Formulaire

Analyse vectorielle

Les relations fournies ci-dessous ne doivent pas être retenues par cœur, sauf celle encadrées.
Les formules de Stockes-Ampère et de Green-Ostrogradski seront aussi à retenir comme des définitions des opérateurs divergence *et* rotationnel.

Les champs vectoriels

COORDONNÉES CARTÉSIENNES

$$\vec{A} = A_x\,\vec{e}_x + A_y\,\vec{e}_y + A_z\,\vec{e}_z$$

COORDONNÉES CYLINDRIQUES

$$\vec{A} = A_r\,\vec{e}_r + A_\theta\,\vec{e}_\theta + A_z\,\vec{e}_z$$

COORDONNÉES SPHÉRIQUES

$$\vec{A} = A_r\,\vec{e}_r + A_\theta\,\vec{e}_\theta + A_\varphi\,\vec{e}_\varphi$$

Le gradient d'un champ scalaire

COORDONNÉES CARTÉSIENNES

$$\overrightarrow{\mathrm{grad}}\,\psi = \vec{\nabla}\psi = \frac{\partial\psi}{\partial x}\,\vec{e}_x + \frac{\partial\psi}{\partial y}\,\vec{e}_y + \frac{\partial\psi}{\partial z}\,\vec{e}_z$$

COORDONNÉES CYLINDRIQUES

$$\overrightarrow{\mathrm{grad}}\,\psi = \vec{\nabla}\psi = \frac{\partial\psi}{\partial r}\,\vec{e}_r + \frac{1}{r}\,\frac{\partial\psi}{\partial\theta}\,\vec{e}_\theta + \frac{\partial\psi}{\partial z}\,\vec{e}_z$$

COORDONNÉES SPHÉRIQUES

$$\overrightarrow{\mathrm{grad}}\,\psi = \vec{\nabla}\psi = \frac{\partial\psi}{\partial r}\,\vec{e}_r + \frac{1}{r}\,\frac{\partial\psi}{\partial\theta}\,\vec{e}_\theta + \frac{1}{r\,\sin\theta}\,\frac{\partial\psi}{\partial\varphi}\,\vec{e}_\varphi$$

La divergence d'un champ vectoriel

FORMULE DE GREEN-OSTROGRADSKI

$$\oiint \vec{A}(P)\,\mathrm{d}\vec{S}_P = \iiint \mathrm{div}\ \vec{E}(M)\,\mathrm{d}\tau_M$$

COORDONNÉES CARTÉSIENNES

$$\mathrm{div}\ \vec{A} = \vec{\nabla}\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

COORDONNÉES CYLINDRIQUES

$$\mathrm{div}\ \vec{A} = \vec{\nabla}\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(r\,A_r) + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

COORDONNÉES SPHÉRIQUES

$$\mathrm{div}\ \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,A_r\right) + \frac{1}{r\,\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\,A_\theta) + \frac{1}{r\,\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

Le rotationnel d'un champ vectoriel

FORMULE DE STOCKES-AMPÈRE

$$\oint \vec{A}(P)\,\mathrm{d}\overrightarrow{OP} = \iint \overrightarrow{\mathrm{rot}}\,\vec{A}(M)\,\mathrm{d}\vec{S}_M$$

COORDONNÉES CARTÉSIENNES

$$\overrightarrow{\mathrm{rot}}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{e}_z$$

COORDONNÉES CYLINDRIQUES

$$\overrightarrow{\mathrm{rot}}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\vec{e}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{e}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(r\,A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\vec{e}_z$$

COORDONNÉES SPHÉRIQUES

$$\overrightarrow{\mathrm{rot}}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \frac{1}{r\,\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta\,A_\varphi) - \frac{\partial A_\theta}{\partial \varphi}\right]\vec{e}_r + \frac{1}{r\,\sin\theta}\times\left[\frac{\partial A_r}{\partial \varphi} - \sin\theta\times\frac{\partial}{\partial r}(r\,A_\varphi)\right]\vec{e}_\theta + \frac{1}{r}\times\left[\frac{\partial}{\partial r}(r\,A_\theta) - \frac{\partial A_r}{\partial \theta}\right]\vec{e}_\varphi$$

Le laplacien d'un champ scalaire

COORDONNÉES CARTÉSIENNES

$$\Delta\psi = \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$$

COORDONNÉES CYLINDRIQUES

$$\Delta\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2}$$

COORDONNÉES SPHÉRIQUES

$$\Delta\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\,\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}$$

Relations vectorielles

$$\begin{aligned}
\overrightarrow{\text{rot}}\left(\overrightarrow{\text{grad}}\,\psi\right) &= \vec{0} \\
\text{div}\left(\overrightarrow{\text{rot}}\,\vec{A}\right) &= 0 \\
\text{div}\left(\overrightarrow{\text{grad}}\,\psi\right) &= \Delta\psi \\
\overrightarrow{\text{rot}}\left(\overrightarrow{\text{rot}}\,\vec{A}\right) &= \overrightarrow{\text{grad}}\left(\text{div}\,\vec{A}\right) - \Delta\vec{A} \\
\overrightarrow{\text{grad}}\,(\phi\times\psi) &= \phi\times\overrightarrow{\text{grad}}\,\psi + \psi\times\overrightarrow{\text{grad}}\,\phi \\
\text{div}\left(\psi\times\vec{A}\right) &= \psi\ \text{div}\,\vec{A} + \vec{A}\cdot\overrightarrow{\text{grad}}\,\psi \\
\overrightarrow{\text{rot}}\left(\psi\times\vec{A}\right) &= \psi\,\overrightarrow{\text{rot}}\,\vec{A} + \left(\overrightarrow{\text{grad}}\,\psi\right)\wedge\vec{A} \\
\text{div}\left(\vec{A}\wedge\vec{B}\right) &= \vec{B}\cdot\overrightarrow{\text{rot}}\,\vec{A} - \vec{A}\cdot\overrightarrow{\text{rot}}\,\vec{B} \\
\overrightarrow{\text{grad}}\left(\vec{A}\cdot\vec{B}\right) &= \vec{A}\wedge\overrightarrow{\text{rot}}\,\vec{B} + \vec{B}\wedge\overrightarrow{\text{rot}}\,\vec{A} + \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} + \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B} \\
\overrightarrow{\text{rot}}\left(\vec{A}\wedge\vec{B}\right) &= \vec{A}\ \text{div}\,\vec{B} - \vec{B}\ \text{div}\,\vec{A} + \left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} - \left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}
\end{aligned}$$

avec :

$$\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A} = \sum_i\sum_j B_j\times\frac{\partial A_i}{\partial x_j}\,\vec{e}_i$$