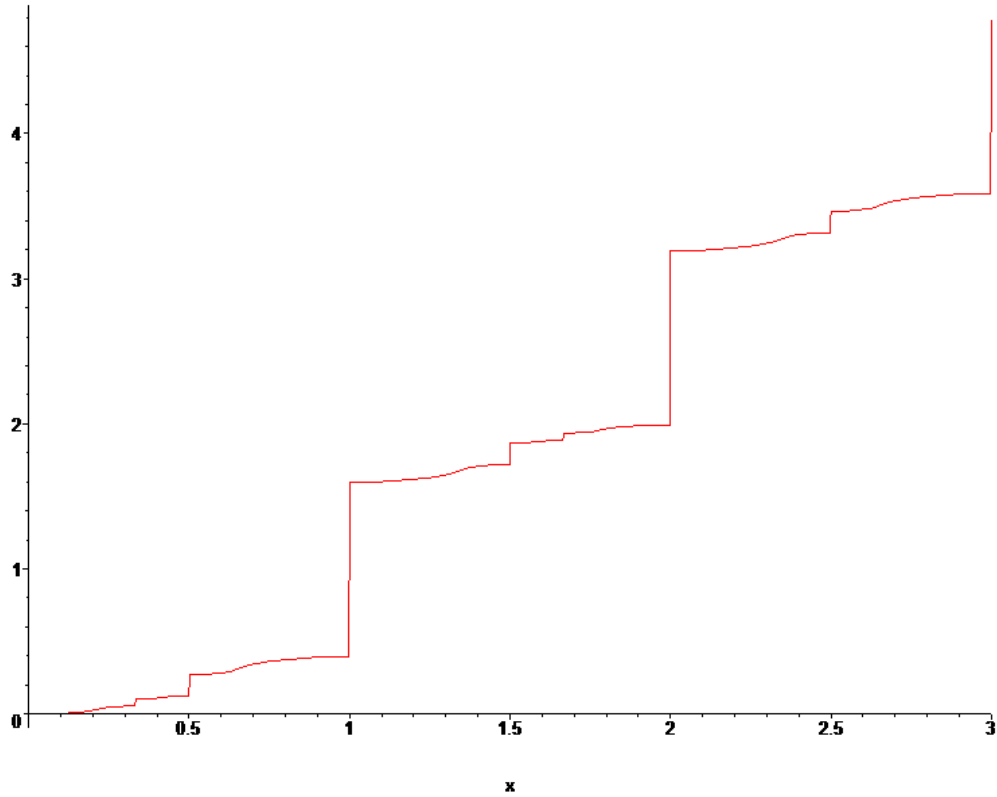


>

> **F:=x->sum(floor(n*x)/n^3,n=1..20);**

$$F := x \rightarrow \sum_{n=1}^{20} \frac{\text{floor}(n x)}{n^3}$$

> **plot(F(x),x=0..3);**



> **F(1/2);**

$$\frac{429707930936037361811563}{1576947742438717801152000}$$

> **F(1/2+0.1);**

F(1/2+0.01);

F(1/2+0.001);

.2898871321

.2724934501

.2724934501

> **F(1/2-0.1);**

F(1/2-0.01);

F(1/2-0.001);

.1148295661

.1228019519

.1228019519

```
> evalf(F(1/2.)-Zeta(3)/8);
```

.1222363372

```
> F(Pi*1.);
```

4.797336695

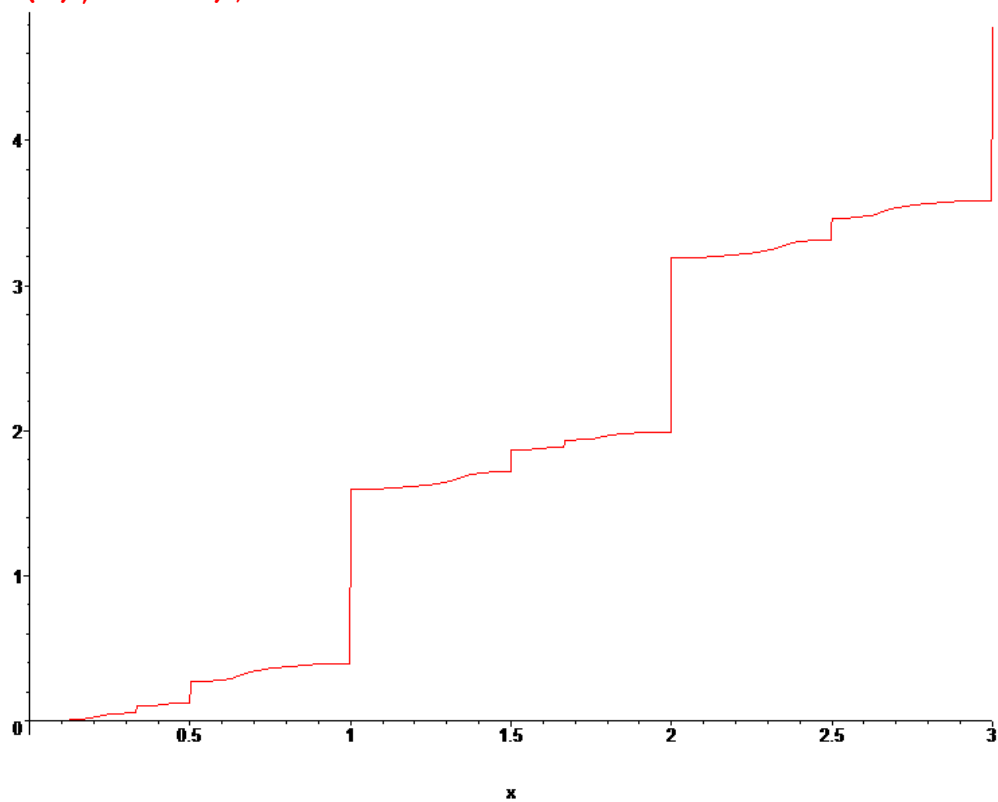
```
> F(Pi+0.1);  
F(Pi+0.01);  
F(Pi+0.001);
```

4.820054776
4.800741579
4.797336695

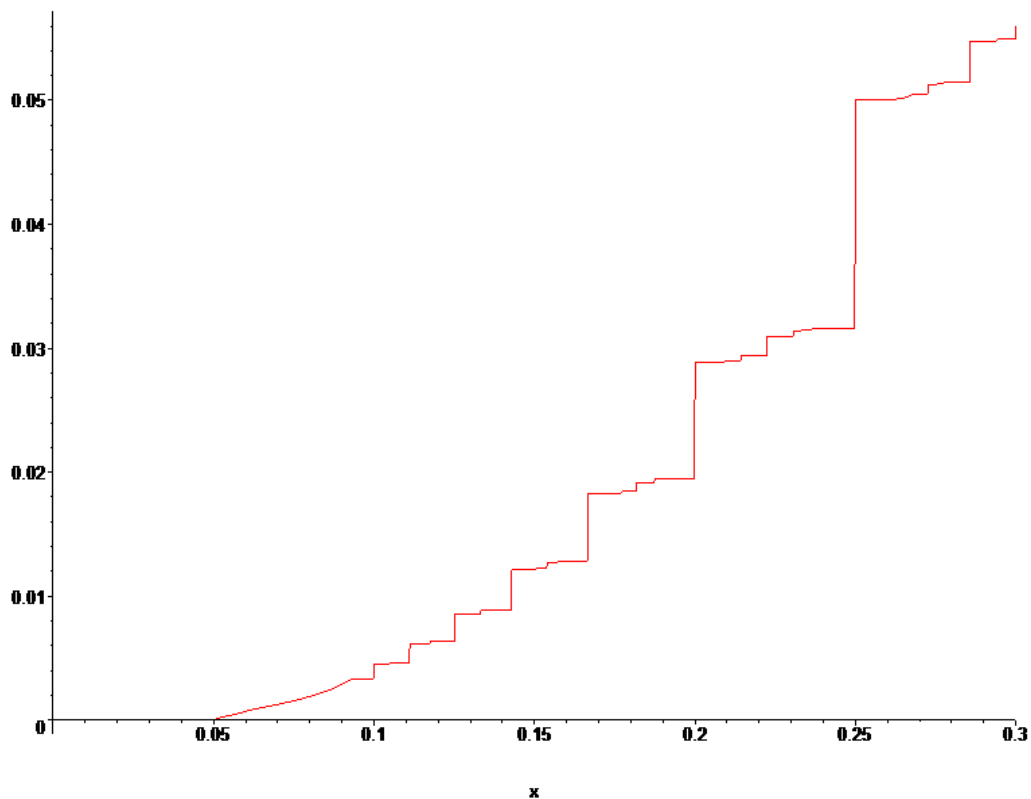
```
> F(Pi-0.1);  
F(Pi-0.01);  
F(Pi-0.001);
```

4.788489732
4.797040399
4.797336695

```
> plot(F(x),x=0..3);
```



```
> plot(F(x),x=0..0.3);
```



```

>
>
> restart;
> a:=proc(n)
if n=0 or n=1 then 1
else a(n-1)+a(n-2);
fi;
end:
>
> seq(a(n), n=0..10);
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

> r1:=(1+sqrt(5))/2; r2:=(1-sqrt(5))/2;
       $r1 := \frac{1}{2} + \frac{1}{2}\sqrt{5}$ 
       $r2 := \frac{1}{2} - \frac{1}{2}\sqrt{5}$ 

> aa:=n->x*r1^n+y*r2^n;
       $aa := n \rightarrow x r1^n + y r2^n$ 

> Solve({aa(0)=1, aa(1)=1}, {x, y});
       $\text{Solve}\left(\left\{x\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right) + y\left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right) = 1, x + y = 1\right\}, \{y, x\}\right)$ 

> solve({aa(0)=1, aa(1)=1}, {x, y});

```

$$\{x = \frac{1}{10}\sqrt{5} + \frac{1}{2}, y = -\frac{1}{10}\sqrt{5} + \frac{1}{2}\}$$

> **assign(%) ;**

> **x,y;**

$$\frac{1}{10}\sqrt{5} + \frac{1}{2}, -\frac{1}{10}\sqrt{5} + \frac{1}{2}$$

> **aa(4) ;**

$$\left(\frac{1}{10}\sqrt{5} + \frac{1}{2}\right)\left(\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^4 + \left(-\frac{1}{10}\sqrt{5} + \frac{1}{2}\right)\left(\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^4$$

> **expand(aa(4)) ;**

5

> **seq(expand(aa(n)), n=0..10) ;**

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

> **x:='x';y:='y' ;**

$x := x$

$y := y$

> **Solve (fx-1-x=x*(fx-1)+x^2*fx,fx) ;**

Solve ($fx - 1 - x = x (fx - 1) + x^2 fx$, fx)

> **solve (fx-1-x=x*(fx-1)+x^2*fx,fx) ;**

$$-\frac{1}{-1+x+x^2}$$

> **f:=x->1/(1-x-x^2) ;**

$$f := x \rightarrow \frac{1}{1-x-x^2}$$

> **series(f(x),x,11) ; #DSE**

$$1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + 21x^7 + 34x^8 + 55x^9 + 89x^{10} + O(x^{11})$$

> **add(a(n)*x^n,n=0..10) ;**

$$1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + 13x^6 + 21x^7 + 34x^8 + 55x^9 + 89x^{10}$$

> **convert(f(x),parfrac,x,sqrt(5)) ; #D.E.S.**

$$-\frac{2}{5} \frac{\sqrt{5}}{2x+1-\sqrt{5}} + \frac{\frac{2}{5}\sqrt{5}}{2x+1+\sqrt{5}}$$

> **s1:=(-1+sqrt(5))/2;s2:=(-1-sqrt(5))/2;**

$$s1 := -\frac{1}{2} + \frac{1}{2}\sqrt{5}$$

$$s2 := -\frac{1}{2} - \frac{1}{2}\sqrt{5}$$

> 1/sqrt(5)/(s1-x)-1/sqrt(5)/(s2-x);

$$\frac{1}{5} \frac{\sqrt{5}}{-\frac{1}{2} + \frac{1}{2}\sqrt{5} - x} - \frac{1}{5} \frac{\sqrt{5}}{-\frac{1}{2} - \frac{1}{2}\sqrt{5} - x}$$

> normal(%);

$$-4 \frac{1}{(2x+1-\sqrt{5})(2x+1+\sqrt{5})}$$

> rationalize(%);

$$-4 \frac{1}{4x^2 + 4x - 4}$$

> normal(%);

$$-\frac{1}{-1+x+x^2}$$

> aaa:=n->1/sqrt(5)*(1/s1)^(n+1)-1/sqrt(5)*(1/s2)^(n+1);

$$aaa := n \rightarrow \frac{\left(\frac{1}{s1}\right)^{(n+1)}}{\sqrt{5}} - \frac{\left(\frac{1}{s2}\right)^{(n+1)}}{\sqrt{5}}$$

> aaa(5);

$$\frac{1}{5} \frac{\sqrt{5}}{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)^6} - \frac{1}{5} \frac{\sqrt{5}}{\left(-\frac{1}{2} - \frac{1}{2}\sqrt{5}\right)^6}$$

> rationalize(%);

$$8$$

> seq(rationalize(aaa(n)),n=0..10);

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89

>

>

>

>

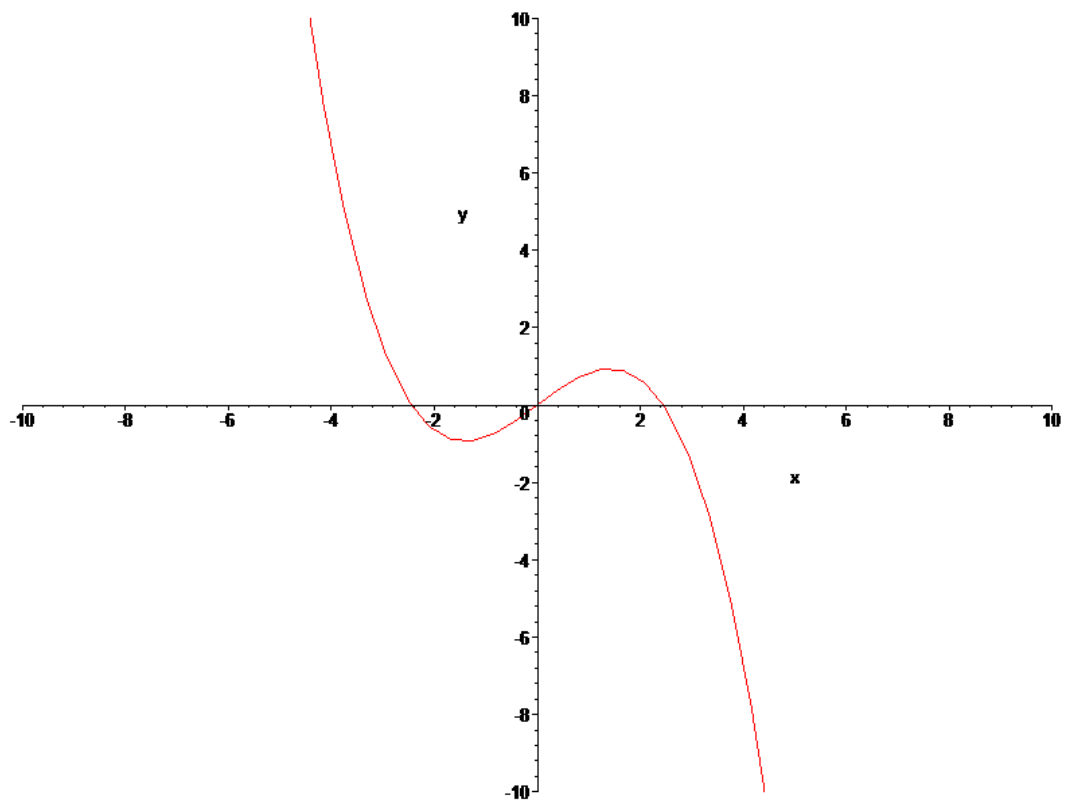
> restart;

>

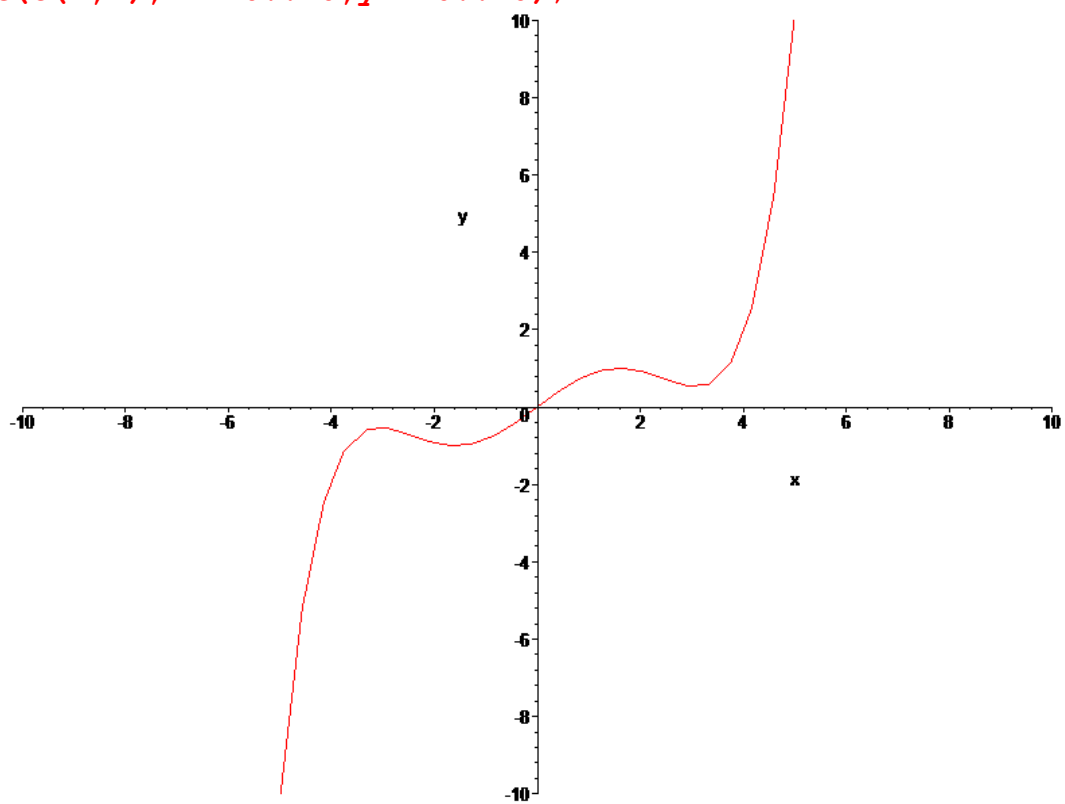
> S:=(x,N)->sum((-1)^n*x^(2*n+1)/(2*n+1)!,n=0..N);

$$S := (x, N) \rightarrow \sum_{n=0}^N \frac{(-1)^n x^{(2n+1)}}{(2n+1)!}$$

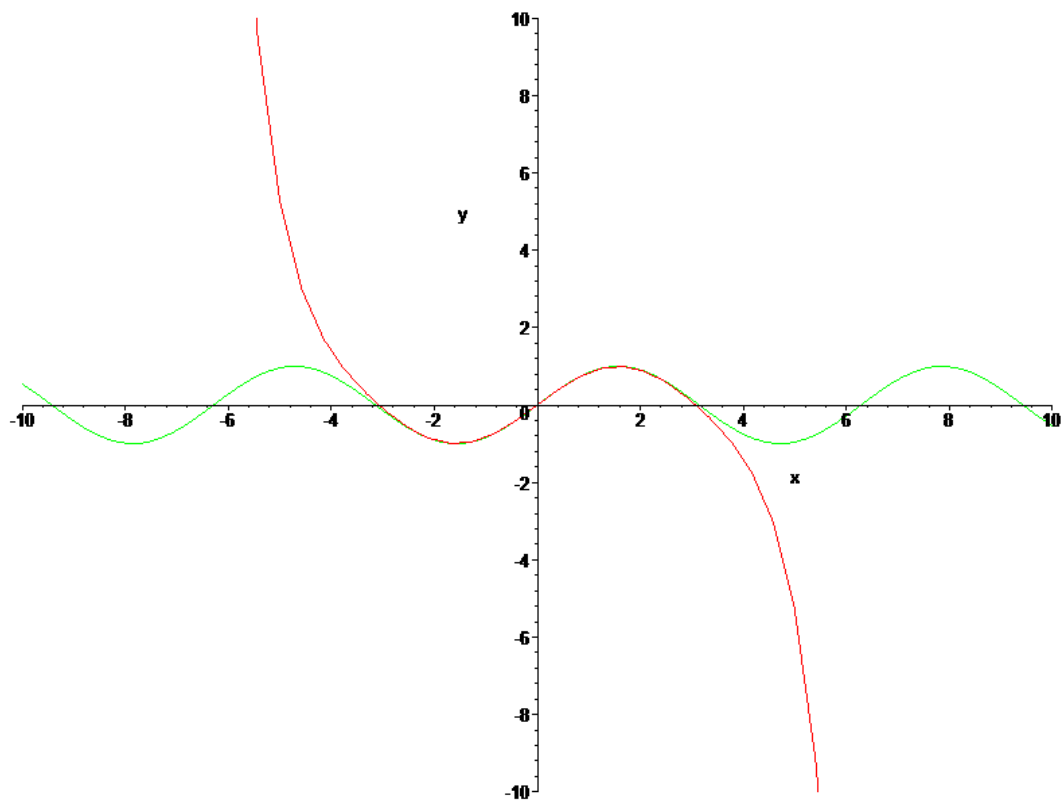
> plot(S(x,1),x=-10..10,y=-10..10);



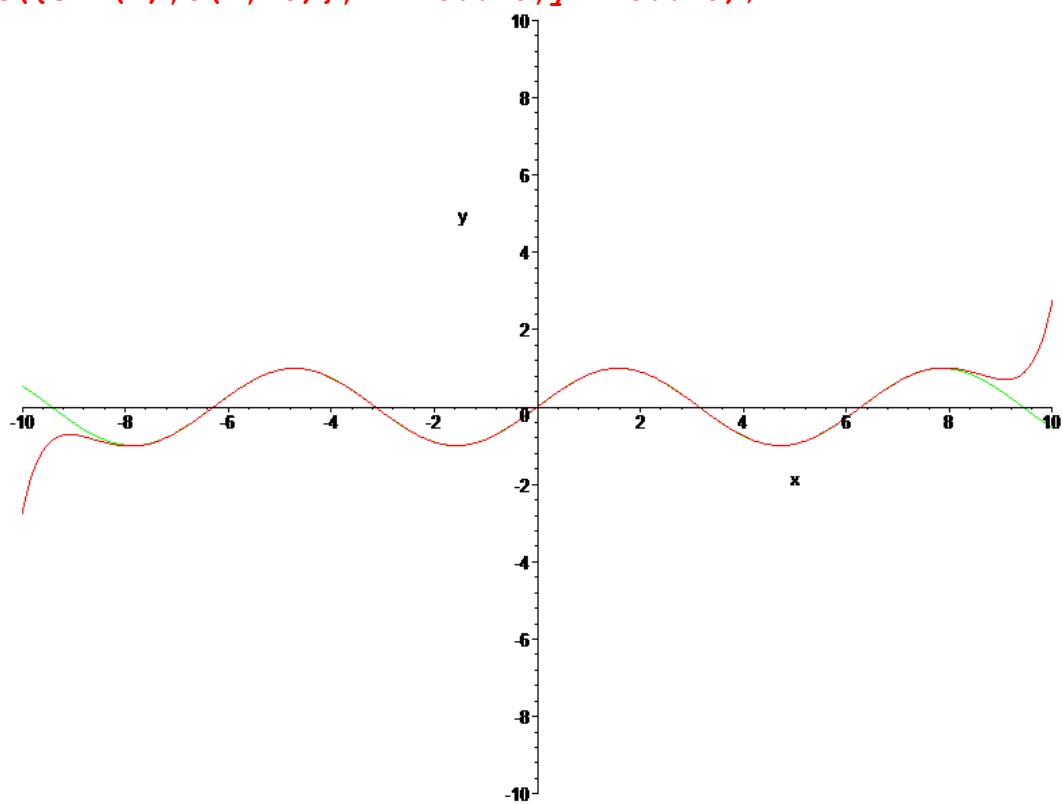
```
> plot(S(x,2),x=-10..10,y=-10..10);
```



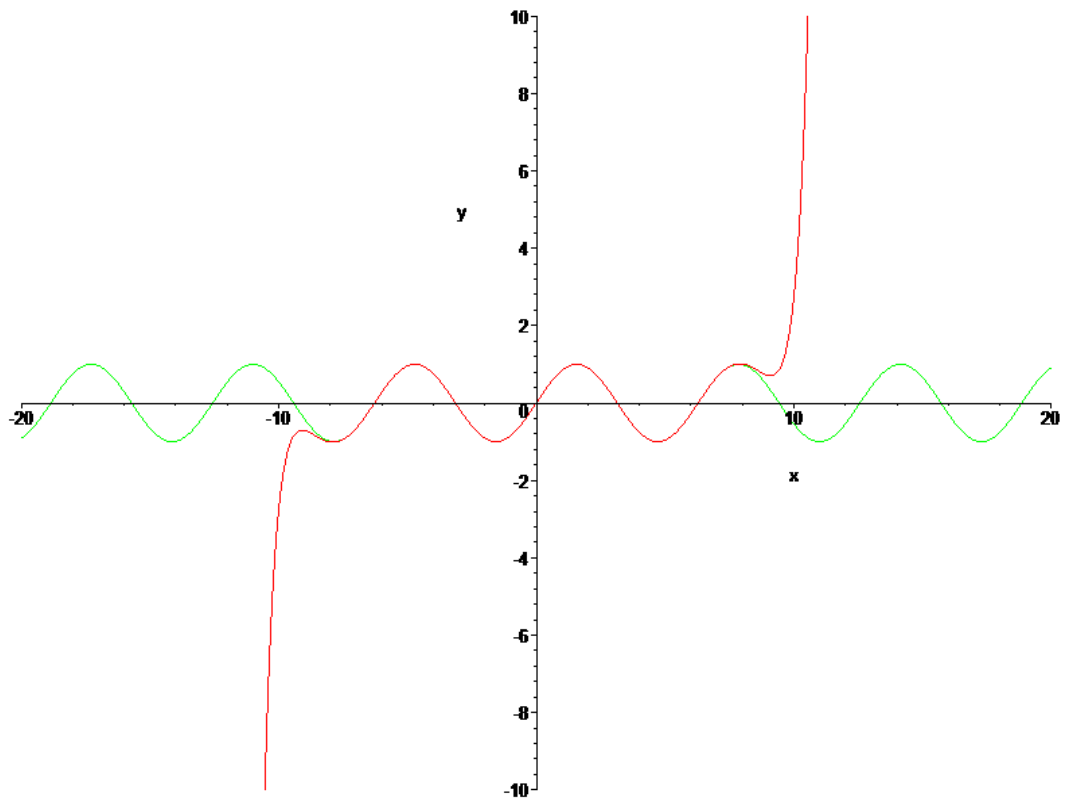
```
> plot({sin(x),S(x,3)},x=-10..10,y=-10..10);
```



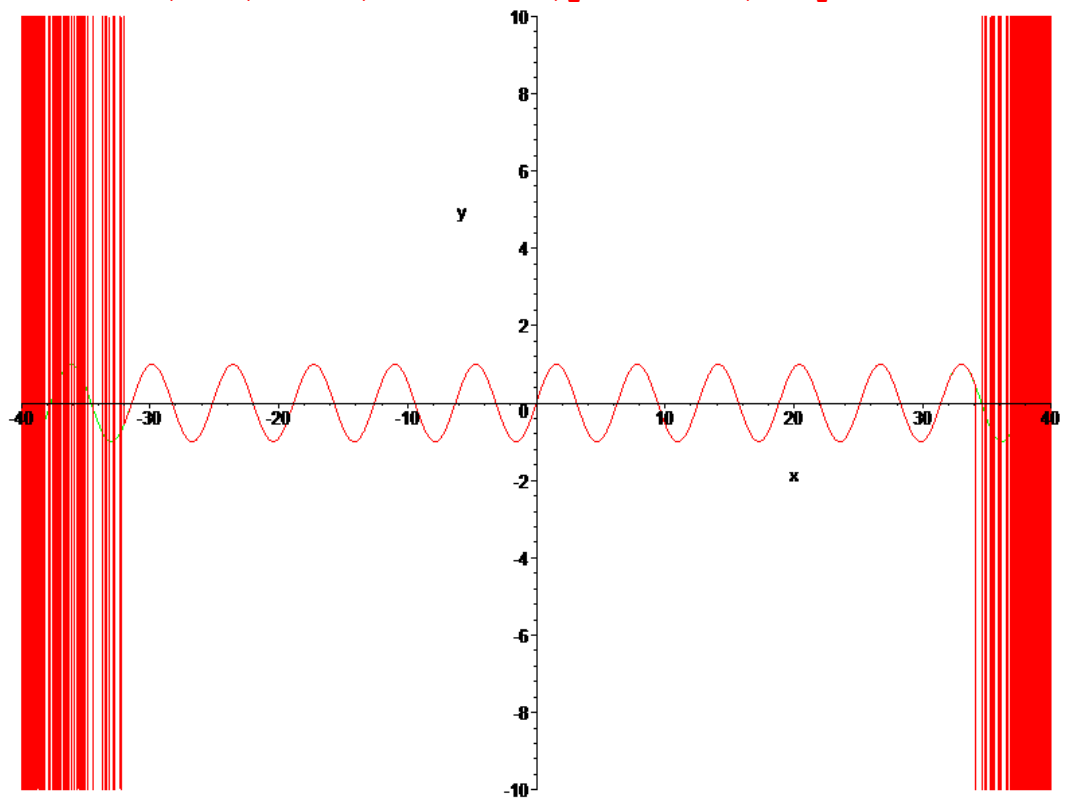
```
> plot({sin(x), S(x, 10)}, x=-10..10, y=-10..10);
```



```
> plot({sin(x), S(x, 10)}, x=-20..20, y=-10..10, numpoints=10000);
```



```
> plot({sin(x), S(x,100)}, x=-40..40, y=-10..10, numpoints=1000);
```

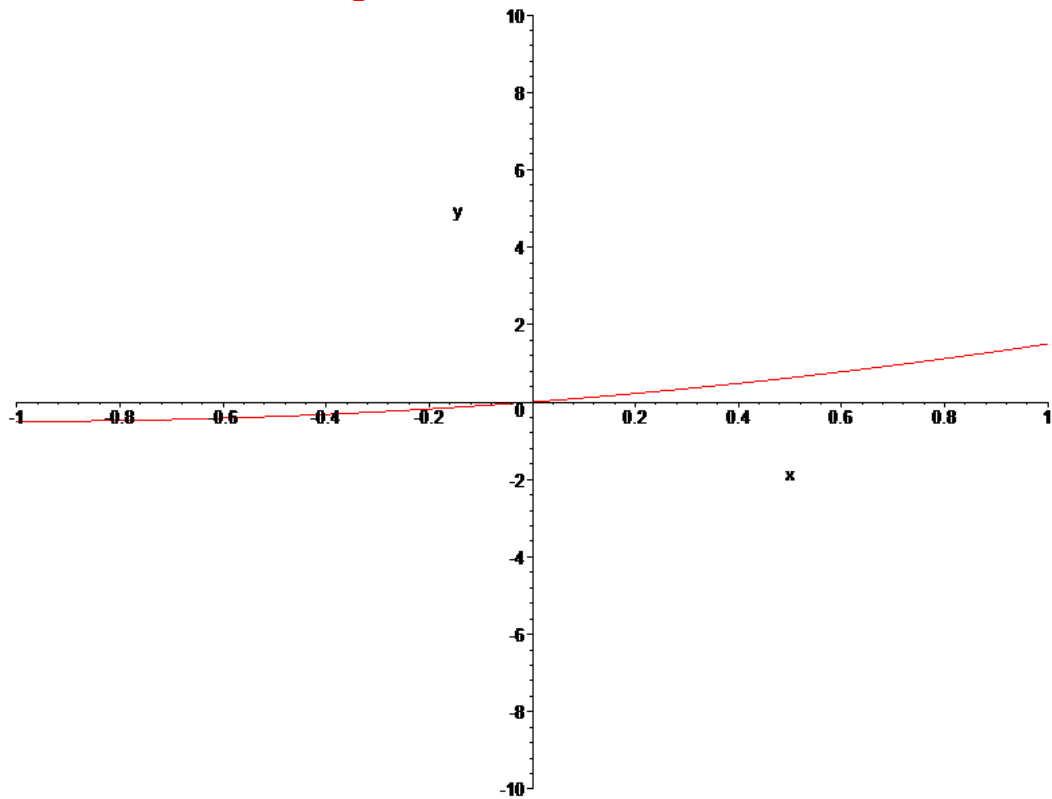


```
>
>
```

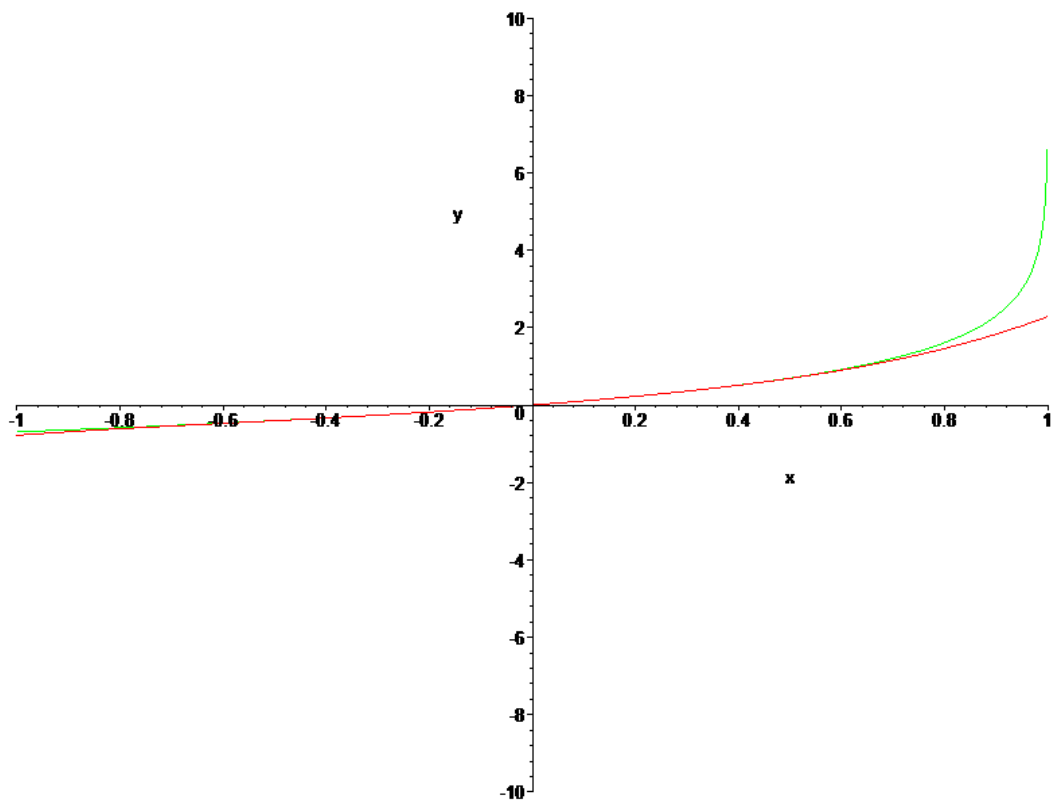
```
> S := (x, N) -> sum (x^ (n) / (n) , n=1 .. N) ;
```

$$S := (x, N) \rightarrow \sum_{n=1}^N \frac{x^n}{n}$$

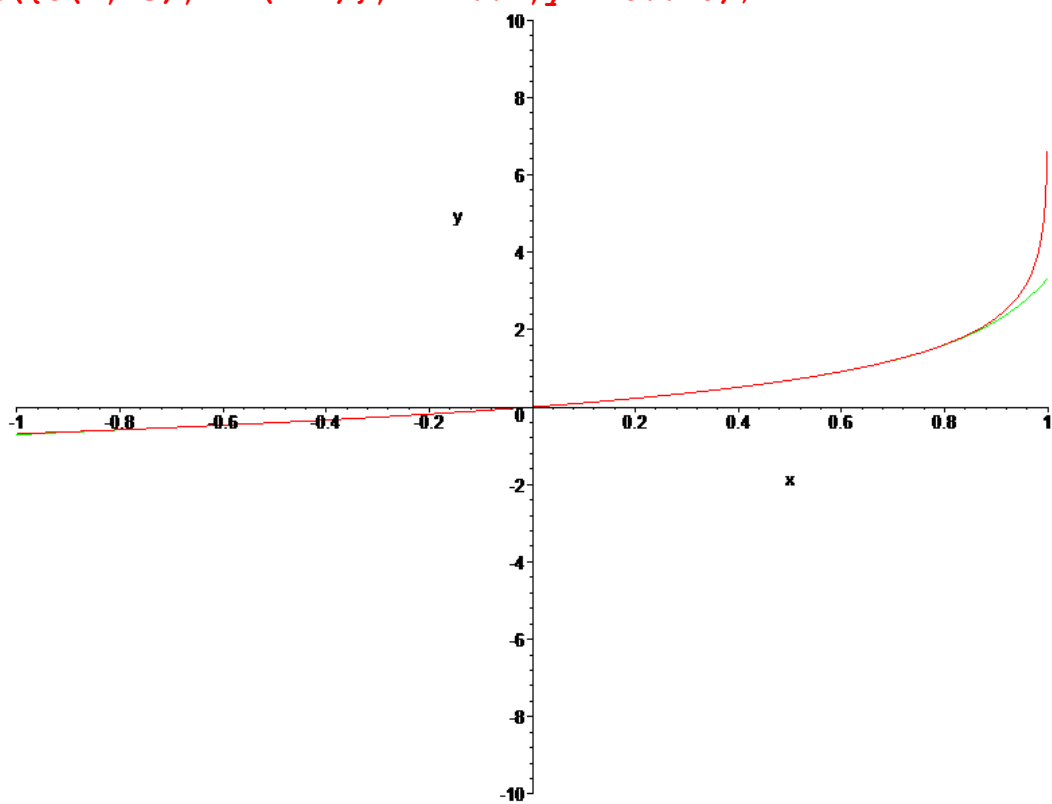
```
> plot ({S (x, 2) }, x=-1 .. 1, y=-10 .. 10) ;
```



```
> plot ({S (x, 5) , -ln (1-x) }, x=-1 .. 1, y=-10 .. 10) ;
```



```
> plot({S(x,15) , -ln(1-x) }, x=-1..1, y=-10..10);
```



```
>
```

```
> restart;
```

```

>
> nb:=proc(n)
local c,i,j,k;
  c:=0;
  for i from 0 to n do
    for j from 0 to n do
      for k from 0 to n do
        if i+2*j+3*k=n then c:=c+1;
        fi;
      od;od;od;
    c;
  end;
>
> seq(nb(n) ,n=0..10) ;
1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14

```

```

> nb(100) ;
884

```

```

> f:=x->1/(1-x)/(1-x^2)/(1-x^3) ;

$$f := x \rightarrow \frac{1}{(1-x)(1-x^2)(1-x^3)}$$


```

```

> series(f(x),x,101) ;
1 + x + 2 x^2 + 3 x^3 + 4 x^4 + 5 x^5 + 7 x^6 + 8 x^7 + 10 x^8 + 12 x^9 + 14 x^10 + 16 x^11 + 19 x^12 +
21 x^13 + 24 x^14 + 27 x^15 + 30 x^16 + 33 x^17 + 37 x^18 + 40 x^19 + 44 x^20 + 48 x^21 + 52 x^22 +
56 x^23 + 61 x^24 + 65 x^25 + 70 x^26 + 75 x^27 + 80 x^28 + 85 x^29 + 91 x^30 + 96 x^31 + 102
x^32 + 108 x^33 + 114 x^34 + 120 x^35 + 127 x^36 + 133 x^37 + 140 x^38 + 147 x^39 + 154 x^40 +
161 x^41 + 169 x^42 + 176 x^43 + 184 x^44 + 192 x^45 + 200 x^46 + 208 x^47 + 217 x^48 + 225
x^49 + 234 x^50 + 243 x^51 + 252 x^52 + 261 x^53 + 271 x^54 + 280 x^55 + 290 x^56 + 300 x^57 +
310 x^58 + 320 x^59 + 331 x^60 + 341 x^61 + 352 x^62 + 363 x^63 + 374 x^64 + 385 x^65 + 397
x^66 + 408 x^67 + 420 x^68 + 432 x^69 + 444 x^70 + 456 x^71 + 469 x^72 + 481 x^73 + 494 x^74 +
507 x^75 + 520 x^76 + 533 x^77 + 547 x^78 + 560 x^79 + 574 x^80 + 588 x^81 + 602 x^82 + 616
x^83 + 631 x^84 + 645 x^85 + 660 x^86 + 675 x^87 + 690 x^88 + 705 x^89 + 721 x^90 + 736 x^91 +
752 x^92 + 768 x^93 + 784 x^94 + 800 x^95 + 817 x^96 + 833 x^97 + 850 x^98 + 867 x^99 + 884
x^100 + O(x^101)

```

```

>
> 1/f(x) ;

$$(1-x)(1-x^2)(1-x^3)$$


```

```

> 1/factor(1/f(x),{I,sqrt(3)}) ;

$$-4 \frac{1}{(2x+1+I\sqrt{3})(2x+1-I\sqrt{3})(x+1)(-1+x)^3}$$


```

>

>

> **convert**(1/factor(1/f(x) , {I,sqrt(3)}) ,parfrac,x) ;#D.E.S.

$$\frac{1}{9} \frac{1+I\sqrt{3}}{2x+1+I\sqrt{3}} - \frac{1}{9} \frac{-1+I\sqrt{3}}{2x+1-I\sqrt{3}} + \frac{\frac{1}{8}}{x+1} - \frac{17}{72} \frac{1}{-1+x} + \frac{\frac{1}{4}}{(-1+x)^2} - \frac{1}{6} \frac{1}{(-1+x)^3}$$

>

> **j:=exp**(2*I*Pi/3) ;

$$j := -\frac{1}{2} + \frac{1}{2} I\sqrt{3}$$

> **a:=n->**(1/6)*(n+1)*(n+2)/2+(1/4)*(n+1)+(17/72)-(1/8)*(-1)^(n+1)+(j/9)*(1/j)^(n+1)+(j^2/9)*(1/j^2)^(n+1) ;

$$a := n \rightarrow \frac{1}{12} (n+1)(n+2) + \frac{1}{4} n + \frac{35}{72} - \frac{1}{8} (-1)^{(n+1)} + \frac{1}{9} j \left(\frac{1}{j} \right)^{(n+1)} + \frac{1}{9} j^2 \left(\frac{1}{j^2} \right)^{(n+1)}$$

> **a**(100) ;

$$\frac{7957}{9} + \frac{\frac{1}{9}}{\left(-\frac{1}{2} + \frac{1}{2} I\sqrt{3}\right)^{100}} + \frac{\frac{1}{9}}{\left(-\frac{1}{2} + \frac{1}{2} I\sqrt{3}\right)^{200}}$$

> **rationalize**(normal(a(100))) ;

$$\frac{-\frac{803469022129495137770981046170581301261101496891396417650688}{221} (1 + I\sqrt{3}) - 803469022129495137770981046170581301261101496891396417650688 + 803469022129495137770981046170581301261101496891396417650688 I\sqrt{3}}{221}$$

> **evalc**(a(100)) ;

884

> **seq**(evalc(a(n)) ,n=0..20) ;

1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, 24, 27, 30, 33, 37, 40, 44

> **seq**(nb(n) ,n=0..20) ;

1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, 24, 27, 30, 33, 37, 40, 44

>

>

> **(x-j)*(x-j^2)*(x-1)^3*(x+1) ;**

$$\left(x + \frac{1}{2} - \frac{1}{2} I\sqrt{3}\right) \left(x - \left(-\frac{1}{2} + \frac{1}{2} I\sqrt{3}\right)^2\right) (-1+x)^3 (x+1)$$

> **expand**(%) ;

$$x^2 - x^4 - x^5 + x^6 + x - 1$$

> **b:=proc**(n)

```
option remember;
  if n<6 then nb(n)
  else b(n-1)+b(n-2)-b(n-4)-b(n-5)+b(n-6);fi;end:
> b(100);
```

884

```
> seq(b(n),n=0..20);
1, 1, 2, 3, 4, 5, 7, 8, 10, 12, 14, 16, 19, 21, 24, 27, 30, 33, 37, 40, 44
```

```
>
>
>
>
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>
>
>
>
```

```
> f:=x->sum(ln(1+1/n)*x^n,n=1..infinity);
```

$$f := x \rightarrow \sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right) x^n$$

```
> f(x);
```

$$\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right) x^n$$

```
> f(-1);
```

$$\sum_{n=1}^{\infty} \ln\left(1 + \frac{1}{n}\right) (-1)^n$$

```
> evalf(%);
```

-.4515827053

```
> ln(2/Pi);
```

$$\ln\left(2 \frac{1}{\pi}\right)$$

```
> evalf(%);
```

-.4515827056

```
>  
> j:=int((-1)^floor(1/x)/x,x=0.001...1);  
j := -.4520827052  
  
> evalf(j);  
-.4520827052  
  
>
```