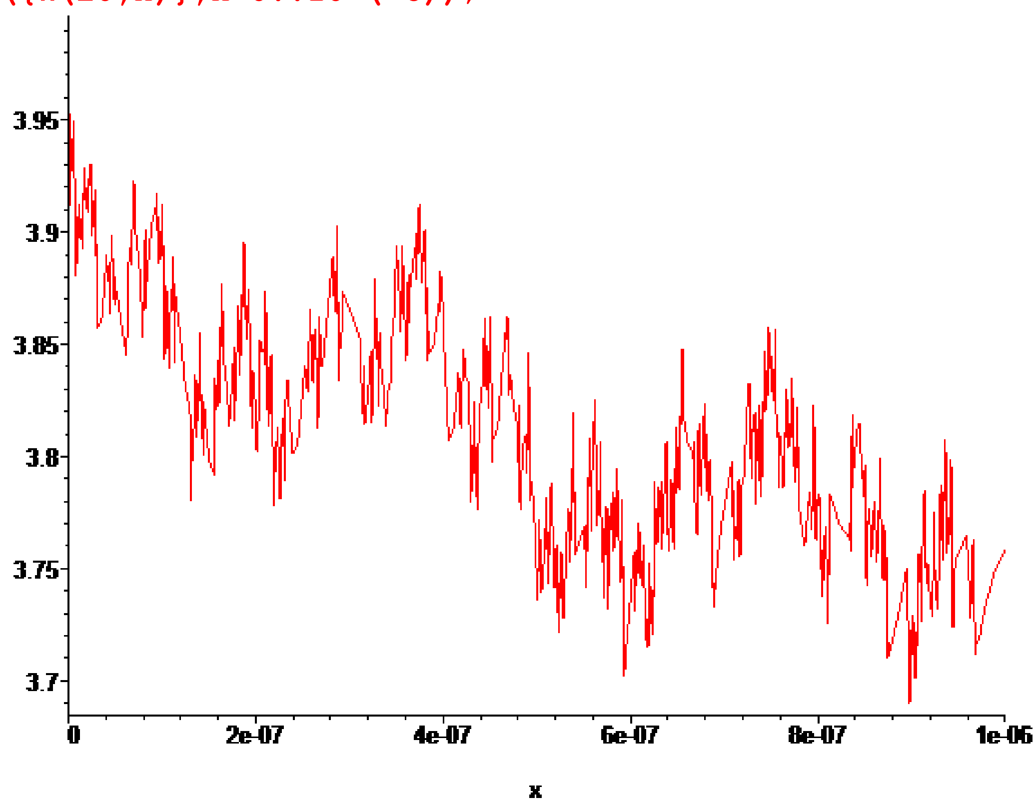

```
> plot({W(10,x)},x=0..10^(-4));
```



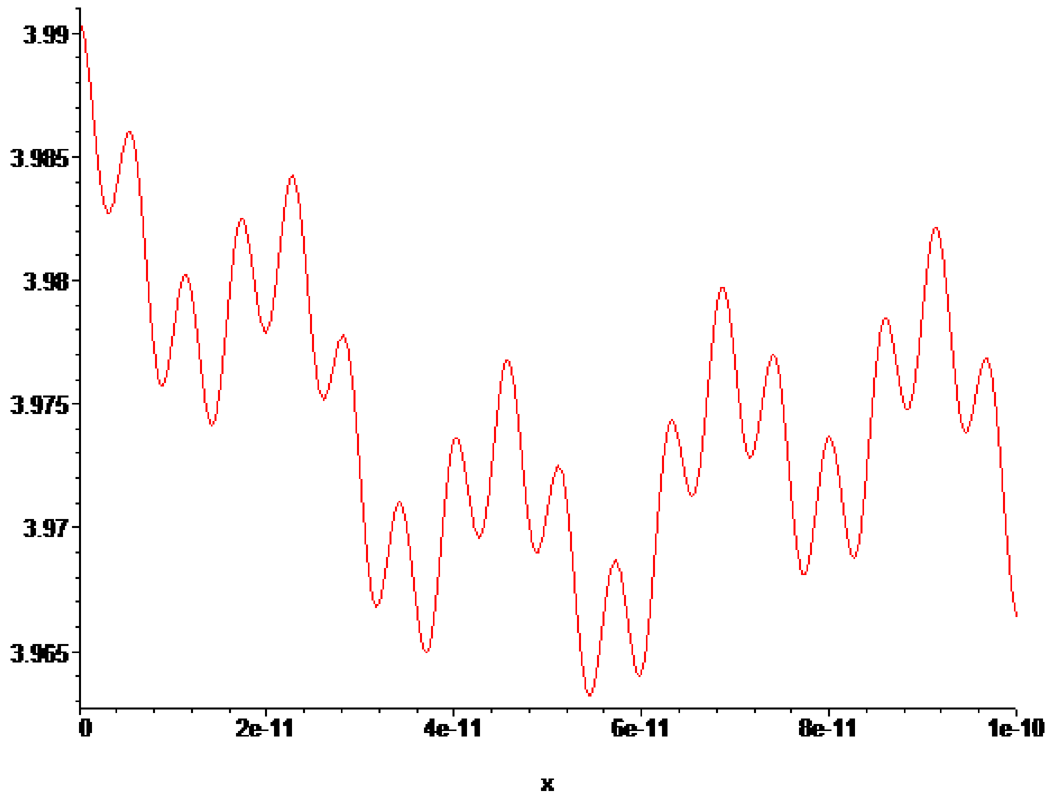

```
> plot({W(20,x)},x=0..10^(-4));
```



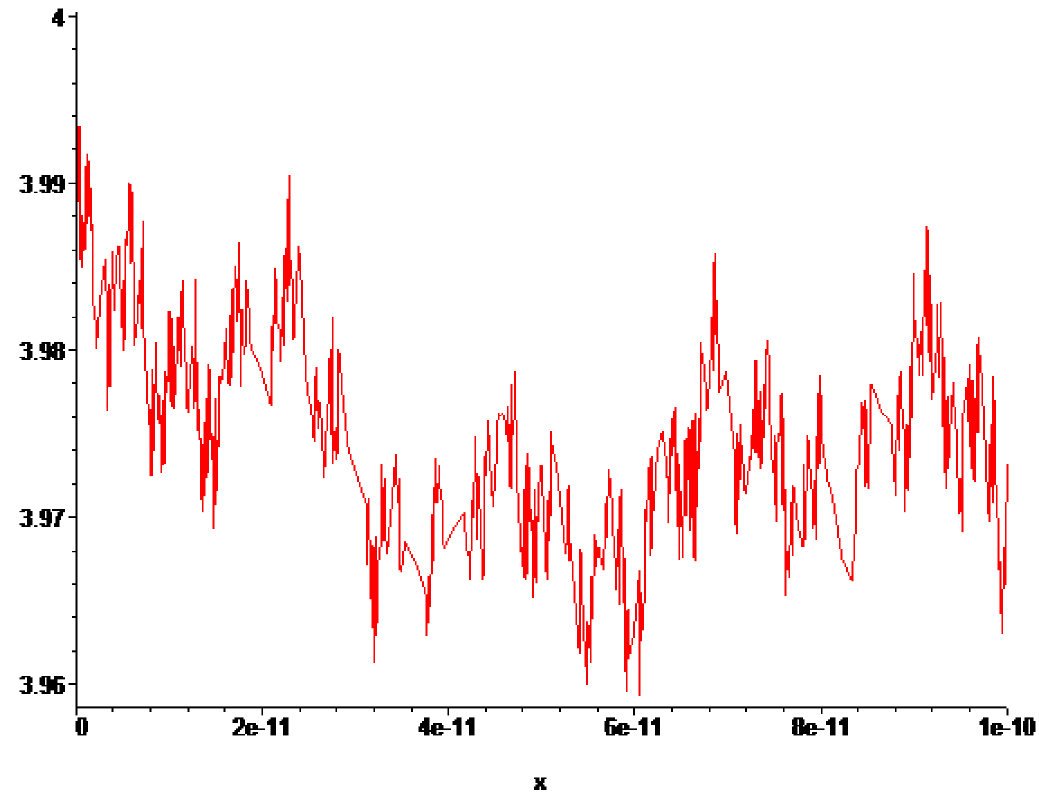

```
> plot({W(20,x)},x=0..10^(-6));
```




```
> plot({W(20,x)},x=0..10^(-10));
```




```
> plot({W(30,x)},x=0..10^(-10));
```




```
> #Etc......
```

>

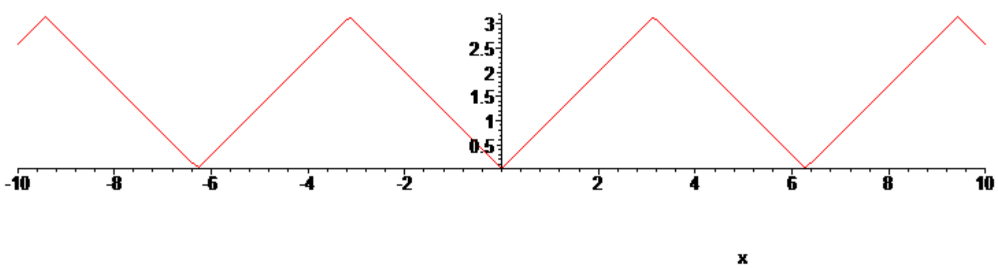
>

> **#EXEMPLE IMPORTANT DES SERIES DE FONCTIONS**

```
> f:=x->2*arcsin(abs(sin((x/2))));
```

$$f := x \to 2\ \arcsin\left(\left|\sin\left(\frac{1}{2}x\right)\right|\right)$$

```
> plot(f(x),x=-10..10,scaling=constrained);
```



>

```
> a:=n->(2/Pi)*int(t*cos(n*t),t=0..Pi);
> 'a(0)'=a(0);
assume(p,integer):
'a(p)'=normal(a(p));
```

$$a := n \to 2\,\frac{\int_0^{\pi} t\cos(n\,t)\,dt}{\pi}$$

$$a(0) = \pi$$

$$a(p) = 2\,\frac{(-1)^{p\sim} - 1}{\pi\, p\sim^2}$$

>

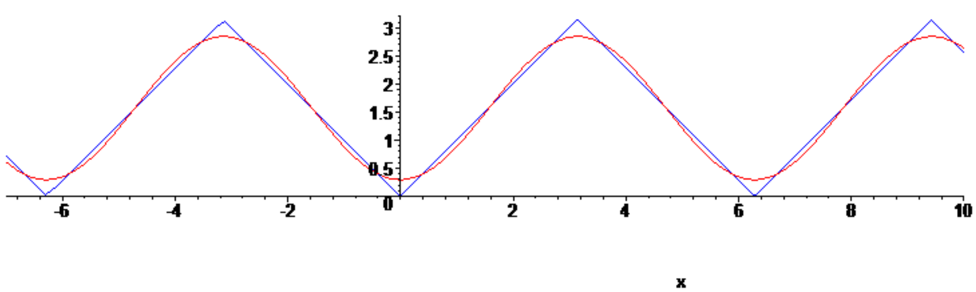
```
> S:=(n,x)->a(0)/2+sum(a(p)*cos(p*x),p=1..n);
```

$$S := (n, x) \rightarrow \frac{1}{2}\,a(0) + \left(\sum_{p=1}^{n} a(p)\cos(p\,x)\right)$$

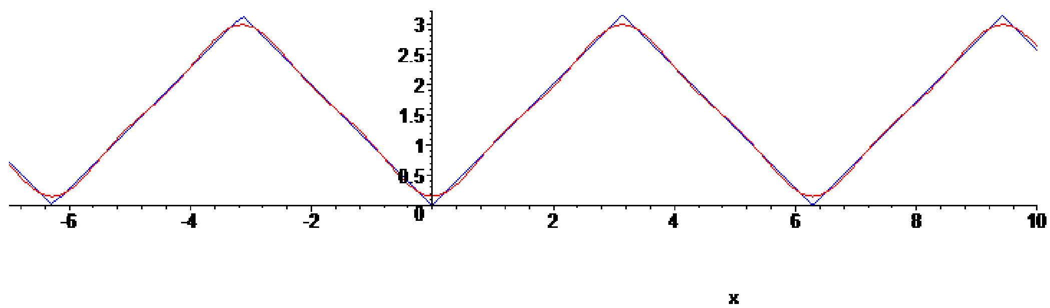
```
> P:=(a,b,n)->
plots[display](plot({S(n,x)},x=a..b,color=red),plot(f(x),x=a..b,color=blue,scaling=constrained));
```

$$P := (a, b, n) \rightarrow plots_{display}(\text{plot}(\{S(n, x)\}, x = a .. b, color = red), \text{plot}(f(x), x = a .. b, color = blue, scaling = constrained))$$

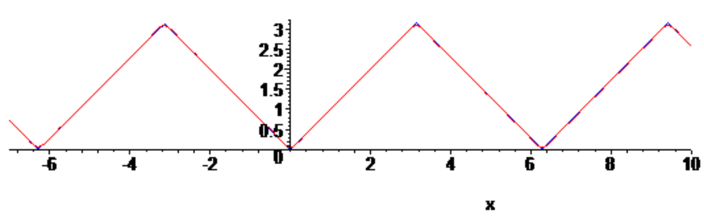
```
> P(-7,10,1);
```



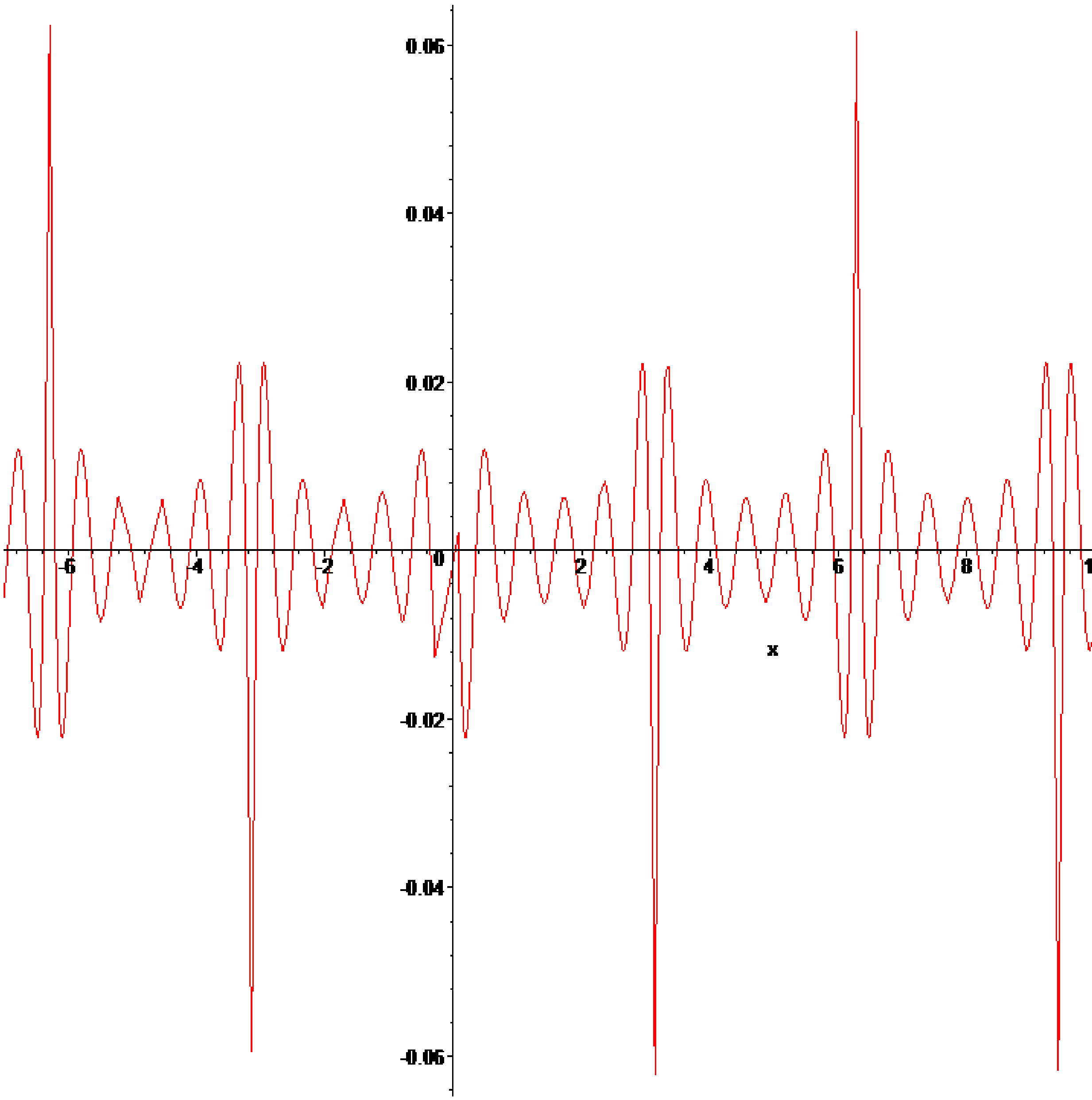
```
> P(-7,10,3);
```



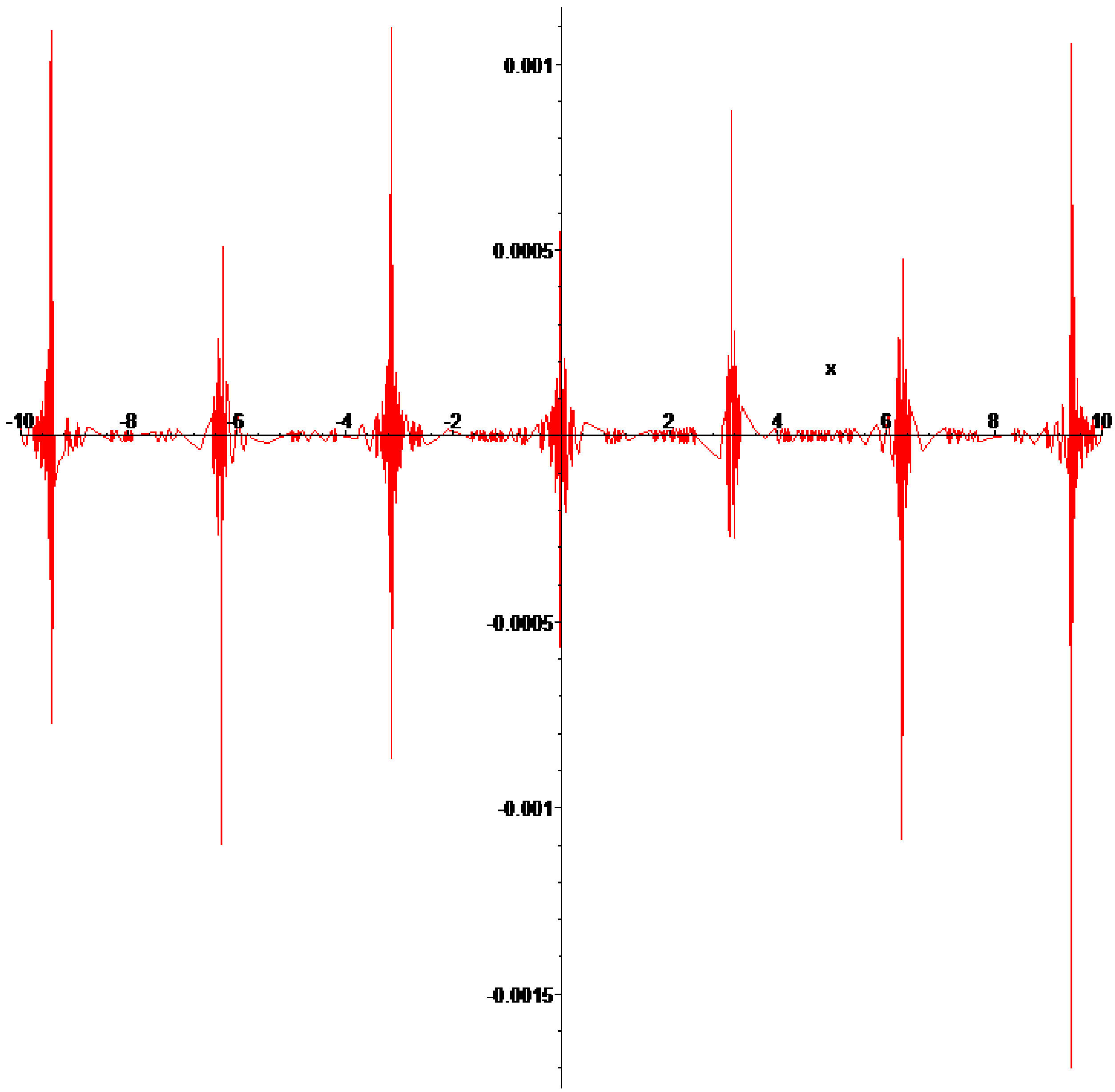

```
> P(-7,10,10);
```



```
> plot(S(10,x)-f(x),x=-7..10);
```




```
> plot(S(200,x)-f(x),x=-10..10);
```




```
> #En x=0
> f(0);
```

$$0$$

```
> a(0)+sum(a(p),p=1..infinity)=0;
```

$$\pi + \left(\sum_{p\sim=1}^{\infty} \left(2 \frac{(-1)^{p\sim} - 1}{\pi\, p\sim^2} \right) \right) = 0$$

```
> 'Donc    ',Sum(1/k^2,k=1..infinity)=Pi^2/6;
```

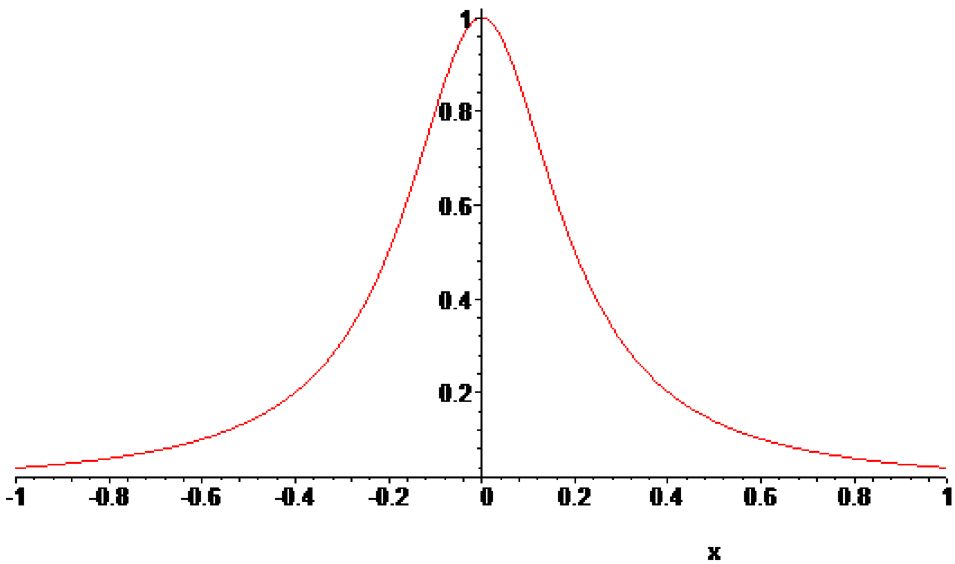
$$Donc, \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{1}{6} \pi^2$$

```
>
```

```
> #STONE-WEIERSTRASS
> f:=x->1/(1+25*x^2);
```

$$f := x \to \frac{1}{1 + 25\,x^2}$$

```
> plot(f(x),x=-1..1,scaling=constrained);
```



```
> p:=proc(f,a,x)  # poly de Lagrange
local paux,aux,i,j;
n:=nops(a);
paux:=0;
for i from 1 to n do
  aux:=1;
  for j from 1 to n do
    if j<>i then aux:=aux*(x-a[j])/(a[i]-a[j]); fi;
  od;
  paux:=paux+f(a[i])*aux;
od;
expand(paux);
end;
Warning, `n` is implicitly declared local to procedure `p`
```

$p := \text{proc}(f, a, x)$

local *paux*, *aux*, *i*, *j*, *n*;

$n := \text{nops}(a);$

$paux := 0;$

for *i* to *n* do

$aux := 1;$

for *j* to *n* do if $j \neq i$ then $aux := aux \times (x - a[j])/(a[i] - a[j])$ end if

end do;

$paux := paux + f(a[i]) \times aux$

end do;

$\text{expand}(paux)$

end proc

```
> a5:=[seq(i/5,i=-5..5)];
a20:=[seq(i/20,i=-20..20)];
```

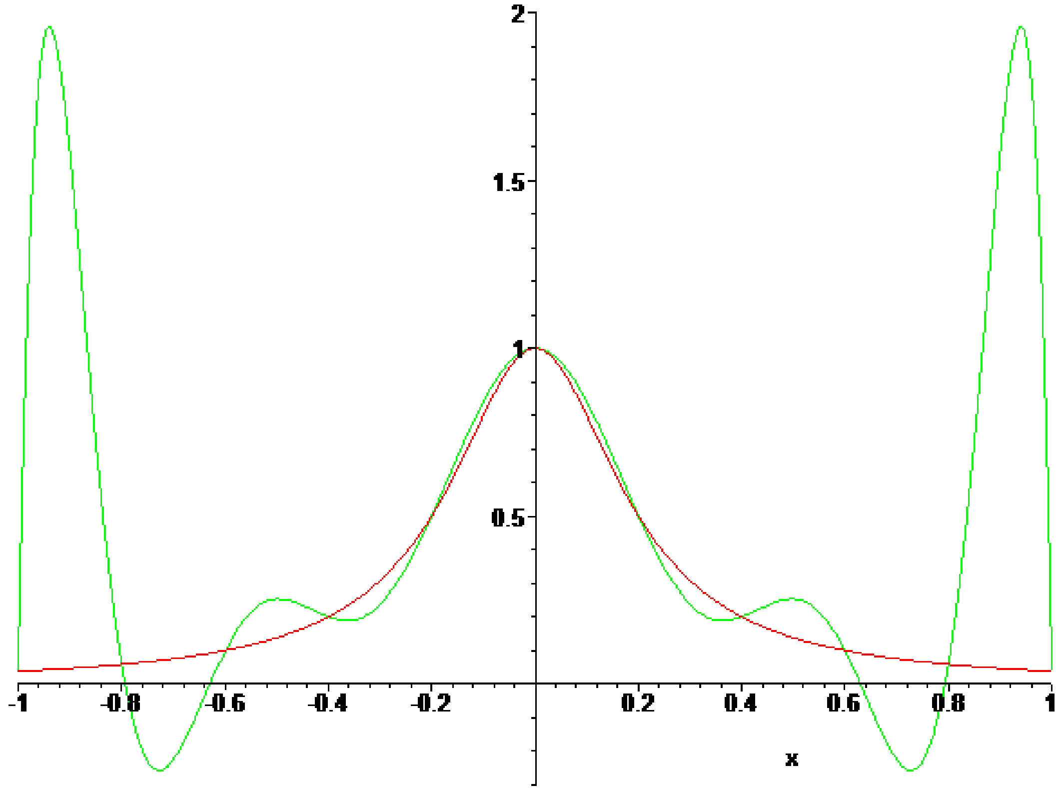
$$a5 := \left[-1, \frac{-4}{5}, \frac{-3}{5}, \frac{-2}{5}, \frac{-1}{5}, 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right]$$

$$a20 := \left[-1, \frac{-19}{20}, \frac{-9}{10}, \frac{-17}{20}, \frac{-4}{5}, \frac{-3}{4}, \frac{-7}{10}, \frac{-13}{20}, \frac{-3}{5}, \frac{-11}{20}, \frac{-1}{2}, \frac{-9}{20}, \frac{-2}{5}, \frac{-7}{20}, \frac{-3}{10}, \frac{-1}{4}, \frac{-1}{5}, \frac{-3}{20}, \frac{-1}{10}, \frac{-1}{20}, 0, \frac{1}{20}, \frac{1}{10}, \frac{3}{20}, \frac{1}{5}, \frac{1}{4}, \frac{3}{10}, \frac{7}{20}, \frac{2}{5}, \frac{9}{20}, \frac{1}{2}, \frac{11}{20}, \frac{3}{5}, \frac{13}{20}, \frac{7}{10}, \frac{3}{4}, \frac{4}{5}, \frac{17}{20}, \frac{9}{10}, \frac{19}{20}, 1\right]$$

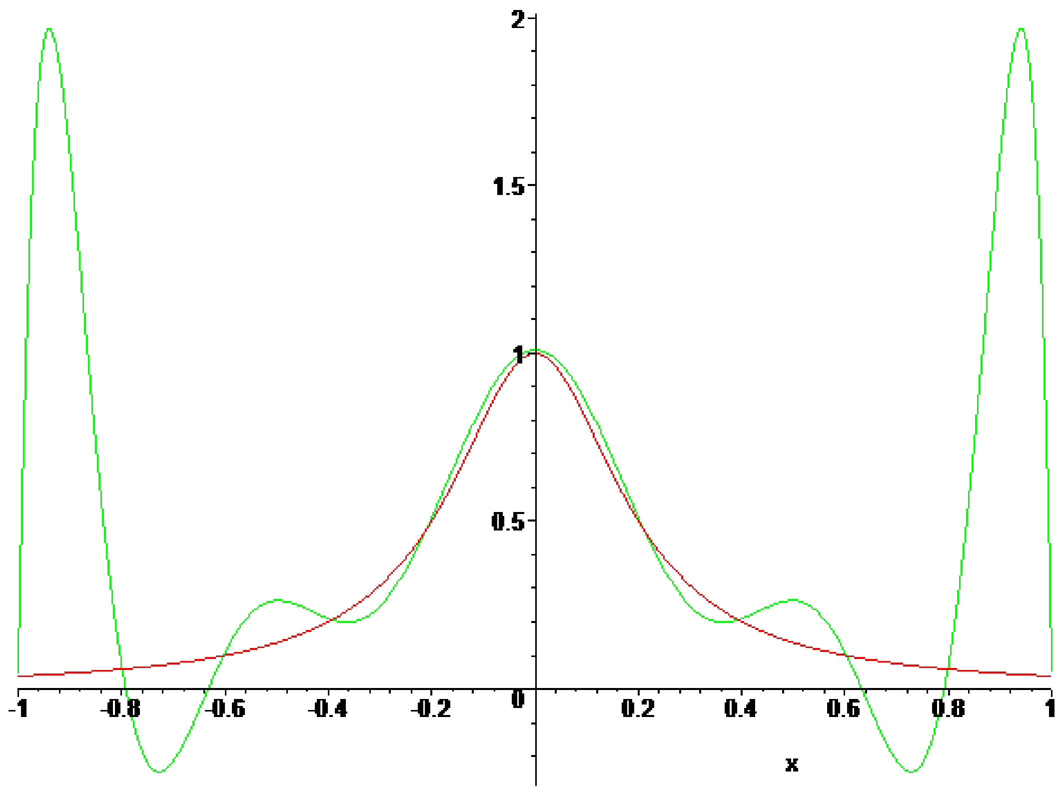
```
> p(f,a5,x);
```

$$1 - \frac{3725}{221}x^2 + \frac{109375}{221}x^8 - \frac{390625}{1768}x^{10} - \frac{51875}{136}x^6 + \frac{54525}{442}x^4$$

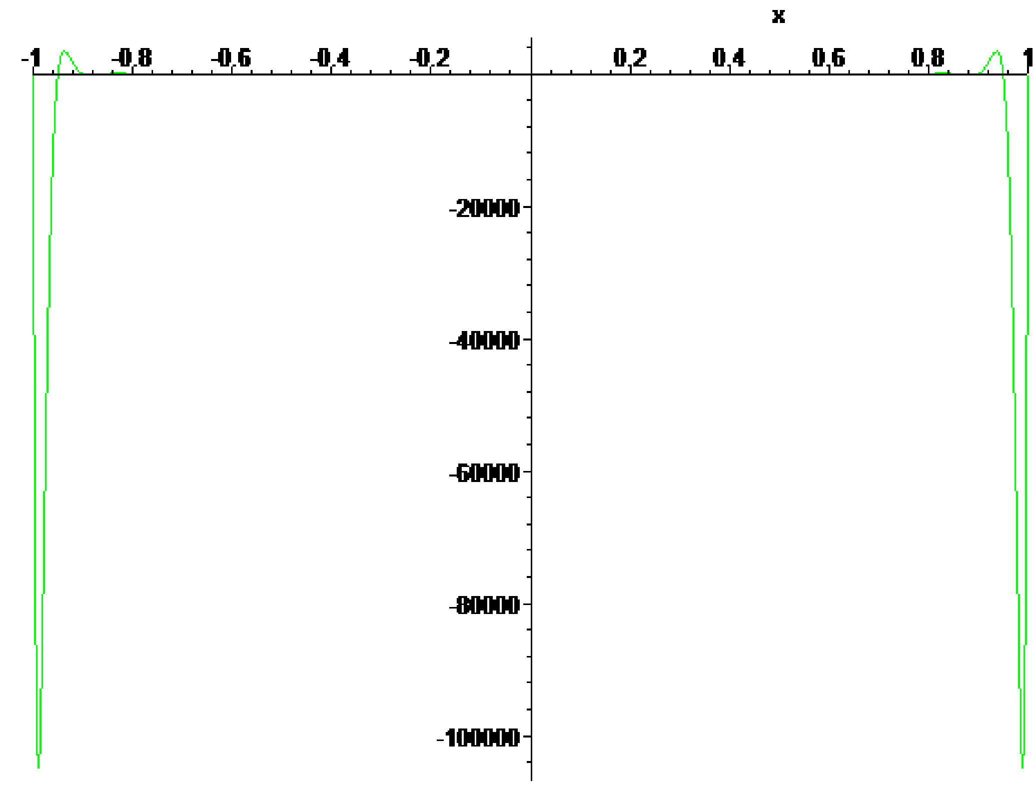
```
>
> plot({f(x),p(f,a5,x)},x=-1..1);
```



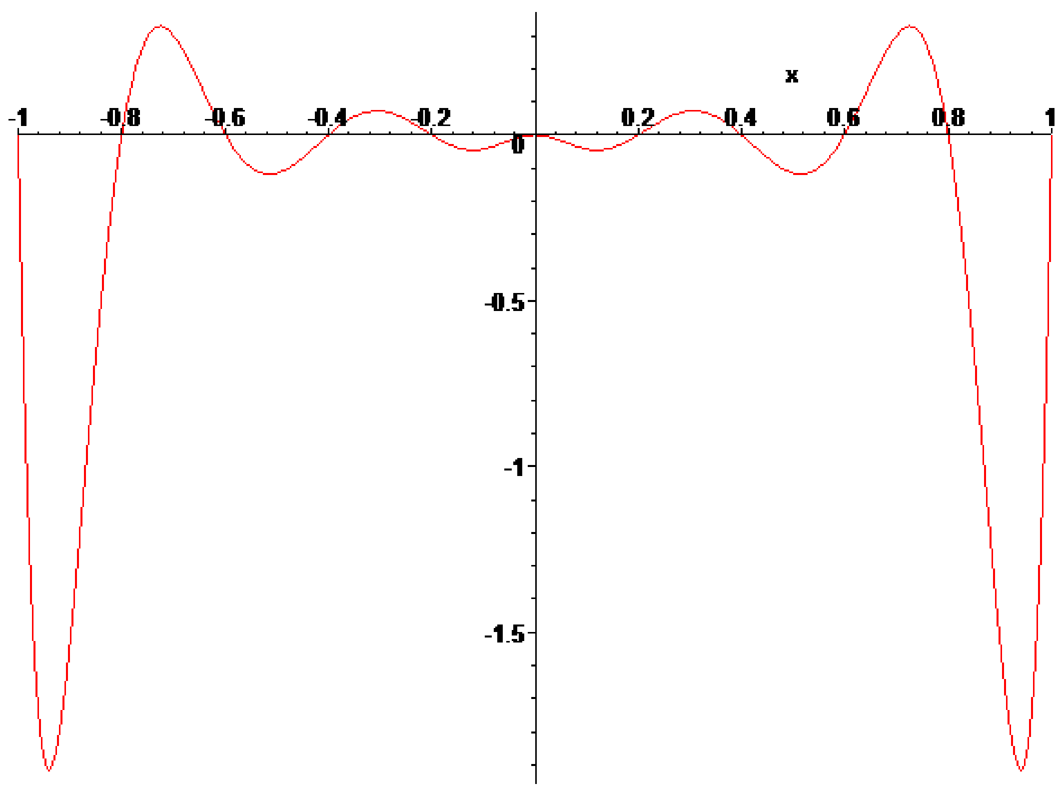

```
> plot({f(x),p(f,a5,x)+0.01},x=-1..1);
```



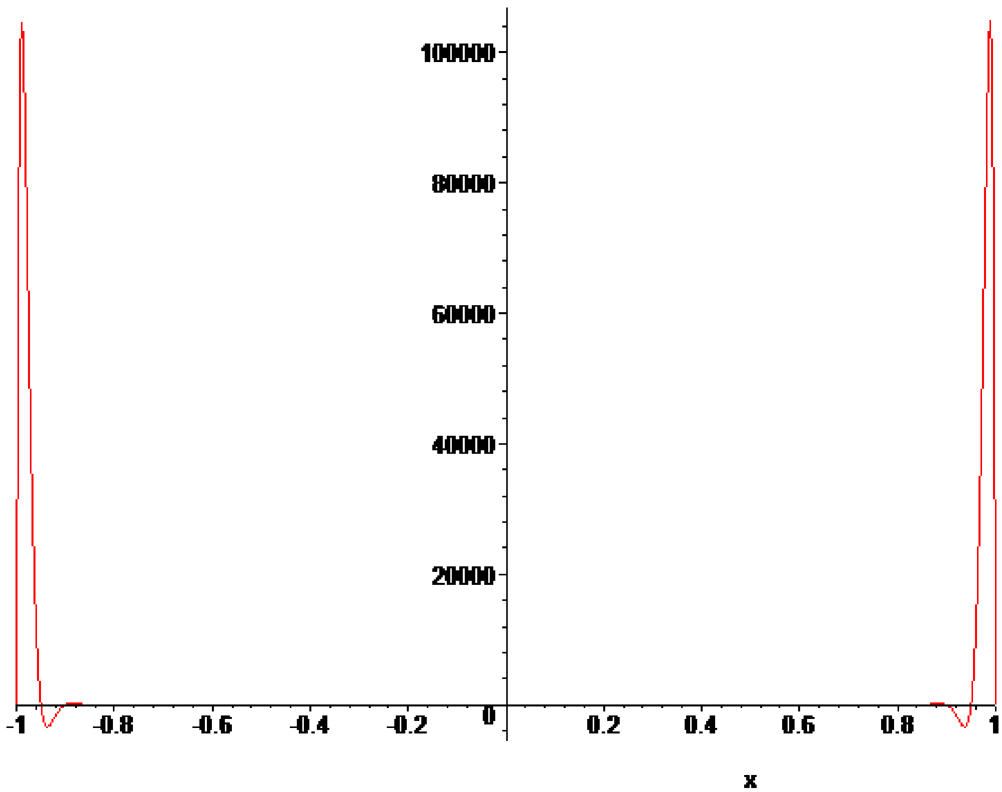

```
> plot({f(x),p(f,a20,x)},x=-1..1);
```




```
> plot({f(x)-p(f,a5,x)},x=-1..1);
```




```
> plot({f(x)-p(f,a20,x)},x=-1..1);
```




```
> t:=n->[seq((cos((2*k-1)*Pi/2/n)),k=1..n)];
```

$$t := n \rightarrow \left[\operatorname{seq}\left(\cos\left(\frac{1}{2} \frac{(2\,k-1)\,\pi}{n} \right), k = 1 \,..\, n \right) \right]$$

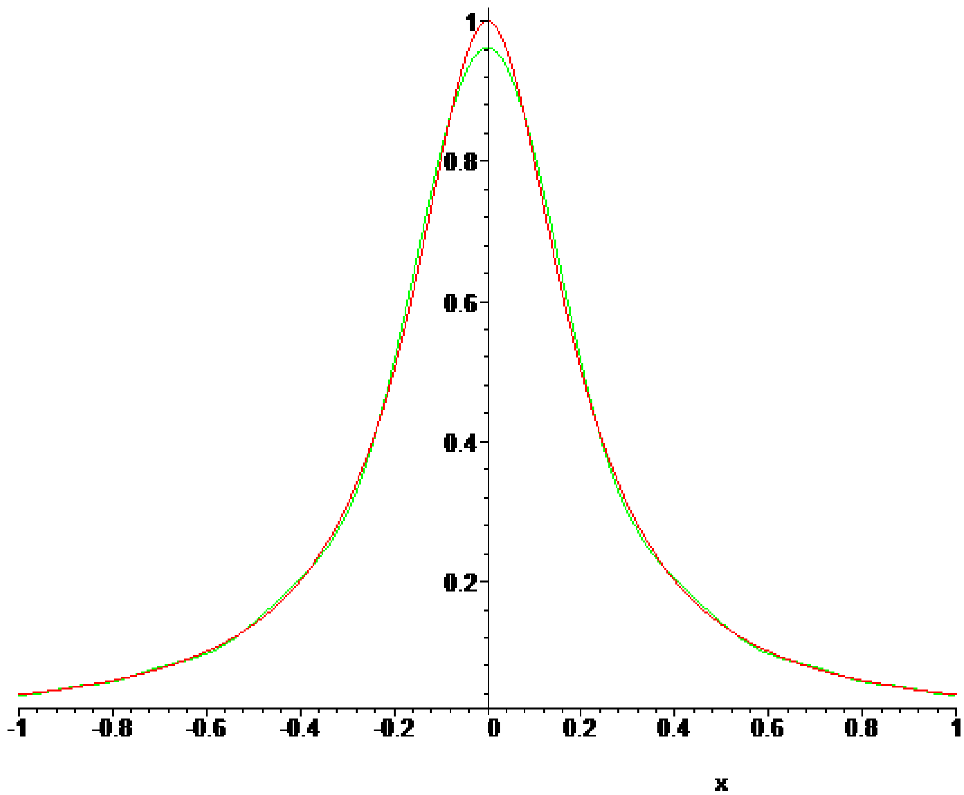
```
> sort(t(10.));
```

[cos(.2500000000 π), cos(.1500000000 π), cos(.05000000000 π), cos(.6500000000 π),
cos(.4500000000 π), cos(.5500000000 π), cos(.9500000000 π),
cos(.3500000000 π), cos(.8500000000 π), cos(.7500000000 π)]

```
> p(f,t(3),x);
```

$$-\frac{100}{79}x^2 + 1$$

```
> plot({f(x),p(f,t(20),x)},x=-1..1);
```




```
> plot({f(x),p(f,t(30),x)},x=-1..1);
```

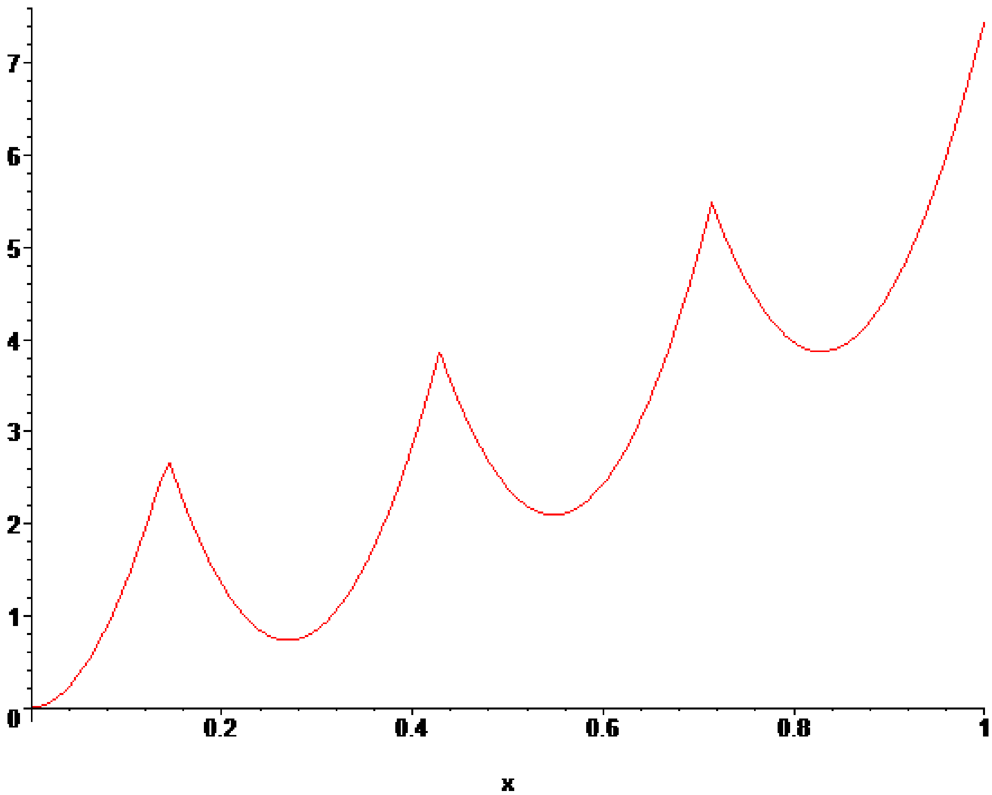
System error, , "ran out of memory"

```
>
>
>
>
> restart;
> #Bernstein
> B:=(n,x,f)->sum(binomial(n,k)*f(k/n)*x^k*(1-x)^(n-k),k=0..n);
```

$$B := (n, x, f) \rightarrow \sum_{k=0}^{n} \text{binomial}(n, k)\, f\left(\frac{k}{n}\right) x^k (1-x)^{(n-k)}$$

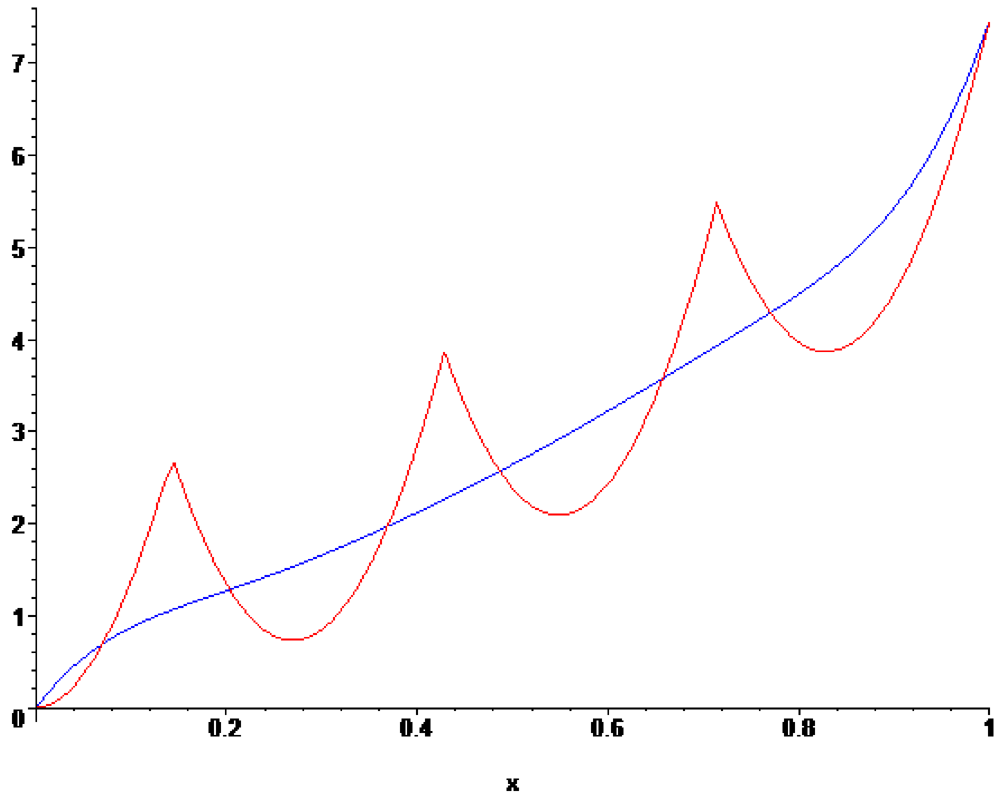
```
> f:=x-
>arcsin(sin(11*x))^2+5*sqrt(x^3);plot(f(x),x=0..1,color=red);
```

$$f := x \rightarrow \arcsin(\sin(11\,x))^2 + 5\sqrt{x^3}$$



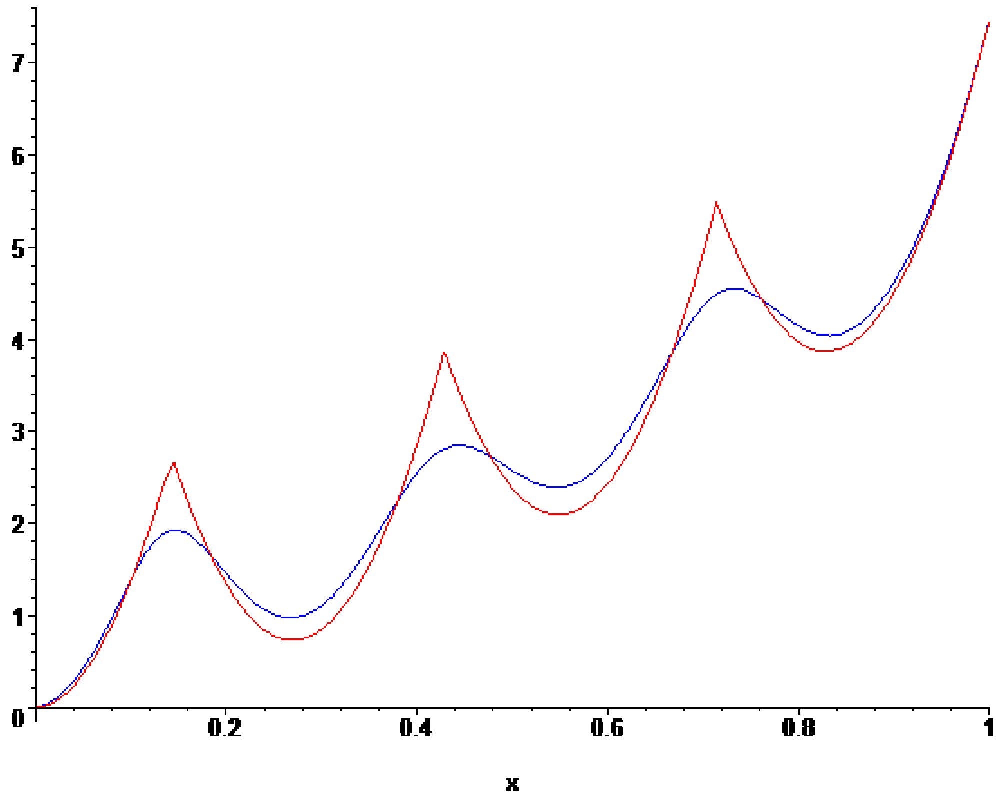

```
> n:=10;
p1:=plot(f(x),x=0..1,color=red):
p2:=plot(B(n,x,f),x=0..1,color=blue):
plots[display](p1,p2);
```

$$n := 10$$



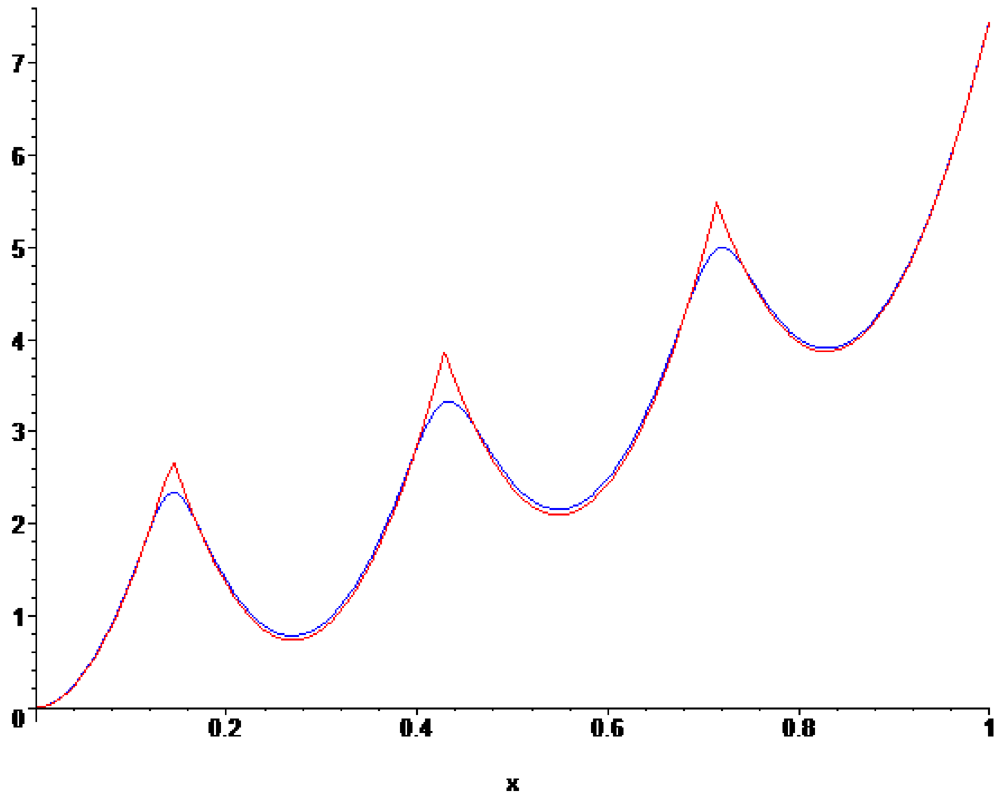

```
> n:=100;
p1:=plot(f(x),x=0..1,color=red):
p2:=plot(B(n,x,f),x=0..1,color=blue):
plots[display](p1,p2);
```

$$n := 100$$




```
> n:=500;
p1:=plot(f(x),x=0..1,color=red):
p2:=plot(B(n,x,f),x=0..1,color=blue):
plots[display](p1,p2);
```

$$n := 500$$



Hommage à Pierre Bezier

```
> F:=x->a*abs(x)+b*abs(x-1)+c*abs(x-3)+d*abs(x-4)+e*abs(x-8)+f*abs(x-9)+g*abs(x-12)+h*abs(x-13);
```

$$F := x \to a\,|x| + b\,|x-1| + c\,|x-3| + d\,|x-4| + e\,|x-8| + f\,|x-9| + g\,|x-12| + h\,|x-13|$$

```
>
eq:={F(0)=0,F(1)=2,F(3)=2,F(4)=4,F(8)=4,F(9)=2,F(12)=2,F(13)=0};
```

$$eq := \{b + 3\,c + 4\,d + 8\,e + 9\,f + 12\,g + 13\,h = 0,\ 4\,a + 3\,b + c + 4\,e + 5\,f + 8\,g + 9\,h = 4,\ 8\,a + 7\,b + 5\,c + 4\,d + f + 4\,g + 5\,h = 4,\ 9\,a + 8\,b + 6\,c + 5\,d + e + 3\,g + 4\,h = 2,\ 12\,a + 11\,b + 9\,c + 8\,d + 4\,e + 3\,f + h = 2,\ 13\,a + 12\,b + 10\,c + 9\,d + 5\,e + 4\,f + g = 0,\ a + 2\,c + 3\,d + 7\,e + 8\,f + 11\,g + 12\,h = 2,\ 3\,a + 2\,b + d + 5\,e + 6\,f + 9\,g + 10\,h = 2\}$$

```
> solve(eq,{a,b,c,d,e,f,g,h});
```

$$\{a = 1, b = -1, h = 1, f = 1, d = -1, g = -1, e = -1, c = 1\}$$

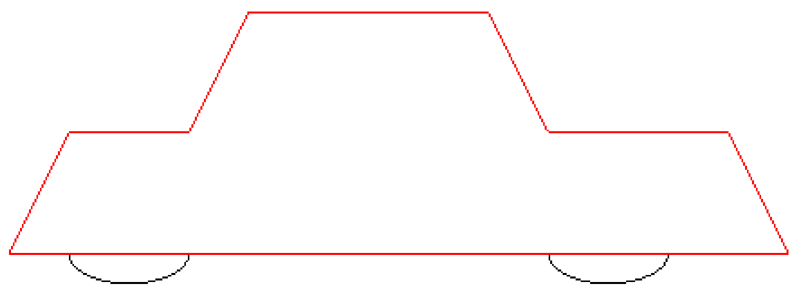
```
> assign(%);
>
>
> F(x);
```

$$|x| - |x-1| + |x-3| - |x-4| - |x-8| + |x-9| - |x-12| + |x-13|$$

```
> G:=x->F(13*x)/13;
```

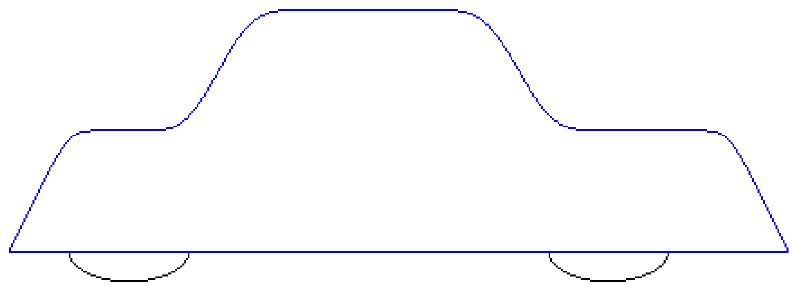
$$G := x \to \frac{1}{13}\,\mathrm{F}(13\,x)$$

```
>
q1:=plot({0,G(x)},x=0..1,scaling=constrained,axes=none,color=red,
numpoints=1000):
q2:=plot(-sqrt(1-(13*x-2)^2)/26,x=1/13..3/13,scaling=constrained,
color=black):
q3:=plot(-sqrt(1-(13*x-
10)^2)/26,x=9/13..11/13,scaling=constrained,
color=black):
plots[display](q1,q2,q3);
```



```
> n:=350;
p1:=plot({G(x)},x=0..1,color=red,scaling=constrained):
p2:=plot({0,B(n,x,G)},x=0..1,color=blue,scaling=constrained,axes=
none):
plots[display](p2,p2,q2,q3);
```

$$n := 350$$



>
> **#Cantor mathcurve**
>