Électronique

Exercice 1

1. $2v + RC\,\dfrac{\mathrm{d}v}{\mathrm{d}t} = u + RC\,\dfrac{\mathrm{d}u}{\mathrm{d}t}$
2. $v(t) = \dfrac{U_0}{2}\left(1 + \mathrm{e}^{-t/\tau}\right)$
3. $\Delta\mathcal{E} = \dfrac{CU_0^2}{8}$

Exercice 2

1. $\dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2} + 2\lambda\omega_0\,\dfrac{\mathrm{d}s}{\mathrm{d}t} + \omega_0^2\,s = 0$
2. $\dfrac{R+r}{2RrC} < \omega_0$

Exercice 3

$C = 5.10^{-4}$ F et $L = \dfrac{1}{C\omega_0^2} = 1$ mH

Exercice 4

1. Filtre passe-haut
2. $\underline{H}(\mathrm{j}\omega) = \dfrac{L\omega\mathrm{j}}{R + L\omega\mathrm{j}} \Rightarrow s + \tau\,\dfrac{\mathrm{d}s}{\mathrm{d}t} = \tau\,\dfrac{\mathrm{d}e}{\mathrm{d}t}$
3. $s(t) = E_0\,\mathrm{e}^{-t/\tau}$
4. $R \gg L\omega \Rightarrow s \simeq \tau\,\dfrac{\mathrm{d}e}{\mathrm{d}t}$

Exercice 5

1. a- $\underline{H}(\mathrm{j}\omega) = \dfrac{1}{1 + RC\omega\mathrm{j}}$

   b- Filtre passe-bas

   c- $f_c = 0,16$ Hz $\Rightarrow v \simeq \dfrac{kV_1V_2}{2}\,\cos\varphi$
2. $\cos\phi = \dfrac{V_c}{\sqrt{V_a\,V_b}} \Rightarrow \phi \simeq 1,26$ rad

Exercice 6

1. $\underline{H}(\mathrm{j}\omega) = \dfrac{1}{1 - x^2 + jQ_0 x}$

Exercice 7

1. a- $s(t) \simeq \dfrac{4}{\pi}\sin(\omega_0 t) + \dfrac{4}{3\pi} \times \dfrac{1}{\sqrt{1 + \left(\dfrac{8Q}{3}\right)^2}}\sin(3\omega_0 t - \varphi_3) = S_1\,\sin(\omega_0 t) + S_3\,\sin(3\omega_0 t - \varphi_3)$ avec $\varphi_3 = \dfrac{8Q}{3}$

   b- $U = \sqrt{S_1^2 + S_3^2} \Rightarrow U \simeq \dfrac{4}{\pi} \times \sqrt{1 + \dfrac{1}{9 + 64Q^2}}$

   c- $Q = 2,86$
2. $U = 0,45$ V

Exercice 8

1. $R_\infty = \dfrac{R_1 + \sqrt{R_1^2 + 4\,R_1 R_2}}{2}$ et $\lambda = \dfrac{1+\sqrt{5}}{2}$
2. $R_n = R_\infty$

Exercice 9

$$\underline{H}(\mathrm{j}\omega) = \frac{\beta\,\mathrm{j}}{(1+\mathrm{j}X)^2 + \beta^2}$$

Exercice 10

1. $\dfrac{\mathrm{d}y}{\mathrm{d}t} \simeq \dfrac{1}{\tau}\,x$
2. $\underline{H}(\mathrm{j}\omega) = \dfrac{T_H}{2\tau\,\mathrm{j}} \times \dfrac{1}{\tan(\pi f\,T_H)} \simeq \dfrac{1}{\mathrm{j}\omega\tau}$ lorsque $f \ll f_H$.

Exercice 11

2. $r_{\text{éq}} = \dfrac{2R_1R_2 + R_1R_3 + R_2R_3}{R_1 + R_2 + 2R_3}$

Exercice 12

$$s(t) \simeq \frac{4E}{3\pi}\,\sin(3\omega_0 t + \varphi) = 4,2 \times \sin(3,12.10^4 \times t + \varphi)$$

Électronique

Électronique - ALI

Exercice 1

$\underline{H}(\mathrm{j}\omega) = -\mathrm{j}\omega\tau \Rightarrow s = -\tau \dfrac{\mathrm{d}e}{\mathrm{d}t}$.

Saturation à haute fréquence.

Exercice 2

$$\underline{H}(\mathrm{j}\omega) = \frac{1}{\dfrac{\underline{Z}_2 + \underline{Z}_1}{\underline{Z}_2} \times \dfrac{\underline{Z}_3}{\underline{Z}_3 + \underline{Z}_4} - \dfrac{\underline{Z}_1}{\underline{Z}_2}}$$

Exercice 3

1. $\underline{H}(\mathrm{j}\omega) = \dfrac{R_1 + R_2}{R_1} \times \dfrac{1}{1 + RC\omega\mathrm{j}} - 1$
2. $V_{20} = V_{10}$ et $\varphi = 2\,\arctan(RC\omega)$

Exercice 4

$i = \dfrac{e}{R}$

Exercice 5

1. $e = -Ri$
2. a- $\dfrac{\mathrm{d}^2 e}{\mathrm{d}t^2} + \dfrac{r - R}{L}\dfrac{\mathrm{d}e}{\mathrm{d}t} + \dfrac{1}{LC}\,e = 0$

   b- Tension sinusoïdale pour $r = R$

Exercice 6

1. a- $\underline{H}(\mathrm{j}\omega) = \dfrac{RC\omega\mathrm{j}}{1 + 3RC\omega\mathrm{j} + (RC\omega\mathrm{j})^2}$ et $e^- = \dfrac{R_1}{R_1 + R_2}\,s$

   b- $\omega_0 = \dfrac{1}{RC}$ et $\alpha = 2 - \dfrac{R_2}{R_1}$

   c- Tension sinusoïdale si $R_2 = 2R_1$

   d- $s(t) = \varepsilon\,\cos(\omega_0 t)$

   e- $s = \dfrac{\varepsilon}{X_2 - X_1}\left(X_2\,\mathrm{e}^{X_1 t} - X_1\,\mathrm{e}^{X_2 t}\right)$ où $X_{1,2} = -\dfrac{\alpha\omega_0}{2} \pm \dfrac{\omega_0}{2}\,(\alpha^2 - 4)^{1/2}$

   f- $s(t) \simeq \varepsilon\,\mathrm{e}^{-\alpha\omega_0 t/2}\,\cos(\omega_0 t)$
2. a- $e^+ = \pm\dfrac{R_1\,V_{\mathrm{sat}}}{R_1 + R_2}$

   b- $\dfrac{\mathrm{d}^2 e^+}{\mathrm{d}t^2} + 3\omega_0\,\dfrac{\mathrm{d}e^+}{\mathrm{d}t} + \omega_0^2\,e^+ = 0$

   c- $e^+ = A\,\mathrm{e}^{X_1 t} + B\,\mathrm{e}^{X_2 t}$ où $X_{1,2} = -\dfrac{3\omega_0}{2} + \pm\dfrac{\omega_0\sqrt{5}}{2}$

Exercice 7

1. $V_0 = \dfrac{R_1\,V_{\text{sat}}}{R_2}$
2. $\dfrac{\mathrm{d}e}{\mathrm{d}t} = -\dfrac{1}{\tau}\,s$ où $\tau = RC$
3. a- $e_1(t) = V_0 - \dfrac{V_{\text{sat}}}{\tau} \times t$

   b- En B : s bascule de $+V_{\text{sat}}$ à $-V_{\text{sat}}$

   c- $e_2(t) = -V_0 + \dfrac{V_{\text{sat}}}{\tau} \times (t - T_1)$

   d- En A : s bascule de $-V_{\text{sat}}$ à $+V_{\text{sat}}$
4. b- $T = \dfrac{4R_1}{R_2} \times RC$

Exercice 8

1. $k = \dfrac{R_1 - 2R_2}{3R_1}$; $\alpha = \dfrac{R_2}{R_1 - 2R_2}$; $\beta = \dfrac{1}{3}$
2. $R_1 = 5R_2$

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Exercice 9

1. a- $\underline{s} = -\dfrac{1}{R_0 C\omega \mathrm{j}}\,\underline{e} \Rightarrow$ Circuit intégrateur ou passe-bas.

   b- $\underline{Y} = \dfrac{1}{\underline{Z}} = \dfrac{1}{R_0} + \dfrac{1}{R_1} + \dfrac{1}{R_0 R_1 C\omega \mathrm{j}}$
2. a- $\underline{Y}_{\text{tot}} = \dfrac{1}{R_0} + \dfrac{1}{R_1} - \dfrac{R_3}{R_2 R_4} + \dfrac{1}{R_0 R_1 C\omega \mathrm{j}}$

   b- $L = R_0 R_1 C$ si $\dfrac{1}{R_0} + \dfrac{1}{R_1} = \dfrac{R_3}{R_2 R_4}$

   c- $L = 10$ H

Électrostatique

Loi de Coulomb et lignes de champ

Exercice 1

1. Réponse c
2. Réponse d
3. Réponses b et d

Exercice 2

$q_B = -q_A \times \left(\frac{13}{8}\right)^{3/2} = 2,2\ \mu\text{C}$

Exercice 3

1. $\vec{E}(O) = \vec{0}$
2. $\vec{E}(O) = \frac{-\sigma}{8\varepsilon_0}\,\vec{u}_z$

Exercice 4

1. $C = \pi a^2$
2. $Q = e \times [1 - 7\,\exp(-6)] \simeq 0,98\,e$

Exercice 5

$\vec{E} = -\frac{\sigma}{2\varepsilon_0}\,(\sin\theta_2 - \sin\theta_1)\,\vec{e}_z$

Exercice 6

1. $\phi = \frac{q}{2\varepsilon_0}\,(\cos\varphi_0' - \cos\varphi_0)$
2. b- $K = -\frac{2a}{\sqrt{a^2+c^2}}$

   c- $\alpha = \arccos\left(\frac{2a}{\sqrt{a^2+c^2}} - 1\right)$

Théorème de Gauss et calculs de potentiels

Exercice 1

1. Réponse b
2. Réponse a
3. Réponse c
4. Réponse c
5. Réponse a

Exercice 2

1. $E(r > R) = \frac{Q}{4\pi\varepsilon_0 r^2}$ et $E(r < R) = \frac{Q}{4\pi\varepsilon_0 R^3}\,r$
2. $\frac{\mathrm{d}^2\overrightarrow{OM}}{\mathrm{d}t^2} + \frac{eQ}{4\pi\varepsilon_0 m R^3}\,\overrightarrow{OM} = \vec{0}$

Exercice 3

$E_0\,r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho(u)\,\mathrm{d}u \Rightarrow \rho = \frac{2\varepsilon_0 E_0}{r}$

Exercice 4

$V(0) = 2E_0 \times R$

Exercice 6

1. $E = \dfrac{\sigma R}{\varepsilon_0 r}$

Exercice 8

$V(M) = \dfrac{Q}{4\pi\varepsilon_0 r}$

Exercice 10

2. $\vec{E} = -\dfrac{\sigma_0}{3\varepsilon_0}\,\vec{u}_z$

Exercice 11

$\dfrac{1}{r}\,\dfrac{\mathrm{d}^2}{\mathrm{d}r^2}(rV) = \dfrac{2n_0 q}{\varepsilon_0}\,\sinh\left(\dfrac{qV}{kT}\right) \Rightarrow V(r) = \dfrac{A}{r}\,\mathrm{e}^{-\alpha r}$ où $\alpha = \sqrt{\dfrac{2n_0 q^2}{\varepsilon_0 kT}}$

Exercice 12

1. $V = \dfrac{\sigma R}{\varepsilon_0}$

Exercice 13

1. $\vec{g}_1 = -\dfrac{4\pi\,Gh\,\mu_1}{2}\,\vec{u}_z$

Capacité et énergie

Exercice 1

1. $\vec{E} = \dfrac{Q}{2\pi\varepsilon_0 \ell r}\,\vec{u}_r$
2. $V_1 - V_2 = \dfrac{Q}{2\pi\varepsilon_0\ell}\,\ln\left(\dfrac{R_2}{R_1}\right)$

Exercice 2

1. Réponses a et c
2. Réponses a et d
3. Réponse c
4. Réponse d

Exercice 3

1. $V_P = \dfrac{-e\,\ln 2}{2\pi\varepsilon_0 a}$
2. $V_N = \dfrac{e\,\ln 2}{2\pi\varepsilon_0 a}$
3. $\mathcal{E} = -\dfrac{e^2\,\ln 2}{2\pi\varepsilon_0 a} = 684\ \mathrm{kJ.mol^{-1}}$

Dipôles électrostatiques

Exercice 1

1. $V = \dfrac{p}{4\pi\varepsilon_0 r^2}$
2. $E_r = \dfrac{p}{4\pi\varepsilon_0} \times \dfrac{2\cos\theta}{r^3}$
3. $\phi_E = 0$

Exercice 2

1. $a = \sqrt[3]{\dfrac{p}{4\pi\varepsilon_0 E}} \Rightarrow V = V_0 = 0$
2. $E_r = 3E$ et $E_\theta = 0$

Exercice 3

1. $\vec{F}_q = \dfrac{pq}{4\pi\varepsilon_0 r^3} \begin{pmatrix} 2\cos\theta \\ \sin\theta \end{pmatrix}$
2. $\vec{F}_q = -\vec{F}_p$
3. $\begin{cases} pq > 0 \Rightarrow \theta_{\text{eq}} = \pi \\ pq < 0 \Rightarrow \theta_{\text{eq}} = 0 \end{cases}$

Exercice 4

1. $\mathcal{E}_p = \dfrac{p_1 p_2}{4\pi\varepsilon_0 r^3} (\sin\theta_1 \, \sin\theta_2 - 2\cos\theta_1 \, \cos\theta_2)$
2.

θ_{eq}	$-\pi/2$	0	$\pi/2$	π
équilibre	instable	stable	instable	stable

Exercice 5

$\mu = 4\pi\varepsilon_0 R^3 \, \vec{E} = \alpha \, \vec{E}$

Exercice 6

1. Équilibres en $r = 0$ et en $r \to \infty$.
2. $v_0 \geqslant \sqrt{\dfrac{e^2}{4\pi\varepsilon_0 R^3} (R^2 - r_0^2)}$
3. Si $r > R$: $r_0 = \sqrt{\dfrac{e}{4\pi\varepsilon_0 E_0}}$ (instable) et si $r < R$: $r_0 = \dfrac{4\pi\varepsilon_0 R^3 E_0}{e} \Rightarrow \alpha = 4\pi R^3$ (stable).

Exercice 7

$V(M) = \dfrac{qa^2}{4\pi\varepsilon_0 r^3} (1 - 3\, \cos^2\theta)$

Exercice 8

1. $\vec{f}_+ = -\dfrac{3\, p_1\, p_2}{2\pi\varepsilon_0 r^4} \, \vec{e}_z$ et $\vec{f}_- = \dfrac{3\, p_1\, p_2}{2\pi\varepsilon_0 r^4} \, \vec{e}_z$

3. $\left\langle \vec{f} \right\rangle = -\dfrac{3\, p_1^2\, p_2^2\, kT}{4\pi^2\varepsilon_0^2\, r^7} \, \vec{e}_z$

Exercice 9

1. $E_z = \dfrac{\lambda R}{2\varepsilon_0} \times z\,(z^2 + R^2)^{-3/2}$
2. $z_1 = \dfrac{R}{\sqrt{2}}$ et $z_2 = -\dfrac{R}{\sqrt{2}}$
3. $\omega = \sqrt{\dfrac{8\lambda p}{3^{5/2}\, \varepsilon_0 m R^3}}$

Magnétostatique

Théorème d'Ampère

Exercice 1

$$\begin{cases} |y| < \dfrac{a}{2} \Rightarrow \vec{B} = \mu_0 J_0 y\, \vec{e}_z \\ y > \dfrac{a}{2} \Rightarrow \vec{B} = \mu_0 \dfrac{J_0 a}{2} \vec{e}_z \\ y < -\dfrac{a}{2} \Rightarrow \vec{B} = -\mu_0 \dfrac{J_0 a}{2} \vec{e}_z \end{cases}$$

Exercice 2

- $r < R_1 \Rightarrow B = \dfrac{\mu_0 I}{2\pi R_1^2}\, r$
- $R_1 < r < R_2 \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$
- $R_2 < r < R_3 \Rightarrow B = \dfrac{\mu_0 I}{2\pi r} \times \dfrac{R_3^2 - r^2}{R_3^2 - R_2^2}$
- $r > R_3 \Rightarrow B = 0$

Exercice 3

$$I_{\text{int}}(r) = 2\pi \int_{u=0}^{r} j(u)\, u\, \mathrm{d}u \Rightarrow \begin{cases} j(r < a) = \dfrac{B_0}{\mu_0} \left(\dfrac{2}{a} - \dfrac{3r}{a^2} \right) \\ j(r > a) = 0 \end{cases}$$

Exercice 4

Réponse a

Exercice 5

1. $\vec{B} = \dfrac{\mu_0 I}{2\pi a}\, \vec{u}_\theta$
2. a- $B = \dfrac{\mu_0 I}{2\pi} \arctan\left(\dfrac{L}{r}\right)$

   b- $B = \dfrac{\mu_0 I}{2\pi} \times \dfrac{1}{L}$

Exercice 6

1. $\vec{B} = B(r,z)\, \vec{e}_\theta$
2. $\vec{B}(M) = \dfrac{\mu_0 N}{2\pi r}\, I\, \vec{e}_\theta$ à l'intérieur de la bobine et $B = 0$ à l'extérieur.
3. $L = \dfrac{\mu_0 N^2 h}{2\pi} \ln\left(\dfrac{r_2}{r_1}\right)$

Exercice 7

$\vec{B}_{\text{int}}(r < R) = \mu_0\, j_s\, \sin\alpha\, \vec{u}_z$ et $\vec{B}_{\text{ext}}(r > R) = \mu_0\, j_s\, \dfrac{R\cos\alpha}{r}\, \vec{u}_\theta$

Exercice 8

1. $\vec{j} = \rho\, \omega\, r\, \sin\theta\, \vec{e}_\varphi$
2. $I = \dfrac{2}{3}\, \rho\omega\, R^3$

Dipôles magnétique

Exercice 1

1. $\vec{\Gamma} = iabB_0 \sin\theta\, \vec{u}_z$
2. $\vec{m} = iabS\, \vec{n} \Rightarrow \vec{\Gamma} = iabSB_0 \sin\theta\, \vec{u}_z$

Exercice 2

1. $M = \mu_0 ns \cos\theta$
2. $e = -\dfrac{\mathrm{d}\phi_{m/S}}{\mathrm{d}t}$
3. $e = \mu_0 nm\omega \sin(\omega t)$

Exercice 3

1. $\vec{B} = \begin{pmatrix} 0 \\ -\dfrac{\mu_0 m}{4\pi x^3} \end{pmatrix}$ et $\vec{m}_1 = \begin{pmatrix} m_1 \cos\theta \\ m_1 \sin\theta \end{pmatrix}$
2. Équilibre stable pour $\theta = -\dfrac{\pi}{2}$
3. $F_x = -\dfrac{3\mu_0 m^2}{4\pi x^4}$

Exercice 4

1. $\dfrac{\mathrm{d}r}{r} = 2\, \dfrac{\cos\varphi\, \mathrm{d}\varphi}{\sin\varphi}$
3. $B = B_0 \times \dfrac{\sqrt{3\cos^2\varphi + 1}}{\sin\varphi}$ où $B_0 = \dfrac{\mu_0\, m_T}{4\pi r_0^3}$
4. $B_{\text{ min}} = B_0$ pour $\varphi = \pm\dfrac{\pi}{2}$
7. a- $\alpha_{\text{éq}} = 0$

   b- $\ddot{\alpha} + \dfrac{mB_h}{J} \sin\alpha = 0$

   c- $\tau_0 = 2\pi\sqrt{\dfrac{J}{mB_h}}$

   d- $B_h = B_e \times \dfrac{1-\rho^2}{1+\rho^2}$ avec $\rho = \dfrac{\tau_1}{\tau_2}$

   e- $B_h = 2{,}43.10^{-5}$ T

Exercice 5

1. $B_r = -\dfrac{\partial V}{\partial r} = \dfrac{3\mu_0 m}{2\pi r^3} \times (3\cos^2\theta - 1)$ et $B_\theta = -\dfrac{1}{r}\dfrac{\partial V}{\partial\theta} = \dfrac{3\mu_0 m}{2\pi r^3} \times 2\sin\theta\cos\theta$
2. $r = r_0\sqrt{\sin^4\theta\, |\cos\theta|}$

Exercice 6

1. $\vec{\Gamma} = \vec{m} \wedge \vec{B}_0 = \dfrac{ea^2}{3}\, \vec{B}_0 \wedge \vec{m}$
2. Mouvement de précession de vitesse angulaire $\Omega_p = \dfrac{5e}{6M}\, B_0$

Exercice 7

$\phi = \dfrac{\mu_0 m}{2} \times \dfrac{R^2}{(R^2+d^2)^{3/2}}$

Électromagnétisme

Équations de Maxwell - Induction

Exercice 1

1. $\overrightarrow{\mathcal{M}} = -\dfrac{IBa^2}{2}\,\vec{u}_z$

3. $e = \dfrac{Ba^2\omega}{2}$

4. $J\,\dfrac{\mathrm{d}\omega}{\mathrm{d}t} + \lambda\omega = \Gamma_m - \dfrac{IBa^2}{2}$ et $L\,\dfrac{\mathrm{d}I}{\mathrm{d}t} + RI = E + \dfrac{Ba^2\omega}{2}$

5. $LJ\,\dfrac{\mathrm{d}^2 I}{\mathrm{d}t^2} + (JR + \lambda L)\,\dfrac{\mathrm{d}I}{\mathrm{d}t} + \left(\lambda R + \dfrac{B^2a^4}{4}\right) I = \dfrac{a^2 B}{2}\,\Gamma_m + \lambda E$

6. $I = \dfrac{2a^2B}{B^2a^4 + 4\lambda R}$

Exercice 2

1. $\sin\theta_e = \dfrac{IBa}{mg}$

2. $T_0 = 2\pi\sqrt{\dfrac{2a}{3g\cos\theta_e}}$

Exercice 3

1. $v = v_0\,\left(1 - \mathrm{e}^{-t/\tau}\right)$ où $\tau = \dfrac{mR}{a^2B^2}$

2. $\begin{cases} \mathcal{E}_{\text{Joule}} = \dfrac{1}{2}mv_0^2 \\ W_{\text{ext}} = mv_0^2 \qquad \Rightarrow W_{\text{ext}} = \mathcal{E}_{\text{Joule}} + \Delta\mathcal{E}_c \\ \Delta\mathcal{E}_c = \dfrac{1}{2}mv_0^2 \end{cases}$

Exercice 4

$E(|x| < \dfrac{a}{2}) = \dfrac{\rho x}{\varepsilon_0}$ et $E(x > \dfrac{a}{2}) = \dfrac{\rho a}{2\varepsilon_0}$

Exercice 5

$\langle\Gamma_z\rangle = \dfrac{S^2B_0^2}{R} \times (\omega - \omega_0)$

Exercice 6

1. $\dfrac{\mathrm{d}v}{\mathrm{d}t} + \dfrac{1}{\tau}\,v = 0 \Rightarrow v = V_0\,\mathrm{e}^{-t/\tau}$

2. $\Delta\mathcal{E}_c = -\dfrac{1}{2}\,mV_0^2$

Exercice 7

1. $\omega_0 = \sqrt{\dfrac{3g}{2a}}$

2. $\dfrac{1}{3}\,ma^2\,\ddot{\theta} + \dfrac{a^4B^2}{4R}\,\dot{\theta} + \dfrac{mga}{2}\,\theta = 0$

Exercice 8

1. $\begin{cases} \underline{e} = \left(\dfrac{1}{C\omega\mathrm{j}} + R + L\omega\mathrm{j}\right)\,\underline{i}_1 + M\omega\mathrm{j}\,\underline{i}_2 \\ 0 = M\omega\mathrm{j}\,\underline{i}_1 + (L_0\omega\mathrm{j} + R_0)\,\underline{i}_2 \end{cases}$

Exercice 9

1. $\vec{f} = -i\ell B\,\vec{u}_z \Rightarrow m\ddot{z} + h\dot{z} + kz = F - i\ell B$
2. $L\,\dfrac{\mathrm{d}i}{\mathrm{d}t} + Ri + \dfrac{q}{C} = E + v\ell B$
3. $U = \dfrac{1}{2}\,mv^2 + \dfrac{1}{2}\,kz^2 + \dfrac{1}{2}\,Li^2 + \dfrac{q^2}{2C}$
4. $\underline{E} + \underline{v}\,\ell B = \underline{Z}\,\underline{i}$ et $\underline{F} - \underline{i}\,\ell B = \underline{\zeta}\,\underline{v}$
5. $\underline{v} = -\dfrac{\ell B}{\underline{\zeta}}\,\underline{i} = -\dfrac{\ell B\,\underline{E}}{\underline{Z}\,\underline{\zeta} + \ell^2 B^2}$
6. $\underline{i} = \dfrac{\ell B\,\underline{F}}{\underline{\zeta}\,\underline{Z} + \ell^2 B^2}$

Exercice 10

2. $\operatorname{div}\vec{E} = \dfrac{\rho}{\varepsilon_0}$ (Maxwell-Gauss) et $\vec{E} = -\overrightarrow{\mathrm{grad}}\,V$ car $\overrightarrow{\mathrm{rot}}\,\vec{E} = \vec{0}$ (Maxwell-Faraday)
3. $E_r = -\dfrac{\partial V}{\partial r} = -f'g\,;\ E_\theta = -\dfrac{1}{r}\,fg'\,;\ E_z = 0$
4. Équation de Laplace : $\Delta V = 0 \Rightarrow f'g + rf''g + \dfrac{1}{r}\,fg'' = 0$
5. $r \times \left(\dfrac{f'}{f} + r\,\dfrac{f''}{f}\right) = -\dfrac{g''}{g} = K$
6. $g(0) = 0 = g(2\pi) \Rightarrow K = \dfrac{n^2}{4} > 0 \Rightarrow g(\theta) = G_0\,\sin\left(\dfrac{\theta}{2}\right)$
7. $4rf' + 4r^2 f'' - f = 0 \Rightarrow f = a\,\sqrt{r} + \dfrac{b}{\sqrt{r}} \Rightarrow V(r,\theta) = V_0 + A\,\sqrt{r}\,\sin\left(\dfrac{\theta}{2}\right)$

Exercice 11

1. $\vec{F}_{MN} = -I\ell\,B\,\vec{u}_z \Rightarrow \mathcal{M}_O(\vec{F}) = \overrightarrow{OG} \wedge \vec{F}_{MN} = RI\ell B\,\vec{u}_z$
2. $RI\ell B = mgR'$
3. $B = 19{,}6.10^{-3}$ T

Exercice 12

1. $\operatorname{div}\vec{E} = 0$ et $\operatorname{div}\vec{B} = 0$
2. $f' = \dfrac{1}{\tau} \times g$
3. $g' = \dfrac{1}{c^2\tau} \times f$
4. $f'' - k^2\,f = 0$ avec $k = \dfrac{1}{\sqrt{c\tau}} \Rightarrow f(z) = E_0\,\mathrm{e}^{-kz}$
5. $\vec{E}(M,t) = E_0\,\mathrm{e}^{-kz}\,\mathrm{e}^{-t/\tau}\,\vec{e}_x$ et $\vec{B}(M,t) = -\dfrac{E_0}{c}\,\mathrm{e}^{-kz}\,\mathrm{e}^{-t/\tau}\,\vec{e}_y$

Exercice 13

1. $J_0 = -eN$
2. $J_0 = \rho(x)\,v(x)\,;\ \dfrac{\mathrm{d}^2 V}{\mathrm{d}x^2} = -\dfrac{\rho(x)}{\varepsilon_0}\,;\ \dfrac{1}{2}\,mv^2 = e\,V(x)$
3. $\dfrac{\mathrm{d}^2 V}{\mathrm{d}x^2} = \dfrac{C}{\sqrt{V}}$ où $C = -\dfrac{J_0}{\varepsilon_0}\,\sqrt{\dfrac{m}{2e}}$
4. $B = -\dfrac{4\varepsilon_0}{9}\,\sqrt{\dfrac{2e}{m}} = -2{,}3.10^{-6}$ SI
5. $n = \dfrac{2}{3}$
6. $v_f = \sqrt{\dfrac{2eu}{m}} = 6.10^6$ m.s^{-1} et $T = \dfrac{3L}{v_f} = 5.10^{-9}$ s

Exercice 14

$$V(x,y) = \sum_{n=1}^{\infty} \frac{2V_0}{n\pi}\ [1-(-1)^n]\ \sin\left(\frac{n\pi x}{a}\right)\ \mathrm{e}^{-n\pi y/a}$$

Exercice 15

1. $\delta q = -\mathrm{d}Q \Rightarrow \vec{\jmath} = \dfrac{-\dot{Q}}{4\pi r^2}\,\vec{e}_r$
2. $\operatorname{div}\vec{\jmath} = 0 \Rightarrow \dfrac{\partial \rho}{\partial t} = 0$
3. $\vec{E}(r,t) = \dfrac{Q+q}{4\pi\varepsilon_0 r^2}\,\vec{e}_r$

Exercice 16

1. $v(t) = v_\ell\,(1-\mathrm{e}^{-t/\tau})$ où $\tau = \dfrac{mR}{a^2B^2}$ et $v_\ell = \dfrac{\mu g \rho}{B^2} = g\tau$
2. $\begin{cases} \mathcal{P}_p = mgv \\ \mathcal{P}_J = mv^2/\tau \\ \dfrac{\mathrm{d}\mathcal{E}_c}{\mathrm{d}t} = mv\,\dfrac{\mathrm{d}v}{\mathrm{d}t} \end{cases} \Rightarrow \dfrac{\mathrm{d}\mathcal{E}_c}{\mathrm{d}t} + \mathcal{P}_J = \mathcal{P}_p \Rightarrow v_\ell = g\tau$ en régime stationnaire

Exercice 17

1. $B(\rho < a) = \dfrac{\mu_0 I \rho}{2\pi a^2}$ et $B(\rho > a) = \dfrac{\mu_0 I}{2\pi\rho}$
2. a- $\vec{B}'(M \in \mathcal{S}) = \dfrac{\mu_0 N I'}{2\pi\rho'}\,\vec{e}_\theta$ et $\vec{B}'(M \notin \mathcal{S}) = \vec{0}$

   b- $\phi' = \dfrac{\mu_0 N b I'}{2\pi}\,\ln\left(\dfrac{c+b}{c-b}\right) \Rightarrow L' = \dfrac{\mu_0 N b}{2\pi}\,\ln\left(\dfrac{c+b}{c-b}\right)$
3. $\Phi = \dfrac{N\mu_0 I b}{2\pi}\,\ln\left(\dfrac{c+b}{c-b}\right) \Rightarrow M = \dfrac{N\mu_0 b}{2\pi}\,\ln\left(\dfrac{c+b}{c-b}\right)$
4. $\phi_{\mathcal{S}/\mathcal{C}} = MI' \Rightarrow M = \dfrac{\mu_0 N b}{2\pi}\,\ln\left(\dfrac{c+b}{c-b}\right)$

Exercice 18

1. $\omega_1 + \omega_2 = \text{cte} = \omega_0\,;\ \dfrac{\mathrm{d}^2 i}{\mathrm{d}t^2} + \Omega^2\, i = 0\,;\ \dfrac{\mathrm{d}^2\omega_2'}{\mathrm{d}t^2} + \Omega^2\,\omega_2' = 0$ où $\omega_2' = \dfrac{\mathrm{d}\omega_2}{\mathrm{d}t}$
2. $\omega_2(t) = \dfrac{\omega_0}{2}\,\dfrac{\Omega_1^2}{\Omega^2}\,[1-\cos(\Omega t)]$ et $\omega_1 = \omega_0 - \omega_2$

Conducteurs ohmiques

Exercice 1

1. $\delta R = \dfrac{1}{\pi\sigma}\,\dfrac{\mathrm{d}z}{a^2 z^2}$
2. $R = \dfrac{1}{2\pi\sigma a}\,\ln\left(\dfrac{a+b}{a-b}\right)$

Exercice 2

$R = \dfrac{\pi}{a\sigma\,\ln(r_2/r_1)}$

Exercice 3

3. $E_\theta = \dfrac{\omega B_0}{2}\,\sin(\omega t)$
4. $P_m = \dfrac{\sigma\omega^2 B_0^2 L S^2}{16\pi}$
5. $P'_m = \dfrac{P_m}{N}$
6. $P'_m = P_m$

Exercice 4

1. • $\vec{\jmath} = \dfrac{\sigma\omega B_0}{2}\,r\,\sin(\omega t)\,\vec{e}_\theta$
   • $P(r) = \langle \mathcal{P}_J(r,t)\rangle = \dfrac{\sigma B_0^2\omega^2}{8}\,r^2$
   • $\overline{P} = \langle P(r)\rangle_{\text{espace}} = \dfrac{\sigma B_0^2\omega^2 a^2}{16}$
2. • $B_1 = \dfrac{\mu_0\sigma\omega B_0}{4}\,(a^2 - r^2)\,\sin(\omega t)$
   • $a < \sqrt{\dfrac{2}{5\omega\mu_0\sigma}} = 4,5\ \text{mm}$

Exercice 5

2. a- $V_H = \dfrac{R_H I B}{b}$
   b- $R_H = -\dfrac{1}{ne} = -\dfrac{M_{\text{Cu}}}{\mu\mathcal{N}e} = -7,36.10^{-11}\ \text{m}^3.\text{C}^{-1}$
   c- Dans un semi-conducteur n est plus faible donc R_H est plus élevé.
3. b- $\tan\theta_H = \gamma B R_H$

Exercice 6

1. $V(r) = V_1 + \dfrac{V_2 - V_1}{\ln(r_2/r_1)}\,\ln\left(\dfrac{r}{r_1}\right) \Rightarrow E = \dfrac{V_1 - V_2}{\ln(r_2/r_1)} \times \dfrac{1}{r}$
2. $e\vec{E} + \lambda\,\vec{v} + e\,\vec{v}\wedge\vec{B} = \vec{0} \Rightarrow \begin{cases} v_r = \dfrac{-\lambda e E}{\lambda^2 + e^2B^2} \\ v_\theta = -\dfrac{-e^2BE}{\lambda^2 + e^2B^2} \end{cases}$ et $\begin{cases} j_r = \dfrac{ne^2\lambda}{\lambda^2 + e^2B^2} \\ j_\theta = \dfrac{ne^3BE}{\lambda^2 + e^2B^2} \end{cases}$
3. $I = \dfrac{ne^2\lambda}{\lambda^2 + e^2B^2} \times 2\pi h \times \dfrac{V_1 - V_2}{\ln(r_2/r_1)} \Rightarrow R = \dfrac{\lambda^2 + e^2B^2}{ne^2\lambda}\,\dfrac{\ln(r_2/r_1)}{2\pi h}$
   $\varepsilon = \dfrac{e^2B^2}{\lambda^2} = 7,9.10^{-11}$ et $R_0 = 112\ \Omega$

Exercice 7

1. $\vec{E}(M,t) = \dfrac{Q(t)+q}{4\pi\varepsilon_0 r^2}\,\vec{e}_r$ et $\vec{B}(M,t) = \vec{0}$
2. $-\dfrac{\mathrm{d}\dot{Q}}{\mathrm{d}t} = \dfrac{\gamma}{\varepsilon_0}\,\dot{Q} \Rightarrow \dot{Q}(t) = -\dfrac{Q_0}{\tau}\,\mathrm{e}^{-t/\tau}$ et $Q(t) = Q_0\,\mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{\varepsilon_0}{\gamma}$
3. $\mathcal{E}_{\text{Joule}} = \dfrac{Q_0^2}{8\pi\varepsilon_0 R}$ pour $t \in [0;\infty[$

Exercice 8

1. $j = \dfrac{I}{\pi a^2} \Rightarrow \rho = \dfrac{I}{\pi a^2 v}$
2. $\vec{E}(r) = \dfrac{\rho}{2\varepsilon_0}\,r\,\vec{u}_r = \dfrac{I}{2\pi a^2 v\varepsilon_0}\,\vec{r}$
3. $\vec{B} = \dfrac{\mu_0 I}{2\pi a^2}\,r\,\vec{u}_\theta = \dfrac{\mu_0 I}{2\pi a^2}\,\vec{u}_z \wedge \overrightarrow{OM}$
4. $\operatorname{div}\vec{E} = \dfrac{\rho}{\varepsilon_0}$ et $\operatorname{div}\vec{B} = 0$
5. $\overrightarrow{\operatorname{rot}}\,\vec{E} = \mu_0\,\vec{j}$; $\operatorname{div}\vec{E} = \dfrac{\rho}{\varepsilon_0}$; $\vec{j} = \rho\,\vec{v}$
6. $\vec{F} = q\,(1-\beta^2)\,\vec{E} \Rightarrow \vec{F} \begin{cases} \text{élargit les faisceaux si } q > 0 \\ \text{rétrécit les faisceaux si } q < 0 \end{cases}$
7. $F_m \simeq F_e \Rightarrow F \simeq 0$

Électromagnétisme

Énergie électromagnétique

Exercice 1

1. $B_{\text{int}}(r) = B_{\text{int}}(0)$ et $B_{\text{ext}}(r) = B_{\text{ext}}(r') \, \forall (r, r') > R$
3. $B_{\text{int}} = \mu_0 n i$
4. $\Lambda = \dfrac{L}{\ell} = \mu_0 n^2 \pi R^2$

Exercice 2

1. Quand t augmente, Q_1 diminue et Q_2 augmente, jusqu'à ce que $Q_{1\infty} = 0$ et $Q_{2\infty} = Q$
4. $\overrightarrow{\text{rot}}\, \vec{B} = \mu_0 \, \vec{\jmath} + \mu_0 \varepsilon_0 \, \dfrac{\partial \vec{E}}{\partial t} = \vec{0} \Rightarrow \sigma \, \vec{E} + \varepsilon_0 \, \dfrac{\partial \vec{E}}{\partial t} = \vec{0}$
5. $E(r,t) = E(r,0)\, \mathrm{e}^{-t/\tau} = E_0(r)\, \mathrm{e}^{-t/\tau}$ où $E_0(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ et $\tau = \dfrac{\varepsilon_0}{\sigma}$
6. $\text{div}\, \vec{E} = \dfrac{\rho}{\varepsilon_0} \Rightarrow \rho = 0$
7. $E(r < r_1) = 0$ et $E(r > r_2) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$
8. $p(r,t) = \dfrac{\delta P}{\mathrm{d}\tau} = \dfrac{\sigma Q^2}{(4\pi\varepsilon_0)^2 r^4}\, \mathrm{e}^{-2t/\tau} \Rightarrow W_J = \dfrac{Q^2}{8\pi\varepsilon_0} \left(\dfrac{1}{r_1} - \dfrac{1}{r_2} \right)$
9. $W_0 = \dfrac{Q^2}{8\pi\varepsilon_0 r_1}$ et $W_1 = \dfrac{Q^2}{8\pi\varepsilon_0 r_2}$. Il s'ensuit que : $W_0 = W_J + W_1$

Exercice 3

1. $\vec{B} = \dfrac{\mu_0 \varepsilon_0}{2} \, \dfrac{\mathrm{d}E}{\mathrm{d}t} \, \vec{u}_\theta \Rightarrow \mathcal{E}_t = \varepsilon_0 \pi r^2 h \, \dfrac{E_0^2}{2}$
2. $\mathcal{E}_{\text{mag}} = \mathcal{E}_t$

Exercice 4

1. $\vec{\Pi} = \dfrac{k}{\mu_0 \omega} \left[E_{0x}^2 \, \cos^2(kz - \omega t + \varphi_1) + E_{0y}^2 \, \cos^2(kz - \omega t + \varphi_2) \right] \vec{u}_z$
2. $\langle P \rangle = \dfrac{kS}{\mu_0 \omega} \times \dfrac{E_{0x}^2 + E_{0y}^2}{2}$
3. $\vec{S} \cdot \Re \left\{ \vec{\Pi} \right\} = \dfrac{kS}{2\mu_0 \omega} \, (E_{0x}^2 + E_{0y}^2)$

Exercice 5

1. $E_\theta = \dfrac{B_0 r}{2\tau} \, \mathrm{e}^{-t/\tau} \ll B_0$ si $\dfrac{a}{\tau} \ll 1$
2. $\vec{\Pi} = \dfrac{B_0^2 r}{2\tau \mu_0} \, \mathrm{e}^{-2t/\tau} \, \vec{e}_r \Rightarrow \mathcal{E}_{\text{traverse}} = \dfrac{B_0^2 \pi a^2 \ell}{2\mu_0} \, (1 - \mathrm{e}^{-2t/\tau})$ et $\mathcal{E}_{\text{intérieur}}(t = 0) = \mathcal{E}_{\text{intérieur}}(t) + \mathcal{E}_{\text{traverse}}$

Exercice 6

1. $B_\theta = \dfrac{\mu_0 \, I(z)}{2\pi r} \, \mathrm{e}^{\mathrm{i}\omega t}$
2. $E_r = \dfrac{I'(z)}{2\pi\varepsilon_0 r \mathrm{i}\omega} \, \mathrm{e}^{\mathrm{i}\omega t}$
3. $I'' + k^2 I = 0 \Rightarrow I = I_0 \, \mathrm{e}^{-\mathrm{i}kz} + I_1 \, \mathrm{e}^{\mathrm{i}kz}$
4. $\vec{\Pi} = \dfrac{-I_0^2}{(2\pi r)^2 \varepsilon_0 c^2} \, \cos^2(\omega t - kz) \, \vec{u}_z \Rightarrow \langle \mathcal{P} \rangle = \dfrac{I_0^2}{4\pi\varepsilon_0 c} \left(\dfrac{1}{a} - \dfrac{1}{b} \right)$

Ondes planes progressives sinusoïdales

Exercice 1

1. $\Delta\vec{E} - \dfrac{1}{c^2}\dfrac{\partial^2\vec{E}}{\partial t^2} = \vec{0}$
2. a- $\omega = kc$

   b- $\vec{c} = -c\,\vec{u}_x$

   c- Polarisation rectiligne

   e- $B_z = -\dfrac{k}{\omega}\,E_0\,\cos(\omega t + kx)$ et $\vec{\Pi} = -\dfrac{E_0^2}{\mu_0 c}\,\cos^2(\omega t + kx)\,\vec{u}_x$

   f- $\underline{\vec{B}} = -\dfrac{E_0}{c}\,\mathrm{e}^{\mathrm{i}(\omega t + kx)}\,\vec{u}_z$

   g- $\langle\mathcal{P}\rangle = \dfrac{E_0^2 S}{2\mu_0 c}$

Exercice 2

$E_0 = \sqrt{\dfrac{2\mu_0 c}{S}\,P} = 86,8\ \mathrm{kV.m^{-1}}$ et $B_0 = \dfrac{E_0}{c} = 2,9.10^{-4}\ \mathrm{T}$

Exercice 3

1. • $E_z = E_0\,\cos(y\,k_y)\,\cos\phi$ où $\phi = \omega t - x\,k_x$
   • $B_x = \dfrac{k_y E_0}{\omega}\,\sin(y\,k_y)\,\sin\phi$ et $B_y = -\dfrac{k_x E_0}{\omega}\,\cos(y\,k_y)\,\cos\phi$
2. $\Pi_x = \dfrac{E_0^2 k_x}{\mu_0\omega}\,\cos^2(y\,k_y)\,\cos^2\phi$ et $\Pi_y = \dfrac{E_0^2 k_y}{4\mu_0\omega}\,\sin(2y\,k_y)\,\sin(2\phi)$
3. $v_\phi = \dfrac{c}{\cos\theta}$

Exercice 4

1. $B_\theta = \dfrac{\mu_0 b\,\dot{\sigma}}{r}\,z$
2. $\vec{\Pi} = -\dfrac{b^2\,\sigma\,\dot{\sigma}}{\varepsilon_0 r^2}\,z\,\vec{u}_z \Rightarrow \phi_e = \dfrac{4\pi b^2 h}{\varepsilon_0}\,\sigma\,\dot{\sigma}\,\ln\left(\dfrac{b}{a}\right)$ et $\phi_e = \dfrac{\mathrm{d}\mathcal{E}}{\mathrm{d}t}$

Exercice 5

1. $B_\theta = \dfrac{1}{\omega}\left[\dfrac{\mathrm{d}f}{\mathrm{d}r}\,\mathrm{e}^{\mathrm{i}\phi-\pi/2} - \dfrac{kf}{\omega}\,\varepsilon^{\mathrm{i}\phi}\right]$ où $\phi = \omega t - kr$
2. $\vec{R} = \dfrac{f}{\mu_0\omega}\left[-\dfrac{\mathrm{d}f}{\mathrm{d}r}\,\dfrac{\sin(2\phi)}{2} + kf\,\cos^2\phi\right]\vec{u}_r \Rightarrow \left\langle\vec{R}\right\rangle = \dfrac{k}{2\mu_0\omega}\,f^2\,\vec{u}_r$
3. $f(r) = \dfrac{\mathrm{cte}}{\sqrt{r}}$
4. $\vec{B} \simeq -\dfrac{kA}{\omega r^{1/2}}\,\cos\phi\,\vec{u}_\theta$

Exercice 6

1. $\dfrac{\partial u}{\partial x} = -\Lambda\,\dfrac{\partial i}{\partial t}$
2. $\dfrac{\partial i}{\partial x} = -\Gamma\,\dfrac{\partial u}{\partial t}$
4. $i = \sqrt{\dfrac{\Gamma}{\Lambda}}\,(f-g)$
5. $g(x=0,t) = f(x=0,t) \times \dfrac{\alpha-1}{\alpha+1}$ où $\alpha = R\sqrt{\dfrac{\Gamma}{\Lambda}}$
6. $R_c = \sqrt{\dfrac{\Lambda}{\Gamma}}$

Exercice 7

1. Onde plane, polarisée circulairement se propageant selon $\vec{u}_x$
2. Onde plane, polarisée rectilignement, se propageant selon $-\vec{u}_y$
3. Onde plane, polarisée elliptiquement, se propageant selon $\vec{u}_z$
4. Onde plane, polarisée rectilignement, se propageant selon $\vec{u} = (0, \cos\dfrac{\pi}{6}, \sin\dfrac{\pi}{6})$

Exercice 8

1. $I = I_0\,\cos^2\alpha$ (loi de Malus)
2. $I = \dfrac{E_{0x}^2}{2}\,\cos^2\alpha + \dfrac{E_{0y}^2}{2}\,\sin^2\alpha$
3. $n_x > n_y$ et $(n_x - n_y)c = \dfrac{\lambda_0}{4}$
4. $E'_x = E_{0x}\,\cos(\omega t')$ et $E'_y = E_{0y}\,\cos(\omega t')$ avec $\omega t' = \omega t - k_x c$

Exercice 9

Onde polarisée rectilignement ayant tourné d'un angle $\tan\beta = \dfrac{\omega}{c}\left(\dfrac{n_d - n_g}{2}\right) L$.

Exercice 10

1. $\vec{\phi}_1 = \dfrac{E_{01}^2}{2\mu_0 c}\,\vec{u}$ et $\vec{\phi}_2 = \dfrac{E_{02}^2}{2\mu_0 c}\,\vec{u}$
2. $\vec{\phi} = \vec{\phi}_1 + \vec{\phi}_2 + \dfrac{\vec{u}}{\mu_0 c^2} \times \vec{E}_{01}\cdot\vec{E}_{01}\,\langle\cos\left[(\omega_1 - \omega_2)t + \theta\right]\rangle$.
   Les interférences ($\vec{\phi} \neq \vec{\phi}_1 + \vec{\phi}_2$) apparaissent si $\vec{E}_{01}\cdot\vec{E}_{02} \neq 0$ et si $\omega_1 = \omega_2$.

Exercice 11

1. $\begin{cases} E_x = E_0\,\cos(kz - \omega t) \\ E_y = \varepsilon E_0\,\sin(kz - \omega t) \end{cases}$ et $\begin{cases} B_x = -\varepsilon\dfrac{E_0}{c}\,\sin(kz - \omega t) \\ B_y = \dfrac{E_0}{c}\,\cos(kz - \omega t) \end{cases}$ où $\varepsilon = \pm 1$
2. $\overrightarrow{\Pi} = \dfrac{E_0^2}{\mu_0 c}\,\vec{u}_z$
3. $\underline{\vec{E}} = \underline{\vec{E}}_0\,\mathrm{e}^{\mathrm{i}\phi}$; $\underline{\vec{B}} = \dfrac{\vec{u}_z}{c} \wedge \vec{E}_0\,\mathrm{e}^{\mathrm{i}\phi}$; $\overrightarrow{\Pi} = \dfrac{\vec{u}_z}{\mu_0 c}\,E_0^2$ où $\phi = kz - \omega t$

Électromagnétisme

Propagation en milieux conducteurs

Exercice 1

1. $\underline{E} = \displaystyle\int_{\omega_1}^{\omega_2} \frac{\mathrm{d}\omega}{2\,\Delta\omega}\, E_0\, \mathrm{e}^{\mathrm{i}(\omega t - kz)}$
2. $k(\omega) = k_0 + \varepsilon k'$
3. $\mathcal{E}(z,t) = E_0\, \mathrm{sin}_c\left[(t - k'z)\,\Delta\omega\right]$
4. $v_g = \dfrac{1}{k'} = \dfrac{\mathrm{d}\omega}{\mathrm{d}k}$

Exercice 2

1. $\Delta\vec{B} - \mu_0\varepsilon_0\, \dfrac{\partial^2\vec{B}}{\partial t^2} = \mu_0\lambda\,\vec{B}$
2. $B = B_0\, \mathrm{e}^{-kx} + B_0'\, \mathrm{e}^{kx}$ avec $k = \sqrt{\mu_0\lambda}$
3. $k^2 = \dfrac{\omega^2 - \omega_c^2}{c^2}$ où $\omega_c = \sqrt{\dfrac{\lambda}{\varepsilon_0}}$

Exercice 3

1. $n^- = n \times \left(1 - \dfrac{\partial u}{\partial z}\right) \Rightarrow \rho = ne\,\dfrac{\partial u}{\partial z}$
2. $E = \dfrac{ne}{\varepsilon_0}\, u \Rightarrow m\,\dfrac{\mathrm{d}^2u}{\mathrm{d}t^2} = -\dfrac{ne^2}{\varepsilon_0}\, u$
   $f_{p1} = 9$ MHz et $f_{p2} = 2,8.10^{11}$ Hz

Exercice 4

1. $\begin{cases} \mathrm{div}\,\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \mathrm{div}\,\vec{\jmath} + \dfrac{\partial\rho}{\partial t} = 0 \\ \vec{\jmath} = \gamma\,\vec{E} \end{cases} \Rightarrow \rho = \rho_0\, \mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{\varepsilon_0}{\gamma} = 1,5.10^{-19}$ s
2. $\dfrac{j_d}{j} = \dfrac{f}{10^{18}} \ll 1$
3. $j_x = J_0\, \mathrm{e}^{-z/\delta}\, \cos(\omega t - \dfrac{z}{\delta})$
4. $E_x = \dfrac{J_0}{\gamma}\, \mathrm{e}^{-z/\delta}\, \cos\phi$ et $B_y = \dfrac{J_0}{\gamma\omega\delta}\, \mathrm{e}^{-z/\delta}\,(\sin\phi + \cos\phi)$
5.
   - $\langle u_e\rangle = \dfrac{\varepsilon_0 J_0^2}{4\gamma^2}\, \mathrm{e}^{-2z/\delta}$
   - $\langle u_m\rangle = \dfrac{J_0^2}{2\gamma^2\mu_0\delta^2\omega^2}\, \mathrm{e}^{-2z/\delta}$
   - $\dfrac{\langle u_m\rangle}{\langle u_e\rangle} = \dfrac{1}{\omega\tau} \gg 1$
6.
   - $\langle\mathcal{P}_J\rangle = \dfrac{J_0^2 ab\delta}{4\gamma}$
   - $\left\langle\vec{\Pi}\right\rangle = \dfrac{J_0^2}{2\gamma^2\delta\omega\mu_0}\, \mathrm{e}^{-2z/\delta}\, \vec{u}_z \Rightarrow \langle\mathcal{P}_t\rangle = \dfrac{J_0^2 ab}{2\mu_0\gamma^2\delta\omega} \Rightarrow \dfrac{\langle\mathcal{P}_t\rangle}{\langle P_J\rangle} = 1$

Exercice 5

1. $\begin{cases} \overrightarrow{\mathrm{rot}}\,\vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \overrightarrow{\mathrm{rot}}\,\vec{B} = \mu_0 \gamma \vec{E} \end{cases}$ et $\begin{cases} \mathrm{div}\,\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \mathrm{div}\,\vec{B} = 0 \end{cases}$

2. $\Delta \vec{E} = \mu_0 \gamma \dfrac{\partial \vec{E}}{\partial t} \Rightarrow \Delta E_x = \mu_0 \gamma \dfrac{\partial E_x}{\partial t}$

3. a- $E_x = E_0\,\mathrm{e}^{\mathrm{i}(kz - \omega t)}$

   b- $k^2 = \mu_0 \gamma \omega\,\mathrm{e}^{\mathrm{i}\pi/2} \Rightarrow k = \dfrac{1+\mathrm{i}}{\delta}$

   c- $E_x = E_0\,\mathrm{e}^{-z/\delta}\,\cos(z/\delta - \omega t) = E_0\,\mathrm{e}^{-z/\delta}\,\cos\phi$

4. $E_x \to 0$

5. $B_y = \dfrac{E_0}{\delta\omega}\,\mathrm{e}^{-z/\delta}\,(\cos\phi - \sin\phi)$ et $E_x = E_0\,\mathrm{e}^{-z/\delta}\,\cos\phi$

6. $\vec{\Pi} = \dfrac{E_0^2}{\mu_0 \delta \omega}\,\mathrm{e}^{-2z/\delta}\left(\cos^2\phi - \dfrac{\sin(2\phi)}{2}\right)\vec{e}_z \Rightarrow \left\langle \vec{\Pi} \right\rangle = \dfrac{E_0^2}{2\pi_0 \delta\omega}\,\mathrm{e}^{-2z/\delta}\,\vec{e}_z$ et $\langle \mathcal{P}_t \rangle = \dfrac{E_0^2 ab}{2\mu_0 \delta \omega}$

7. $\langle \mathcal{P}_J \rangle = \dfrac{\gamma E_0^2 ab\delta}{4} \Rightarrow \langle \mathcal{P}_J \rangle = \langle \mathcal{P}_t \rangle$

Exercice 6

1. $k^2 = \dfrac{\omega^2}{c^2} - \dfrac{\pi^2}{a^2} \Rightarrow v_\phi = \dfrac{c}{\sqrt{1 - \dfrac{c^2\pi^2}{a^2\omega^2}}}$ et $v_g = c\sqrt{1 - \dfrac{\pi^2 c^2}{a^2\omega^2}}$. Le milieu est dispersif.

2. $B_x = -\dfrac{aE_0}{\pi\omega}\cos\left(\dfrac{\pi z}{a}\right)\sin\phi$ et $B_z = \dfrac{kE_0}{\omega}\sin\left(\dfrac{\pi z}{a}\right)\cos\phi$ où $\phi = kx - \omega t$

3. $\langle \Pi_x \rangle = \dfrac{kE_0^2}{4\mu_0\omega}$; $\langle \Pi_z \rangle = 0$; $\langle u \rangle = \dfrac{E_0^2}{4\mu_0 c^2}$; $v_e = v_g$

4. $E_y = 2E_0\,\sin\left(\dfrac{\pi z}{a}\right)\cos(kx - \omega t) \times \cos\left(\dfrac{\Delta\omega}{2}\,t - \dfrac{\Delta\omega}{2}\,k' \times x\right)$ où $k' = \dfrac{\mathrm{d}k}{\mathrm{d}\omega} \Rightarrow v_{\text{env.}} = v_g$.

Exercice 7

1. $\begin{cases} \dot{v}_z = 0 \\ \dot{v}_x = \dfrac{q}{m}E_x + \dfrac{qB_t}{m}v_y \\ \dot{v}_y = \dfrac{q}{m}E_y - \dfrac{qB_t}{m}v_x \end{cases} \Rightarrow K = -\dfrac{\mathrm{i}e\varepsilon\,B_t}{m}$

2. $\dfrac{\partial E^*}{\partial z} = \mathrm{i}\varepsilon\,\dfrac{\partial B^*}{\partial t}$ et $\mathrm{i}\varepsilon\,\dfrac{\partial B^*}{\partial z} = \mu_0\,j^* + \mu_0\varepsilon_0\,\dfrac{\partial E^*}{\partial t}$

3. $\omega_B = \dfrac{eB_t}{m}$

4. a- $\begin{cases} \text{polarisation circulaire gauche si } \varepsilon = +1 \\ \text{polarisation circulaire droite si } \varepsilon = -1 \end{cases}$

   b- Propagation si $\omega > \dfrac{-\varepsilon\,\omega_B + \sqrt{\omega_B^2 + 4\omega_p^2}}{2}$

5. Polarisation rectiligne selon Ox.

Exercice 8

1. $\varepsilon_0 \dfrac{\partial \vec{E}_1}{\partial t} = en_0 \left(\vec{v}_{e1} - \vec{v}_{p1}\right)$
2. $m_e \dfrac{\partial \vec{v}_{e1}}{\partial t} = -e\,\vec{E}_1 \Rightarrow \vec{\jmath} \simeq en_0\,\vec{v}_{e1}$
3. $\omega^2 = \omega_p^2 = \dfrac{n_0\,e^2}{m_e \varepsilon_0}$
4. $\vec{\Pi} = \vec{0}$ et $\langle u \rangle = \dfrac{\varepsilon_0 E_0^2}{4}$. Donc l'énergie électromagnétique reste confinée.

Exercice 9

I- Partie A

1. $\operatorname{div} \vec{E} = \dfrac{\rho}{\varepsilon_0}$; $\overrightarrow{\operatorname{rot}}\,\vec{B} = \mu_0\,\vec{\jmath} + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t}$; $\vec{\jmath} = \gamma\,\vec{E}$
2. $\rho(t) = \rho_0\,\mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{\varepsilon_0}{\gamma} = 7,1.10^{-19}$ s
3. $\left|\dfrac{j_d}{j}\right| \simeq 4,4.10^{-18} \times f \ll 1$
4. $\overrightarrow{\operatorname{rot}}\,\vec{B} = \mu_0\,\vec{\jmath}$

II- Partie B

1. $\delta = \sqrt{\dfrac{2}{\mu_0 \gamma \omega}} = 10^{-2}$ m pour $f = 100$ Hz et $\delta = 2.10^{-4}$ m pour $f = 480$ kHz.
2. $\underline{j}_y = \dfrac{B_0}{\mu_0 \delta}\,(1+\mathrm{i}) \times \dfrac{\sinh\left[(1+\mathrm{i})\,\dfrac{z}{\delta}\right]}{\cosh\left[(1+\mathrm{i})\,\dfrac{b}{\delta}\right]}\,\mathrm{e}^{\mathrm{i}\omega t}$
3. $A = \dfrac{B_0}{2b\gamma\mu_0}$

Électromagnétisme

Réflexion sur un conducteur

Exercice 1

1. $B_x = -\dfrac{f'}{\omega}\sin(\omega t - kx)$ et $B_y = -\dfrac{kf}{\omega}\cos(\omega t - kx)$
2. $f(y) = E_0 \sin(Ky)$ avec $K = \sqrt{\dfrac{\omega^2}{c^2} - k^2}$
3. $j_{sz} = \dfrac{KE_0}{\omega}\sin(\omega t - kx)$ et $\sigma = 0$

Exercice 2

1. $\underline{E}_z(x=0) = \underline{E}_z(x=L) = 0$
2. $E_2 = -E_1$ et $f_n = n\,f_1$ où $f_1 = \dfrac{c}{2L}$
3. a- $\underline{E}_n = -2E_1\,\mathrm{i}\,\mathrm{e}^{\mathrm{i}\omega t}\sin\left(\dfrac{n\pi x}{L}\right)$

   c- $x_p = p\,\dfrac{L}{n} \Rightarrow \Delta x = \dfrac{L}{n}$

   d- $\underline{B}_y = -\dfrac{2E_1}{c}\,\mathrm{e}^{\mathrm{i}\omega t}\cos(kx)$
4. • $\vec{\Pi} = \dfrac{E_1^2}{\mu_0 c}\sin(2\omega t)\,\sin(2kx)\,\vec{e}_x$
   • $u_{em} = 2E_1^2\varepsilon_0\left[\sin^2(\omega t)\,\sin^2(kx) + \cos^2(\omega t)\,\cos^2(kx)\right]$

Exercice 3

1. $\begin{cases} k_x = n_x\,\dfrac{\pi}{a} \\ k_y = n_y\,\dfrac{\pi}{a}\end{cases} \Rightarrow \omega = c\sqrt{k_x^2 + k_y^2}$ et $E_z = E_0\sin(k_x x)\,\sin(k_y y)\,\cos\phi$ où $\phi = \omega t - \varphi$
2. $B_x = -\dfrac{k_y E_0}{\omega}\sin(k_x x)\,\cos(k_y y)\,\sin\phi$ et $B_y = \dfrac{E_0 k_x}{\omega}\cos(k_x x)\,\sin(k_y y)\,\sin\phi$
3. $\mathcal{E}_{\text{élec}} = \mathcal{E}_{\text{mag}} = \dfrac{\varepsilon_0 E_0^2 a^3}{16}$

Exercice 4

1. $\underline{E}_z(x,t) = 0 \Rightarrow \underline{E}_y(x,t) = \dfrac{\sigma(x,t)}{\varepsilon_0}$ et $\begin{cases}\underline{B}_x = 0 \\ \underline{B}_y = 0\end{cases} \Rightarrow \underline{B}_z(x,t) = \mu_0\,j_s(x,t)$
2. $\mathrm{d}U_e = \dfrac{\sigma^2}{2\varepsilon_0}\,ab\,\mathrm{d}x$; $\mathrm{d}U_m = \dfrac{\mu_0}{2}\,j_s^2 ab\,\mathrm{d}x$; $\Gamma = \varepsilon_0\,\dfrac{a}{b}$; $\Lambda = \mu_0\,\dfrac{b}{a}$
3. $\dfrac{\partial\sigma}{\partial x} = -\mu_0\varepsilon_0\,\dfrac{\partial j_s}{\partial t}$ et $\dfrac{\partial j_s}{\partial x} = -\dfrac{\partial\sigma}{\partial t}$
4. $\underline{B}_z = \dfrac{\mu_0 I_0}{a}\,\mathrm{e}^{\mathrm{i}(\omega t - kx)}$ et $\underline{E}_y = \dfrac{I_0 k}{a\omega\varepsilon_0}\,\mathrm{e}^{\mathrm{i}(\omega t - kx)}$

Exercice 5

1. $\begin{cases} \underline{E}_{ix} = E_0\,\mathrm{e}^{\mathrm{i}(\omega t - kz)} \\ \underline{E}_{iy} = \mathrm{i}E_0\,\mathrm{e}^{\mathrm{i}(\omega t - kz)} \end{cases}$ et $\begin{cases} \underline{E}_{rx} = -E_0\,\mathrm{e}^{\mathrm{i}(\omega t + kz)} \\ \underline{E}_{ry} = -\mathrm{i}E_0\,\mathrm{e}^{\mathrm{i}(\omega t + kz)} \end{cases}$ et $\begin{cases} \underline{E}_{x} = -2\mathrm{i}E_0\,\mathrm{e}^{\mathrm{i}\omega t}\,\sin(kz) \\ \underline{E}_{y} = 2E_0\,\mathrm{e}^{\mathrm{i}\omega t}\,\sin(kz) \end{cases}$

2. $\begin{cases} \underline{B}_{ix} = -\dfrac{\mathrm{i}E_0}{c}\mathrm{e}^{\mathrm{i}(\omega t - kz)} \\ \underline{B}_{iy} = \dfrac{E_0}{c}\mathrm{e}^{\mathrm{i}(\omega t - kz)} \end{cases}$ et $\begin{cases} \underline{B}_{rx} = -\dfrac{\mathrm{i}E_0}{c}\mathrm{e}^{\mathrm{i}(\omega t + kz)} \\ \underline{B}_{ry} = \dfrac{E_0}{c}\mathrm{e}^{\mathrm{i}(\omega t + kz)} \end{cases}$ et $\begin{cases} \underline{B}_{x} = -\dfrac{2\mathrm{i}E_0}{c}\mathrm{e}^{\mathrm{i}\omega t}\,\cos(kz) \\ \underline{B}_{y} = \dfrac{2E_0}{c}\mathrm{e}^{\mathrm{i}\omega t}\,\cos(kz) \end{cases}$

3. $\vec{R} = \vec{0}$; $u_e = 2\varepsilon_0 E_0^2\,\sin^2(kz)$; $u_m = 2\varepsilon_0 E_0^2\,\cos^2(kz)$; $u = u_m + u_e = 2\varepsilon_0 E_0^2$

4. $j_{sy} = -\dfrac{2E_0}{\mu_0 c}\,\sin(\omega t)$ et $j_{sx} = \dfrac{2E_0}{\mu_0 c}\,\cos(\omega t)$

Exercice 6

1. $\dfrac{\partial^2 E}{\partial z^2} - \dfrac{1}{c^2}\dfrac{\partial^2 E}{\partial t^2} = 0$ et $E(z=0,t) = E(z=L,t) = 0$

2. $f_n(z) = F_{0n}\,\sin\left(\dfrac{n\pi z}{L}\right)$ avec $\omega_n = \dfrac{n\pi c}{L}$

6. $E(z,t) = \displaystyle\sum_{n=1}^{\infty} E_{0n}\,\cos(\omega_n t)\,\sin\left(\dfrac{n\pi z}{L}\right)$ où $E_{0n} = \dfrac{4E_0}{n\pi}\,\sin\left(\dfrac{n\pi a}{L}\right)\,\sin\left[\dfrac{n\pi}{L}\,(z_0 + \dfrac{a}{2})\right]$

Exercice 7

1. $\vec{\underline{B}}_i = \dfrac{E_0}{c}\begin{pmatrix}\cos\alpha \\ 0 \\ \sin\alpha\end{pmatrix}\,\mathrm{e}^{\mathrm{i}\phi}$ où $\phi = \vec{k}\cdot\overrightarrow{OM} - \omega t$ et $\vec{k} = \begin{pmatrix} k\,\sin\alpha \\ 0 \\ -k\,\cos\alpha\end{pmatrix}$

$\vec{\underline{E}}_r = \begin{pmatrix} 0 \\ -E_0 \\ 0\end{pmatrix}\,\mathrm{e}^{\mathrm{i}\phi'}$; $\vec{\underline{B}}_r = \dfrac{E_0}{c}\begin{pmatrix}\cos\alpha \\ 0 \\ -\sin\alpha\end{pmatrix}\,\mathrm{e}^{\mathrm{i}\phi'}$ où $\phi' = \vec{k}_r\cdot\overrightarrow{OM} - \omega t$ avec $\vec{k}_r = \begin{pmatrix} k\,\sin\alpha \\ 0 \\ k\,\cos\alpha\end{pmatrix}$

2. $\sigma = 0$ et $j_s = \dfrac{2E_0}{\mu_0 c}\,\cos\alpha \times \cos(k\,\sin\alpha\,x - \omega t)\,\vec{u}_y$

Exercice 8

1. $\rho(t) = \rho_0\,\mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{\varepsilon_0}{\gamma} \simeq 10^{-18}$ s

2. $\Delta\vec{E} = \mu_0\gamma\,\dfrac{\partial\vec{E}}{\partial t} = \vec{0}$ et $\Delta\vec{B} = \mu_0\gamma\,\dfrac{\partial\vec{B}}{\partial t} = \vec{0}$

3. $k = \dfrac{1+\mathrm{i}}{\delta}$ où $\delta = \sqrt{\dfrac{2}{\mu_0\gamma\omega}}$

4. a- $\underline{r} = \dfrac{k - (1+\mathrm{i})/\delta}{k + (1+\mathrm{i})/\delta}$ et $\underline{t} = \dfrac{2k}{k + (1+\mathrm{i})/\delta}$

   b- Conducteur parfait $\Rightarrow \underline{r} = -1$ et $\underline{t} = 0$

Exercice 9

1. $\vec{E}_i = \dfrac{c}{n_1}\,\mathrm{e}^{\mathrm{i}\phi}\begin{pmatrix} B_0\,\cos i_1 \\ B_0\,\sin i_1 \\ 0\end{pmatrix}$; $\vec{E}_r = \dfrac{c}{n_1}\,\mathrm{e}^{\mathrm{i}\phi_r}\begin{pmatrix} -B_r\,\cos i_1 \\ B_r\,\sin i_1 \\ 0\end{pmatrix}$; $\vec{E}_t = \dfrac{c}{n_2}\,\mathrm{e}^{\mathrm{i}\phi_t}\begin{pmatrix} B_t\,\cos i_2 \\ B_t\,\sin i_2 \\ 0\end{pmatrix}$

2. $B_r = r\,B_0$ et $B_t = t\,\dfrac{n_2}{n_1}\,B_0$

3. $\tan i_b = \dfrac{n_2}{n_1}$

Rayonnement dipolaire

Exercice 1

A- Modèle de l'électron élastiquement lié

1. a- $\rho(N) = \dfrac{3e}{4\pi a^3}$

   b- $\vec{E} = \dfrac{e}{4\pi\varepsilon_0 a^3}\,\overrightarrow{PN}$

   c- $\vec{F}_{p\to e} = -\dfrac{e^2}{4\pi\varepsilon_0 a^3}\,\overrightarrow{PN}$

2. a- $m\,\dfrac{\mathrm{d}^2\overrightarrow{ON}}{\mathrm{d}^2} = -\dfrac{e^2}{4\pi\varepsilon_0 a^3}\,\overrightarrow{PN}$ et $M\,\dfrac{\mathrm{d}^2\overrightarrow{OP}}{\mathrm{d}t^2} = \dfrac{e^2}{4\pi\varepsilon_0 a^3}\,\overrightarrow{PN}$

   b- $\dfrac{\mathrm{d}^2\vec{R}}{\mathrm{d}t^2} = \left(\dfrac{1}{M}+\dfrac{1}{m}\right)\dfrac{e^2}{4\pi\varepsilon_0 a^3}\,\overrightarrow{PN}$

   c- $\mu = \dfrac{m\times M}{m+M}$ et $\omega_0 = \sqrt{\dfrac{e^2}{4\pi\varepsilon_0 a^3}}$

   d- $\omega_0 = 1,5.10^{16}$ rad.s^{-1} (UV)

B- Polarisation de l'atome

2. $\alpha(\omega) = \dfrac{e}{\mu\sqrt{(\omega_0^2-\omega^2)^2+\Gamma^2\omega^2}}$ et $\sin\psi = \dfrac{-\Gamma\omega}{\sqrt{(\omega_0^2-\omega^2)^2+\Gamma^2\omega^2}}$

Exercice 2

A- Onde électromagnétique rayonnée par l'atome

2. a- $[B] = \left[\dfrac{\mu_0 I}{2\pi r}\right] = [\mu_0]\,\mathrm{C.s^{-1}.m^{-1}}$

   b- $B_\varphi = \dfrac{\mu_0\,\sin\theta}{4\pi rc}\,\ddot{p}(t-\dfrac{r}{c})$ et $E_\theta = \dfrac{\mu_0\,\sin\theta}{4\pi r}\,\ddot{p}(t-\dfrac{r}{c})$

3. $<\vec{\Pi}> = \dfrac{\mu_0 e^2}{16\,\pi^2 c}\,\dfrac{\sin^2\theta}{r^2} < (\ddot{x})^2 > \vec{e}_r$

B- Amortissement des oscillations par rayonnement

1. $F = \dfrac{-e^2\,\dddot{x}}{6\pi\varepsilon_0 c^3}$

2. $\ddot{x} + \omega_0^2\,x - \dfrac{e^2}{6\pi\varepsilon_0\mu c^3}\,\dddot{x} = 0$

3. b- $\Gamma = \dfrac{e^2\omega_0^2}{6\pi\varepsilon_0\mu c^3}$

   c- $\Gamma = 3,70.10^7$ s^{-1}

Exercice 3

3. $P = \dfrac{\omega^4}{24\pi\varepsilon_0 c^3}$

Exercice 4

1. $\vec{B} = -\dfrac{\mu_0\omega^2 p_0}{4\pi r c}\begin{pmatrix} 0 \\ \sin\phi \\ -\cos\theta\,\cos\phi \end{pmatrix}$ et $\vec{E} = \dfrac{\mu_0\omega^2 p_0}{4\pi r}\begin{pmatrix} 0 \\ \cos\theta\,\cos\phi \\ \sin\phi \end{pmatrix}$ où $\phi = \omega t' - \varphi = \omega t - kr - \varphi$
2. $P = \dfrac{\mu_0\omega^4 p_0^2}{6\pi c}$
3. $\mathcal{E}_m = -\dfrac{e^2}{8\pi\varepsilon_0 c}$ et $r = \left[a_0^3 - \dfrac{\mu_0 e^4}{4\pi^2\varepsilon_0 c m^2} \times t\right]^{1/3}$
4. $\tau = \dfrac{4\pi^2\varepsilon_0 m^2 c a_0^3}{\mu_0 e^4} = 1,3.10^{-11}$ s

Exercice 5

1. $\delta\vec{E} = \dfrac{\mathrm{j}\omega\mu_0 I}{4\pi}\,\dfrac{\mathrm{e}^{\mathrm{j}(\omega t - kr)}}{r}\,\sin\theta\,\mathrm{d}z\,\vec{e}_\theta$ et $\delta\vec{B} = \dfrac{\mathrm{j}kI}{4\pi\varepsilon_0 c^2}\,\dfrac{\mathrm{e}^{\mathrm{j}(\omega t - kr)}}{r}\,\sin\theta\,\mathrm{d}z\,\vec{e}_\varphi$
2. a- $NM = r - z\,\cos\theta \Rightarrow \delta = z\,\cos\theta$

   b- $\vec{E}(M,t) = \dfrac{c\mu_0 I_0}{2\pi r}\,\mathrm{e}^{\mathrm{j}(\omega t - kr - \frac{\pi}{2})}\,\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\vec{e}_\theta$
3. $\vec{B}(\vec{r},t) = -\dfrac{\mu_0 I_0}{2\pi r}\,\sin(\omega t - kr)\,\dfrac{\cos\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin\theta}\,\vec{e}_\varphi$
4. $\langle\Pi\rangle = \dfrac{\mu_0 c I_0^2}{8\pi r^2}\,\dfrac{\cos^2\left(\dfrac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$
5. $P = \dfrac{1,22\,\mu_0 c}{4\pi}\,I_0^2$
6. $R_r = \dfrac{1,22\,\mu_0 c}{2\pi} = 73,2\ \Omega$ et $I_0 = \sqrt{\dfrac{2P}{R_r}} = 239,5$ A

Exercice 6

5. $I_0 = q\mathrm{i}\omega$
6. $\vec{E} = -\overrightarrow{\mathrm{grad}}\,V \Rightarrow \overrightarrow{\mathrm{rot}}\,\vec{E} = \vec{0} \neq -\dfrac{\partial\vec{B}}{\partial t}$
8. $A_z = \dfrac{\mu_0}{4\pi r}\,I_0\ell_0\,\mathrm{e}^{\mathrm{i}\omega t'}$ où $t' = t - \dfrac{r}{c}$
10. $\overrightarrow{\mathrm{rot}}\,\vec{e}_\varphi = \dfrac{1}{r\,\sin\theta}\,\vec{e}_z$
12. $\left\langle\vec{\Pi}\right\rangle = \vec{0}$
14. $E_0 = \sqrt{\dfrac{3P}{4\pi\varepsilon_0 c r^2}} = 2,1.10^{-3}$ V.m^{-1}

OPTIQUE

PROPAGATION DE LA LUMIÈRE

Exercice 1

1. $\tau_c = \dfrac{1}{\Delta\nu}$
2. $L_c = c \times \tau_c = \dfrac{c}{\Delta\nu_c}$ et $L_c = \dfrac{\lambda_m^2}{\Delta\lambda}$
3. $\lambda_m = 550$ nm et $\Delta\lambda = 150$ nm
4. $\nu_m = 5,45.10^{14}$ Hz ; $L_c = 2.10^{-6}$ m ; $\tau_c = 7.10^{-15}$ s
5. $\nu_m = 4,66.10^{14}$ Hz ; $L_c = 32$ m ; $\tau_c = 10^{-9}$ s

Exercice 2

1. $(APA') = (AQA')\,\forall(P,Q)$
2. $\delta = PQ \times (n_2 \sin i_2 - n_1 \sin i_1)$
3. $\delta = 0 \Rightarrow n_1 \sin i_1 = n_2 \sin i_2$

Exercice 3

1. $\delta = AB \sin\alpha$ et $\delta' = (SB) - (SA) = -AB \sin\alpha$
2. $\delta = 2ne \cos r \pm \dfrac{\lambda_0}{2}$

Exercice 4

1. $\mathcal{L} = \overrightarrow{AH} \cdot \vec{k} + \overrightarrow{HA'} \cdot \vec{k}'$
2. b- $\mathrm{d}\mathcal{L} = (\overrightarrow{HA'} - \overrightarrow{HA}) \cdot \mathrm{d}\vec{k}'$

Exercice 5

1. $\varphi_g(M) - \varphi(A) = k \left(p + \dfrac{x^2+y^2}{2p}\right)$ et $\varphi_d - \varphi(A') = -k \left(p' + \dfrac{x^2+y^2}{2p'}\right)$
2. $\Delta\varphi_1(x,y) = \varphi_0 + k\,\dfrac{x^2+y^2}{2} \left(\dfrac{1}{p} + \dfrac{1}{p'}\right)$ où $\varphi_0 = \text{cte} = \varphi(A) - \varphi(A') + k\,(p+p')$
3. $\Delta\varphi_2(x,y) = \varphi_0' + \dfrac{k\,(x^2+y^2)}{2} \times \dfrac{1}{f'}$ où $\varphi_0' = \text{cte}$
4. $\dfrac{1}{p'} + \dfrac{1}{p} = \dfrac{1}{f'}$

Optique

Interférences en lumière cohérente

Exercice 1

1. a- $I = I_0\ [1 + \cos(k\delta_0)]$

   b- $\delta = \dfrac{ax}{D}$

   c- $i = \dfrac{\lambda D}{a}$ et $p = \dfrac{ax}{\lambda f}$

   d- $x_4 = 2,2$ mm

2. $i = \dfrac{\lambda D}{a}$ et $x_{\text{centre}} = \dfrac{(n-1)\,eD}{a}$

Exercice 2

1. Raie jaune avec $\Delta\nu = c\,\dfrac{\Delta\lambda}{\lambda^2} = 1,5.10^9$ Hz
2. $\tau_c = \dfrac{1}{\Delta\nu} = 6,6.10^{-10}$ s $\Rightarrow L_c = c\,\tau_c = 0,20$ m
4. $\delta = (n_1 - n_2)\,e + \dfrac{ax}{D}$
5. $\delta_1 = (n_a - 1)\,e$ en $x = 0$
6. $x_{\text{centre}} = \dfrac{(1 - n_a)\,eD}{a}$
8. $n_a = 1 + 23,5 \times \dfrac{\lambda_0}{e} = 1 + 3.10^{-4}$

Exercice 3

1. Franges rectilignes.
2. $i = \dfrac{d\,\lambda_0}{2a}$
3. Frange sombre.

Exercice 4

1. Franges sur $\mathcal{M}_s$.
2. $I = I_0\left[1 + \cos\left(\dfrac{2\pi}{\lambda}\,\dfrac{r^2}{R}\right)\right]$
3. Franges brillantes circulaires, de rayons $r_n = \sqrt{n\lambda\,R^2}$ où $n \in \mathbb{N}$.
4. Défilement de $\dfrac{2y}{\lambda}$ anneaux qui rentrent au centre.

Exercice 5

1. Franges circulaires.
2. $N_{\max} = \dfrac{nd\,\sin\theta}{4\lambda} \Rightarrow \theta = 5^\circ$

Exercice 6

$i = \dfrac{\lambda\,(D+d)}{4\alpha d} = 1$ mm

Exercice 7

1. $O_1S'' = OH = 1$ m et $S_1S_2 = \dfrac{e}{p}\,SH = 2,4$ mm
2. $d = 1,06$ m
3. 15 franges brillantes espacées de $i = 0,23$ mm.

Exercice 8

1. $\overline{S_1S_2} = 10$ cm
2. $\delta \simeq \dfrac{r^2}{2}\left(\dfrac{1}{S_1H} + \dfrac{1}{S_2H}\right)$
3. $p_{\max} = \dfrac{1}{36.10^{-4}\lambda\ell} = 111$ franges brillantes.

Exercice 9

I.A. La recherche d'exoplanètes

1. $I = 2I_0\,[1 + \cos(k_0\delta)]$
3. $\mathcal{C} = 1$
5. $i = \dfrac{\lambda_0 f}{a}$
8. $I = 2I_0\ [1 + \cos(\Delta\varphi)]$ où $\Delta\varphi = k\left(D\,\sin\alpha + \dfrac{Dx}{f}\right) - \pi$
9. $\alpha = 0 \Rightarrow \Delta\varphi = \dfrac{kDx}{f} - \pi \Rightarrow I = 2I_0\left[1 - \cos\left(\dfrac{kDx}{f}\right)\right]$
10. $\alpha = \alpha_p \Rightarrow I = 2I_0\left[1 + \cos\left(\dfrac{kDx}{f}\right)\right] = 4I_0$ en $x = 0$

Optique

Interférences en lumière incohérente

Exercice 1

$x = \dfrac{\lambda_0^2}{2a\,\Delta\lambda} = 7,2$ cm

Exercice 2

$\Delta p = 5 = \frac{ax}{D}\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_6}\right) \Rightarrow a = 1,55.10^{-3}$ m

Exercice 3

1. $I = 2I_0\left[1 + \cos\left(\dfrac{2\pi}{\lambda_0}\dfrac{ax}{D}\right)\right]$
2. a- $I = 4I_0\left[1 + \cos\left(k_0\dfrac{ae}{2D_S}\right) \times \cos\left(k_0\dfrac{ax}{D}\right)\right]$

   b- $\mathcal{V} = \left|\cos\left(k_0\dfrac{ae}{2D_S}\right)\right|$

   c- $\tan\beta = \dfrac{e}{D_S} = \dfrac{\lambda}{\Delta a}$

Exercice 4

1. $I = I_0\,[1 + \mathrm{sin}_c(\pi\delta\,\Delta\sigma)\,\cos(2\pi\sigma_0\,\delta)]$
2. $\delta = \dfrac{n}{\Delta\sigma}$ avec $n \in \mathbb{Z}^*$

Exercice 5

1. $\delta = \dfrac{ax}{D}$
2. $I = I_0\nu_m\sqrt{\pi}\left[1 + \cos\left(\dfrac{2\pi\nu_0 ax}{cD}\right)\,\exp\left[-\left(\dfrac{\pi\nu_m ax}{cD}\right)^2\right]\right]$
3.
   - Si $\nu_m \to 0$, $\mathcal{C} \simeq 1 \Rightarrow I = I_0\nu_m\sqrt{\pi}\,[1 + \cos(k_0\delta)]$ (équivaut à une lumière monochromatique)
   - Si $\nu_m \gg 1$ (lumière totalement incohérente), $\mathcal{C} \simeq 0 \Rightarrow$ brouillage des franges.

Exercice 6

1. $I = \dfrac{4I_0}{\pi} + \dfrac{\pi I_0}{X_0^2} \times \dfrac{1}{\dfrac{\pi^2}{4X_0^2} - \left(\dfrac{ka}{D'}\right)^2} \times \cos\left(\dfrac{kaX_0}{D'}\right)\cos\left(\dfrac{kax}{D}\right)$
2. $\mathcal{C} = \dfrac{1}{1 - \left(\dfrac{2kaX_0}{\pi D'}\right)^2}\cos\left(\dfrac{kaX_0}{D'}\right)$

Optique

Interférences à N ondes

Exercice 1

2. $\theta_m = -\theta_0$
3. $2\,\sin\left(\dfrac{D_m}{2}\right) = \dfrac{k\lambda_0}{a}$
4. $\Delta x = 1,1$ mm

Exercice 2

2. $\delta = na\,\sin\theta$
3. $a \simeq \lambda = 400$ nm ; par exemple : $a = 3\lambda \simeq 1,2\ \mu$m
4. Si $p = 1$, $\theta \simeq 20^\circ$ et si $p = 2$, $\theta \simeq 40^\circ$

Exercice 3

1. $a_\ell = 2\,\dfrac{L\,\Delta\lambda}{\lambda_1} \simeq 10\ \mu$m
2. $L' = \dfrac{\lambda_1}{\Delta\lambda}\,\dfrac{a_\ell}{3} = 11,8$ mm

Exercice 4

1. $\delta = 2a\,\cos i$
2. $\dfrac{\underline{a}_{n+1}}{\underline{a}_n} = r^2\,\mathrm{e}^{-\mathrm{j}\varphi}$
3. $I = \dfrac{J_0}{1 + m\,\sin^2\varphi/2}$ avec $J_0 = \dfrac{I_0}{(1-r^2)^2}$ et $m = \dfrac{4r^2}{(1-r^2)^2} = 90$

Exercice 5

1. $\sin\theta + \sin\theta_i = \dfrac{p\lambda}{a}$
2. $\theta_i = i_0 = 45^\circ$ et $\begin{cases} \theta_1 = \alpha_1 - 45^\circ = -28,3^\circ \\ \theta_2 = \alpha_2 - 45^\circ = 1,4^\circ \end{cases} \Rightarrow \langle a\rangle_{CD} = 1,2\ \mu\text{m et } \langle a\rangle_{DVD} \simeq 0,633\ \mu\text{m}$
3. $N_{\text{CD}} \simeq 680$ Mo et $N_{\text{DVD}} \simeq 2,4$ Go.

Exercice 6

Pics secondaires en $\sin\theta - \sin\theta_i = p\,\dfrac{\lambda}{a} - \dfrac{\lambda}{Pa}$, de largeur $\dfrac{\lambda}{Na}$

Exercice 7

1. a- $\varphi = k_0 a\,\sin\theta$

   b- $\underline{s} = \underline{s}_0\,\mathrm{e}^{\mathrm{i}(N-1)\,\varphi/2} \times \dfrac{\sin(N\varphi/2)}{\sin(\varphi/2)}$

   c- $I = I_0\,\left[\dfrac{\sin(N\varphi/2)}{\sin(\varphi/2)}\right]^2$

   e- $I = I_{\max}$ pour $\sin\theta = \dfrac{p\lambda}{a}$
2. a- $a\,(\sin\theta - \sin\theta_i) = p\lambda$

   b- $\theta \simeq \dfrac{p\lambda}{a}$

   d- $2\,\sin\left(\dfrac{D_m}{2}\right) = \dfrac{p\lambda}{a}$
3. a- $\Delta i_0 = \dfrac{b}{f'}$

   b- $\dfrac{\mathrm{d}i_p}{\mathrm{d}\lambda} = \dfrac{p}{2a\,\cos i_p}$ et $\dfrac{\lambda}{\Delta\lambda} = pN$

Interféromètre de Michelson

Exercice 1

1. $I = I_0 \left[1 + \cos\left(\frac{\pi\,\Delta\lambda\,\delta}{\lambda^2}\right)\cos\left(\frac{2\pi\delta}{\lambda_0}\right)\right]$

2. $N = \frac{\lambda_0}{\Delta\lambda} = 982$ franges

Exercice 2

$\ell = 17,6\ \mu$m

Exercice 3

1. $I = I_0 \left[1 + \cos\left(\frac{2\pi}{\lambda} \times 2\alpha X\right)\right]$ et $i = \frac{\lambda}{2\alpha}$

3. $I = 2I_0 \left[1 + \cos\left(\pi\delta \times \frac{\Delta\lambda}{\lambda^2}\right)\cos\left(\frac{2\pi}{\lambda}\,\delta\right)\right] \Rightarrow \Delta\lambda = 0,6.10^{-9}\ \mu$m et $\lambda'_1 = 0,5895\ \mu$m

Exercice 4

1. $\delta = 2\alpha x + 2e$

2. $I = A\,\Delta\nu \left[1 + \mathrm{sin}_c\left(\frac{\pi\delta\,\Delta\nu}{c}\right)\cos\left(\frac{2\pi\delta}{c}\,\nu_0\right)\right] \Rightarrow \Delta\nu = 1$ GHz

3. $\tau = \frac{1}{\Delta\nu} = 10^{-9}$ s et $L_c = 30$ cm

Exercice 5

1. $e = \frac{2,5\,\lambda_0}{2} = 0,72\ \mu$m

2. $n = 1 + 2,5\,\frac{\lambda_0}{2e} = 1 + 0,75.10^{-3}$

Exercice 6

$\lambda_0 = \frac{\Delta\delta}{N} = 547$ nm et $\Delta\lambda = 18$ nm

Exercice 7

1. $I \propto I_0\pi R^2 + \frac{\lambda f^2 I_0}{e}\sin\left(\frac{\pi e R^2}{\lambda f^2}\right)\cos\left(\frac{4\pi e}{\lambda} - \frac{\pi e R^2}{\lambda f^2}\right)$

2. $\mathcal{C} = \mathrm{sin}_c\left(\frac{\pi e R^2}{\lambda f^2}\right)$

Exercice 8

$\Delta e' = \frac{3\lambda_0}{2\,(n'-1)} = 1,8\ \mu$m

Thermodynamique

Premier principe de la thermodynamique

Exercice 1

1. $W_{\text{irr}} = RT_0 \left(\dfrac{p_0}{p_i} - 1\right)$
2. $W_{\text{rév}} = RT_0 \ln\left(\dfrac{p_0}{p_i}\right)$
3. $W_{\text{rev}} \simeq W_{\text{irr}}$

Exercice 2

1. $W = nR\left(T_i \dfrac{p_0}{p_i} - T_f\right)$
2. $T_f = \dfrac{T_i}{\gamma}\left[1 + (\gamma - 1)\dfrac{p_0}{p_i}\right]$

Exercice 3

1. $P_g \simeq P_0\left(1 - \dfrac{\gamma x}{\ell_0}\right)$ et $P_d \simeq P_0\left(1 + \dfrac{\gamma x}{\ell_0}\right)$
2. $\ddot{x} + \dfrac{2\gamma s P_0}{m\ell_0}\,x = 0$

Exercice 4

1. **Détente adiabatique réversible**
   - $\left(\dfrac{T_1}{T_0}\right) = \left(\dfrac{P_1}{P_0}\right)^{(\gamma-1)/\gamma}$
   - $W = \dfrac{nRT_0}{\gamma - 1}\left(\dfrac{T_1}{T_0} - 1\right)$ et $\Delta S = 0$
   - $T_1 = 398$ K et $|W| = 7\,580$ J
2. **Deuxième type de détente**
   b- $T_2 = 639$ K et $|W| = 4\,477$ J et $\Delta S = 9,9$ J.K^{-1}
   c- $|W|_{\text{rév}} > |W_{\text{irr}}|$

Exercice 5

$T_f = \gamma T_0 = 410$ K (137°C)

Exercice 6

1. $W_{\text{op}} = nRT_1\left[\ln\left(\dfrac{P_2}{P_1}\right) + 1 - \dfrac{P_2}{P_1}\right] = -835$ J
2. $m = \dfrac{nRT_1}{L_f}\ln\left(\dfrac{P_1}{P_2}\right) = 6,8$ g

Exercice 7

4. Pour un gaz parfait :

$$\ell = T^2 \frac{\partial}{\partial T}\left(\frac{p}{T}\right)_V = 0 \Rightarrow \mathrm{d}U = C_V(T)\,\mathrm{d}T \qquad h = -T^2 \frac{\partial}{\partial T}\left(\frac{V}{T}\right)_p = 0 \Rightarrow \mathrm{d}H = C_p(T)\,\mathrm{d}T \text{ et } C_p - C_V = nR$$

Deuxième principe de la thermodynamique

Exercice 1

1. $T_f = \dfrac{\gamma + 1}{2\gamma}\, T_0$
2. $\Delta S = \dfrac{nR}{\gamma - 1}\left[\gamma \ln\left(\dfrac{\gamma+1}{2\gamma}\right) + (\gamma - 1)\ln 2\right] = 0,06 \times \dfrac{nR}{\gamma - 1}$

Exercice 2

- $\Delta S = \Delta S_{AC} = nR \ln\left(\dfrac{p_0}{p_1}\right) = -26,7\ \mathrm{J.K^{-1}}$
- $S_{\mathrm{cr}} = nR\left[\dfrac{p_1}{p_0} - 1 - \ln\left(\dfrac{p_1}{p_0}\right)\right] = 173\ \mathrm{J.K^{-1}}$

Exercice 3

1. $\alpha = \dfrac{p_3 - p_1}{V_3 - V_1}$ et $\beta = \dfrac{p_1 V_3 - p_3 V_1}{V_3 - V_1}$
2. $W = \dfrac{(p_3 - p_1)(V_3 - V_1)}{2}$
3. $\Delta S_{12} = \dfrac{\gamma R}{\gamma - 1} \ln\left(\dfrac{V_3}{V_1}\right)$ et $\Delta S_{23} = \dfrac{R}{\gamma - 1}\ln\left(\dfrac{p_3}{p_1}\right)$ et $\Delta S_{31} = -(\Delta S_{12} + \Delta S_{23})$

Exercice 4

2. $\Delta S = C \ln\left(\dfrac{T_1}{T_0}\right)$
3. $S = S_0 + C \ln T$
4. $S_{\mathrm{cr}} = C\left[\dfrac{T_0}{T_1} - 1 - \ln\left(\dfrac{T_0}{T_1}\right)\right]$
5. $S_{\mathrm{cr}} \geqslant 0$

Exercice 5

$\Delta S = mc \ln\left[\dfrac{(T_1 + 2T_2)^3}{27\, T_1 T_2}\right] = mc \ln\left[\dfrac{1}{27} f\left(\dfrac{T_1}{T_2}\right)\right]$ où $f(x) = x^2 + 6x + 12 + \dfrac{8}{x} \geqslant 27$

Exercice 6

$Q = m\,\Delta s \times T = -396$ kJ et $W = 369$ kJ.

Exercice 7

1. $\Delta U_{AC} = Q_{AC} = 191$ kJ et $S_{\mathrm{cr}} \neq 0 \Rightarrow$ la transformation est isséversible.
2. - Premier type (isotherme + adiabatique réversibles) :

   $$Q'_{AB} = T_0\,\Delta s_{AC} = 49 \text{ kJ et } W'_{AC} = \Delta U_{AC} - Q'_{AC} = 142 \text{ kJ}$$

   - Deuxième type (adiabatique + isotherme réversibles) :

   $$Q''_{AC} = Q''_{BC} = T_f\,\Delta s_{AC} = 61,9 \text{ kJ et } W''_{AC} = \Delta U_{AC} - Q''_{AC} \Rightarrow Q''_{AC} = 129 \text{ kJ}$$

Exercice 8

1. $T_k = T_0\,\alpha^k$
2. $Q_N = nRT_0 \ln\left(\dfrac{P_1}{P_2}\right) \times \dfrac{1 - \alpha^N}{1 - \alpha}$ et $W_N = nRT_0\,(\alpha^N - 1) \times \left[\dfrac{1}{\gamma - 1} + \dfrac{1}{1 - \alpha}\ln\left(\dfrac{P_1}{P_2}\right)\right]$
3. - $Q_\infty = C_p\,(T_N - T_0) = \Delta H$ car P_1 tend vers P_2 (la transformation devient isobare) ;
   - $W_\infty = nR\,(T_0 - T_f)$ et pour une transformation isobare : $W = -p_{\mathrm{ext}}\,(V_2 - V_1) = nR\,(T_0 - T_f)$.

Thermodynamique

Fluide en écoulement, machines thermiques

Exercice 1

$\rho = 1 - a^{1-\gamma} = 0,46$

Exercice 2

1. a- $\dot{Q} = \mathcal{P} + D_m \times \dfrac{v_2^2 - v_1^2}{2} = 144$ kW

   b- $\dfrac{\Delta S}{\Delta t} = \dfrac{D_m R}{M} \ln\left(\dfrac{P_1}{P_2}\right) = 823$ J.K^{-1}.s^{-1} et $\dfrac{S_{\text{cr}}}{\Delta t} = \dfrac{\Delta S}{\Delta t} - \dfrac{\dot{Q}}{T_a} = 340$ J.K^{-1}.s^{-1}

2. a- $T_1 = T_2 + \dfrac{M(\gamma-1)}{\gamma R}\left[\dfrac{v_2^2 - v_1^2}{2} + \dfrac{\mathcal{P}}{D_m}\right] = 333$ K

   b- $\dfrac{S_{\text{cr}}}{\Delta t} = D_m \dfrac{R}{M(\gamma-1)} \ln\left[\left(\dfrac{T_2}{T_1}\right)^{\gamma}\left(\dfrac{P_2}{P_1}\right)^{1-\gamma}\right] = 361$ J.K^{-1}.s^{-1}

Exercice 3

1. $v_\ell = \dfrac{1}{\mu} = 10^{-3}$ m^3.kg^{-1} et $v_g = \dfrac{RT}{pM} = 1,7$ m^3.kg^{-1}
2. a- Mélange liquide-vapeur

   b- $m_\ell = 5,9$ g ; $m_g = 3,2$ g ; $x_v = 0,35$; $V_\ell = 5,9.10^{-3}$ L ; $V_g = 5,4$ L

Exercice 4

1. $D_m(T_s - T_e) + D'_m(T'_s - T'_e) = 0$
2. $D_m(T_s - T_e) + D'_m(T'_s - T'_e) = -\mathcal{P}$
3. $T'_s = T_s = \dfrac{D_m T_e + D'_m T'_e}{D_m + D'_m} = 32$°C
4. • Si $T'_s = T_e = 80$°C : $T_s = T_e + \dfrac{D'_m}{D_m}(T'_e - T'_s) = -160$°C (impossible !)

   • Si $T_s = T'_e = 20$°C : $T'_s = T'_e + \dfrac{D_m}{D'_m}(T_e - T'_s) = 35$°C

Exercice 5

1. $v = \dfrac{RT}{MP} = 0,83$ m^3.kg^{-1} et $v_{\text{lu}} = 0,80$ m^3.kg^{-1}
2. $\mathrm{d}h = c_p\,\mathrm{d}T = 0$
3. $c_p = 10^3$ J.K^{-1}.kg^{-1} et $\gamma = \dfrac{1}{1 - \dfrac{R}{c_p M}} = 1,4$

Exercice 6

1. $T_s = -15$°C
2. Éviter la glace.

Exercice 7

1. $P = 8$ bar et $m = 4$ kg et $m_{\max} = 25$ kg
2. $T_B = 80$°C
3. Éviter l'explosion.
4. $x = 0,5$ et $T \simeq -42$°C

Exercice Concours Central-Supélec

Q1. $\eta = -\dfrac{Q_c}{W}$

Q2. $\eta = \dfrac{T_c}{T_c - T_f}$

Q3. $\eta = 10,2$

Q6. Pas de liquide

Q7. Augmentation de $|q_c| \Rightarrow$ amélioration du COP

Q8. $D_m = 0,44$ kg.s^{-1}

Q9. $\eta = 5,5$

Q10. $\eta_{\text{réel}} = 3,8$

Q11. $\dfrac{\mathrm{d}T}{\mathrm{d}x} + \dfrac{1}{\ell_c}\,T = \dfrac{1}{\ell_c}\,T_e$ où $\ell_c = \dfrac{D_{m0}\,c_\ell}{\alpha}$

Q12. $T = T_e + (T_i - T_e)\,\mathrm{e}^{-x/\ell_e}$

Q14. $\dot{Q}_f = D_{m1}\,c_e\,(T_{S1} - T_{E1}) \Rightarrow D_{m1} = 7,2$ kg.s^{-1}

Q15. $D_{m0} = \dfrac{D_{m1}}{N_m}$

Q16. $L = N_m\,\dfrac{\ell_c}{4}\,\ln\left(\dfrac{T - T_e}{T_f - T_e}\right)$

Q17. $L = 102$ m

Q18. Pour faciliter le remplacement des modules.

Exercice 8

$\eta = 1 - \dfrac{T_f}{T_c}$

Exercice 9

	θ	x_v	p	s	h	v
état 1	212°C	1	20	$6,28$	2 800	$0,1$
état 2	-10°C	$0,80$	$0,5$	$6,28$	2 200	2

Exercice 10

1. $T_f = \sqrt{T_{10}\,T_{20}}$
2. $W_{\text{fourni}} = C\,(\sqrt{T_{10}} - \sqrt{T_{20}})^2$

Exercice 11

1. $\Delta h = w_u + q = h_2 - h_1$
2. $\sigma = \dfrac{R}{M}\,\ln\left(\dfrac{P_1}{P_2}\right) \Rightarrow P_1 > P_2$ pour le passage $(T_1, P_1) \longrightarrow (T_2, P_2)$
3. $\sigma = \dfrac{R}{M\,(\gamma-1)}\,\ln\left[\left(\dfrac{T_2}{T_1}\right)^{\gamma}\left(\dfrac{P_2}{P_1}\right)^{1-\gamma}\right] - \dfrac{\gamma R}{M\,(\gamma-1)} \times \dfrac{T_2 - T_1}{T_0}$
4. $P_1 = P_2 \Rightarrow \sigma = \dfrac{\gamma R}{M\,(\gamma-1)}\left[\ln\left(\dfrac{T_2}{T_1}\right) - \dfrac{T_2 - T_1}{T_0}\right]$ et $\begin{cases} \text{Si } T_0 > T_1 \text{ le gaz se réchauffe } (T_2 > T_1) \\ \text{Si } T_0 < T_1 \text{ le gaz se refroidit } (T_2 < T_1) \end{cases}$

Exercice 12

2. $P_u = D_m\,(h_2 - h_1) = 122$ kW
3. $\eta = \dfrac{h_1 - h_4}{h_2 - h_1} = 5,15$ et $x_4 = \dfrac{h_3 - h_5}{h_1 - h_5} = 0,059$

Exercice 13

2. $\eta = 1 - \tau^{\frac{1-\gamma}{\gamma}}$
3. $\tau_m = \left(\dfrac{T_1}{T_2}\right)^{\frac{\gamma}{2(1-\gamma)}} = 5,46 \Rightarrow \eta_{\max} = 1 - \sqrt{\dfrac{T_1}{T_3}} = 0,38$

Thermodynamique

Conduction thermique

Exercice 1

$$\mu c \, \frac{\partial T}{\partial t} = \frac{\lambda}{r^2} \frac{\partial}{\partial r} \left(r^2 \, \frac{\partial T}{\partial r} \right) \Rightarrow T = T_1 + \frac{R_1 R_2}{R_1 - R_2} \left(T_2 - T_1 \right) \times \left(\frac{1}{r} - \frac{1}{R_1} \right)$$

Exercice 2

$$T = T_{\text{ext}} + \left(T_0 - T_{\text{ext}} \right) \mathrm{e}^{-x/\delta} \text{ avec } \delta = \sqrt{\frac{\lambda R}{2h}}$$

Exercice 3

2. $T(x,t) = T_0 + T_1 \, \sin\left(\dfrac{\pi x}{L}\right) \, \mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{\mu c L^2}{\lambda \pi^2}$
3. $j_Q = -\lambda T_1 \, \dfrac{\pi}{L} \, \cos\left(\dfrac{\pi x}{L}\right) \, \mathrm{e}^{-t/\tau}$

Exercice 4

$$T = T_0 + \theta_0 \, \cos\left(\omega t - \frac{x}{\delta}\right) \, \mathrm{e}^{x/\delta} \text{ avec } \delta = \sqrt{\frac{2K}{\mu c \omega}} \Rightarrow \begin{cases} x_{\text{jour}} = 33 \text{ cm} \\ x_{\text{an}} = 6,3 \text{ cm} \end{cases}$$

Exercice 5

1. a- $j_Q > 0$ car $\dfrac{\mathrm{d}T}{\mathrm{d}z} < 0$

   b- $\mu c \, \dfrac{\partial T}{\partial t} = -K \, \Delta T = 0 \Rightarrow \Delta T = 0 \Rightarrow T = ax + b$

   c- $T(z,t) = \dfrac{T_0(t) - T_e}{h(t)} \, z + T_0$
2. a- $\mathcal{P}' = j_Q \times S$

   b- $T_0(t) = \dfrac{K T_e + \alpha h T_a}{K + \alpha h}$

   c- $T_0 \to T_a$ pour $h \to \infty$
3. a- $j_Q = L \, \dfrac{\mathrm{d}m}{\mathrm{d}t}$

   b- $\dfrac{\mathrm{d}m}{\mathrm{d}t} = \mu \, \dfrac{\mathrm{d}h}{\mathrm{d}t}$

   c- $L\mu \, \dfrac{\mathrm{d}h}{\mathrm{d}t} = \alpha \left(T_0 - T_a \right)$

   d- $h \, \dfrac{\mathrm{d}h}{\mathrm{d}t} + \dfrac{K}{\alpha} \, \dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{K}{\mu L} \left(T_e - T_a \right) \Rightarrow h^2 + 0,1 \, h - 1,3.10^{-7} \times t = 0$
4. $h = \dfrac{-0,1 + \sqrt{0,01 + 4 \times 1,3.10^{-7} \, t}}{2} = 6,7 \text{ cm}$

Exercice 6

$$\mathcal{P} = \frac{4\lambda}{a^2} \times \left(T_{\max} - T_s \right) = 3.10^8 \text{ W.m}^{-3}$$

Exercice 7

1. $T(x) = T_0 + \dfrac{RI^2}{2KS}\left(x - \dfrac{x^2}{L}\right)$

2. $x_m = \dfrac{L}{2} \Rightarrow T_{\max} = T(x_m) = T_0 + \dfrac{RI^2L}{4KS}$

Exercice 8

I- Approche analytique

2. $T = \dfrac{T_2 - T_1}{L}\, x + T_1$

3. a- $T = T_1 + \dfrac{S}{2D}\,(Lx - x^2)$

   b- $x_m = \dfrac{L}{2} \Rightarrow T_m = T_1 + \dfrac{SL^2}{8D} = 450$ K

4. a- $T = T_1 + ax - \dfrac{Sx^2}{2D}$ où $a = \dfrac{T_2 - T_1}{L} + \dfrac{SL}{2D} = 200$ K.m^{-1}

   b- $x_m = \dfrac{aD}{S} = 0,25$ cm et $T_m = T_1 + \dfrac{a^2D}{2S} = 425$ K

Exercice 9

1. $j(x) = I_0\left(\mathrm{e}^{-a/\delta} - \mathrm{e}^{-x/\delta}\right)$

2. $T = T_0 + \dfrac{I_0\delta}{\lambda}\left(1 - \mathrm{e}^{-x/\delta}\right) - \dfrac{I_0}{\lambda}\,\mathrm{e}^{-a/\delta}\, x$

3. $T_0 = T_{\text{ext}} + \dfrac{T_0}{h}\left(1 - \mathrm{e}^{-a/\delta}\right)$

Exercice 10

1. $T_0 = \dfrac{\alpha_1 T_{10} + \alpha_2 T_{20}}{\alpha_1 + \alpha_2}$

2. $\begin{cases} T_1(x,t) = T_0 + (T_{10} - T_0)\,\mathrm{erf}(u) \\ T_2(x,t) = T_0 + (T_0 - T_{20})\,\mathrm{erf}(u) \end{cases} \Rightarrow T_0 = \dfrac{\alpha_1 T_{10} + \alpha_2 T_{20}}{\alpha_1 + \alpha_2}$

Résistance thermique

Exercice 1

1. $T(x) = \dfrac{T_e - T_1}{e}\, x + T_1$
2. $T(e) = \dfrac{\lambda T_1 + heT_0}{he + \lambda}$ et $j_{th} = \dfrac{h\lambda}{he+\lambda}\,(T_1 - T_0) \Rightarrow R_{th} = \dfrac{e}{\lambda S} + \dfrac{1}{hS}$

Exercice 2

1. $\mu c\, \dfrac{\partial T}{\partial t} = \dfrac{\lambda}{r}\, \dfrac{\partial}{\partial r}\left(r\, \dfrac{\partial T}{\partial r}\right)$
3. $T = T_1 + \dfrac{T_2 - T_1}{\ln(R_2/R_1)}\, \ln\left(\dfrac{r}{R_1}\right) \Rightarrow R_{th} = \dfrac{\ln(R_2/R_1)}{2\pi\lambda h}$

Exercice 3

1. $\mu c\, \dfrac{\partial T}{\partial t} = \operatorname{div} \vec{\jmath} = 0$
2. $T(x) = T_1 + \dfrac{T_2 - T_1}{L}\, x$
3. $\dfrac{\mathrm{d}T_1}{\mathrm{d}t} = -\dfrac{\lambda S}{cL}\,(T_1 - T_2)$
4. $\dfrac{\mathrm{d}T_2}{\mathrm{d}t} = \dfrac{\lambda S}{cL}\,(T_1 - T_2)$
5. $T_{1,2} = \dfrac{T_{10} + T_{20}}{2} \pm \dfrac{T_{10} - T_{20}}{2}\, \mathrm{e}^{-t/\tau}$ où $\tau = \dfrac{cL}{\lambda S}$

Exercice 4

1. a- $\phi = 4\pi K\, \dfrac{R_1 R_2}{R_2 - R_1}\,(T_1 - T_2)$

   b- $R_{th} = \dfrac{R_2 - R_1}{4\pi K R_1 R_2}$

3. a- $T = T_1 + \dfrac{R_1 R_2}{R_2 - R_1}\,(T_1 - T_2)\left(\dfrac{1}{r} - \dfrac{1}{R_1}\right)$

   b- $j_Q(R_2) = \dfrac{K R_1}{R_2} \times \dfrac{T_1 - T_2}{R_2 - R_1}$

4. $T_0 = \dfrac{T_1\, R'_{th} + T_2\, R_{th}}{R'_{th} + R_{th}}$

Exercice 5

$$\begin{cases} R_{th} = 0,34 \text{ K.W}^{-1} \\ R'_{th} = 1,14 \text{K.W}^{-1} \end{cases} \Rightarrow \dfrac{\phi'}{\phi} = 30\,\%$$

Exercice 6

$$\phi_{\mathrm{th}} = \dfrac{T_1 - T_0}{R_{\mathrm{th}}} \text{ où } R_{\mathrm{th}} = \dfrac{1}{2\pi\ell}\left[\dfrac{1}{\lambda}\, \ln\left(\dfrac{r_2}{r_1}\right) + \dfrac{1}{hr_2}\right]$$

Exercice 7

1. $T_s = 26,8^\circ\mathrm{C}$
2. a- $T_s = \dfrac{T_e\, G_e + T_i\, G + \phi_0}{G_e + G}$

   b- $\phi_2 > 0\, \forall \phi_0$ mais $\phi_1 > 0$ pour $\phi_0 > h_e S\,(T_i - T_e)$

Mécanique

Changement de référentiel

Exercice 1

$v = 15$ km/h

Exercice 2

1. $\sin\theta = \dfrac{v_e}{v}$
2. $\Delta t = \dfrac{h}{\sqrt{v^2 - v_e^2}}$

Exercice 3

$\alpha > 60^\circ$

Exercice 4

$\ddot{x} = g \times \dfrac{M\,\sin\alpha - m\,\sin\beta}{M + \dfrac{3m}{2}}$

Exercice 5

$\vec{v}_{G/R} = -\dfrac{1}{2}\,\vec{v}_{P/R}$

Exercice 6

1. $\ddot{z} = \dfrac{2T}{m} - g$
2. $v_{B/\mathcal{R}_h} = 2v$ dans le référentiel $\mathcal{R}_h$ de l'homme, et $\mathcal{P}_{\text{homme}} = \dfrac{\mathrm{d}}{\mathrm{d}t}(Ec + Ep)$

Dynamique en référentiel non galiléen

Exercice 1

2. $\ell = \dfrac{mg\,\cos\theta + k\ell_0}{k - m\omega^2\,\sin^2\theta}$
3. $R = m\,\sin\theta \times \left|g - \ell\omega^2\,\cos\theta\right|$

Exercice 2

1. $x(t) = x_0\cosh(\omega t)$
2. $R_y = 2m\omega^2 x_0\,\sinh(\omega t)$ et $R_z = mg$

Exercice 3

1. $\mathcal{E}_{pe} = -\dfrac{mr^2}{2}\,\Omega^2$
2. $\cos\theta_e = \dfrac{2kd + 2mg}{2kd + md\Omega^2}$

Exercice 4

$x = \dfrac{\Omega\,\cos\lambda\,gt^3}{3} = 2$ cm et $y = -\Omega^2\,\dfrac{\sin(2\lambda)}{12}\,gt^4 = 10^{-6}$ m

Exercice 5

3. $\ddot{\rho} + 2\beta\mathrm{i}\,\dot{\rho} + \omega_0^2\rho = 0$
4. $\left\{\begin{array}{l} x(t) = a\,\cos(\beta t)\,\cos(\omega_0 t) \\ y(t) = -a\,\sin(\beta t)\,\cos(\omega_0 t) \end{array}\right.$ avec $\beta = \Omega\,\sin\lambda$

Exercice 6

$$t_1 = \left[\frac{2h \left(1 + \dfrac{m}{M} \sin^2 \alpha \right)}{g \, \sin^2 \alpha \, \left(1 + \dfrac{m}{M} \right)} \right]^{1/2}$$

Exercice 7

1. $Ep = -mg\ell \left(\cos\theta + \dfrac{\eta^2}{2} \sin^2\theta \right)$
2.

$$\begin{aligned}
\eta = 1 \quad &\Rightarrow \quad \theta_{e1} = 0 \text{ (équilibre indifférent)}; \theta_{e2} = \pi \text{ (équilibre instable)} \\
\eta < 1 \quad &\Rightarrow \quad \theta_{e1} = 0 \text{ (équilibre stable)}; \theta_{e2} = \pi \text{ (équilibre instable)} \\
\eta > 1 \quad &\Rightarrow \quad \theta_{e1} = 0 \text{ (équilibre instable)}; \theta_{e2} = \pi \text{ (équilibre instable)}; \theta_{e3} = \arccos(1/\eta^2) \text{ (équilibre stable)}
\end{aligned}$$

Mécanique du solide

Exercice 1

1. Cercle de rayon $L/2$
2. $\tan\alpha > \dfrac{1}{2f}$

Exercice 2

1. $\mathcal{P}_{\text{dissip}} = -fMg\,\dot{x}$
2. $\ddot{x} = \dfrac{g\,(m - fM)}{M + m}$
3. $m > f \times M$
4. $\ddot{x} = g \, \dfrac{m - fM}{M + m + I/R^2}$

Exercice 3

$T = T_0 \, \mathrm{e}^{\mu\theta} = 10^6$ N

Exercice 4

1. $x_1(t) = \dfrac{\mu g m}{k} + 7,5 \, \dfrac{\mu g m}{k} \, \cos(\omega t)$
2. $X_1 = -6,5 \, \dfrac{\mu g m}{k}$
3. $x_2 = -5,5 \, \dfrac{\mu g m}{k} \, \cos(\omega t) - \dfrac{\mu g m}{k} \Rightarrow X_2 = 4,5 \, \dfrac{\mu g m}{k}$
4. • $x_3(t) = \dfrac{\mu g m}{k} + 3,5 \, \dfrac{\mu g m}{k} \, \cos(\omega t) \Rightarrow X_3 = -2,5 \, \dfrac{\mu g m}{k}$
   • $x_4(t) = -1,5 \, \dfrac{\mu g m}{k} \, \cos(\omega t) - \dfrac{\mu g m}{k} \Rightarrow X_4 = 0,5 \, \dfrac{\mu g m}{k}$
5. $x(t) = \mathrm{e}^{-\omega_0 t/2Q} \, [A \, \cos(\omega t) + B \, \sin(\omega t)]$

Exercice 5

1. $L = \dfrac{3}{2}\,(\ell - r)$
2. $\gamma < 3fg$

Exercice 6

$\omega(t) = \omega_0 - 2f \, \dfrac{g}{R} \times \dfrac{1+f}{1+f^2} \times t$

Exercice 7

$f \geqslant \dfrac{\pi}{\pi^2 - 2} \simeq 0,399$

Exercice 8

$T = \dfrac{2}{\omega} \, \left[\dfrac{\pi}{2} + \arctan\left(\dfrac{v_0}{a\omega} \right) \right] + \dfrac{2a}{v_0}$

Physique quantique

Fonctions d'onde

Exercice 1

$A = \sqrt{\dfrac{2}{a}}$

Exercice 2

1. $-\dfrac{\hbar^2}{2m}\,\Delta\kappa + E_p\,\kappa = E\,\kappa$
2. $E_p = \dfrac{-e^2}{4\pi\varepsilon_0 R}$
3. $\kappa(\theta) = A\,\cos(\eta\theta) + B\,\sin(\eta\theta)$
4. • $R = n^2 \times a_0$ où $a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{m_e\,e^2} = 0,52.10^{-10}$ m
   • $E = -\dfrac{E_1}{n^2}$ avec $E_1 = \dfrac{me}{32}\left(\dfrac{e^2}{\pi\varepsilon_0\hbar}\right)^2 = 2,18.10^{-18}$ J $= 13,6$ eV

Exercice 3

1. a- $\delta\mathcal{P} = |\psi_{1s}|^2\,\mathrm{d}\tau$ avec $\mathrm{d}\tau = r^2\mathrm{d}r\,\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$
   b- $\delta\mathcal{P}_r = A^2\,I_\Omega \times \mathrm{e}^{-2r/a_0}r^2\,\mathrm{d}r$ avec $I_\Omega = \displaystyle\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi}\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi$
   c- $F_{1s} = A^2\,I_\Omega \times r^2\,\mathrm{e}^{-2r/a_0}$
   d- $F_{1s}(r = a_0)$ est maximum.
2. • $F_{2s} = B^2\,I_\Omega \times \left(2r - \dfrac{r^2}{a_0}\right)\,\mathrm{e}^{-r/a_0}$
   • Extremum pour $r_1 = 0,76\,a_0$; $r_2 = 2a_0$; $r_3 = 5,24\,a_0$
   • F_{2s} maximum pour $r_3 = 5,24\,a_0$

Exercice 4

1. a- Forme de cloche centrée sur 0.
   b- $\Delta x = \dfrac{\sigma}{2}$
2. $|\psi(x,t)|^2$ se propage à la vitesse $\dfrac{\hbar k_0}{m}$

   $\Delta x(t) = \dfrac{\sigma}{2} \times \sqrt{1 + \left(\dfrac{2\hbar t}{m\sigma^2}\right)^2} \Rightarrow$ le paquet d'ondes s'étale.

Particules dans des potentiels

Exercice 1

$E_n = n^2 \, \dfrac{\pi^2 \hbar^2}{2ma^2}$

Exercice 2

1. $\psi_n(x,t) = \sqrt{\dfrac{2}{a}} \, \sin\left(\dfrac{n\pi x}{a}\right) \, \mathrm{e}^{-\mathrm{i}E_n t/\hbar}$ où $E_n = n^2 \, \dfrac{\pi^2 \hbar^2}{2ma^2}$

3. $n+1$ nœuds pour l'énergie E_n.

4. ΔE_n augmente lorsque a diminue et $\langle E_{\min} \rangle = \dfrac{\hbar}{2ma^2}$

Exercice 3

3. $\begin{cases} A_1 + B_1 = 0 \\ A_1 \, \mathrm{e}^{\mathrm{i}ka} + B_1 \, \mathrm{e}^{-\mathrm{i}ka} = A_2 \, \mathrm{e}^{-a/\delta} \\ \mathrm{i}kA_1 \, \mathrm{e}^{\mathrm{i}ka} - \mathrm{i}kB_1 \, \mathrm{e}^{-\mathrm{i}ka} = -\dfrac{A_2}{\delta} \, \mathrm{e}^{-a/\delta} \end{cases}$

Exercice 4

1. $\dfrac{\mathrm{d}^2\varphi_{1,3}}{\mathrm{d}x^2} - k^2 \, \varphi_{1,3} = 0$ et $\dfrac{\mathrm{d}^2\varphi_2}{\mathrm{d}x^2} + p^2 \, \varphi_2 = 0$.

Exercice 6

1. $\psi_{\text{inc.}} = A \, \mathrm{e}^{\mathrm{i}\,(\vec{k}\cdot\vec{r} - \omega t)}$; $\psi_{\text{ref.}} = B \, \mathrm{e}^{\mathrm{i}\,(\vec{k}_1\cdot\vec{r} - \omega t)}$; $\psi_{\text{trans.}} = C \, \mathrm{e}^{\mathrm{i}\,(\vec{k}_2\cdot\vec{r} - \omega t)}$

3. $k_{1x} = -k_x$ et $k_{2x}^2 = k_x^2 - \dfrac{2mV_0}{\hbar^2}$

4. $\dfrac{B}{A} = \dfrac{k_x - k_{2x}}{k_x + k_{2x}}$ et $\dfrac{C}{A} = \dfrac{2k_x}{k_x + k_{2x}}$

5. $R = \left|\dfrac{B}{A}\right|^2 = \left|\dfrac{k_x - k_{2x}}{k_x + k_{2x}}\right|^2$ et $T = 1 - R = \dfrac{4k_x k_{2x}}{k_x^2 + k_{2x}^2} \neq \dfrac{|C|^2}{|A|^2}$

6. $\theta_1 = \theta$

7. $n = \dfrac{1}{\sqrt{1 - V_0/E}}$

8. $\sin\theta_2 = n \, \sin\theta$

Physique quantique

États non stationnaires

Exercice 1

1. $E_1 = \hbar\,\omega_1$ et $E_2 = \hbar\,\omega_2$
2. $\psi_1(x,t) = \varphi_1(x,t)\,\mathrm{e}^{-\mathrm{i}\omega_1 t}$ et $\psi_2(x,t) = \varphi_2(x)\,\mathrm{e}^{-\mathrm{i}\omega_2 t}$
3. $\rho_1 = |\varphi_1(x)|^2$ et $\rho_2 = |\varphi_2(x)|^2$
4. $\rho(x,t) = |\psi(x,t)|^2 = |\varphi_1|^2 = |\varphi_2|^2 + 2r\,\cos\left[(\omega_2-\omega_1)t+\theta\right] \Rightarrow \omega = \dfrac{E_2 - E_1}{\hbar}$

Exercice 2

1. $\psi(x,t) = \dfrac{1}{\sqrt{2}}\left[\phi_1(x)\,\mathrm{e}^{-\mathrm{i}E_1 t/\hbar} + \phi_2(x)\,\mathrm{e}^{-\mathrm{i}E_2 t/\hbar}\right]$
2. $|\psi|^2 = \dfrac{1}{2}\left[\phi_1^2 + \phi_2^2 + 2\phi_1\phi_2\,\cos(\omega t)\right]$
3. $\Delta t = \dfrac{\pi}{\omega}$

Exercice 3

III.A - Conformations de la molécule d'ammoniac

2. $k_B\,T = 2,3.10^{-3}$ eV $\ll V_0$ et $T_{\text{seuil}} = 3\,000$ K

III.B- Inversion quantique de la molécule d'ammoniac

2. a- $\varphi(x) = 0$ partout ailleurs.

   c- $\displaystyle\int_{x_0}^{x_0+\ell} |\varphi(x)|^2\,\mathrm{d}x = 1$

3. a- $\varphi_{An}(x) = \varphi_0\,\sin\left[\dfrac{n\pi}{\ell}\,(x+x_0)\right]$ où $\varphi_0 = \sqrt{\dfrac{2}{\ell}}$ et $E_n^A = \dfrac{n^2\pi^2\hbar^2}{2m\ell^2}$

   b- $\varphi_{Bn}(x) = \sqrt{\dfrac{2}{\ell}}\,\sin\left[\dfrac{n\pi}{\ell}\,(x-x_0)\right]$ avec $E_n^B = \dfrac{n^2\pi^2\hbar^2}{2m\ell^2}$

   c- $\mathcal{P} = 0$

4. $\varphi_A(x) = \mathcal{A}\,\sin\left[k\,(x+x_0+\ell)\right]$

5. a- $K = \dfrac{\hbar^2}{2m}\,(V_0 - E)$

   b- $\varphi_C(x_0^-) = \varphi_B(x_0^+)$ et $\dfrac{\mathrm{d}\varphi_C}{\mathrm{d}x}(x_0^-) = \dfrac{\mathrm{d}\varphi_B}{\mathrm{d}x}(x_0^+)$

6. a- $\psi(x,t) = \dfrac{1}{\sqrt{2}}\left[\varphi_1^{\text{sym}}\,\mathrm{e}^{-\mathrm{i}E_1^{\text{sym}}t/\hbar} + \varphi_1^{\text{anti}}\,\mathrm{e}^{-\mathrm{i}E_1^{\text{anti}}t/\hbar}\right]$

   b- $|\psi_1|^2 = |\psi_2|^2$

   c- $\psi(x,t) = \dfrac{1}{\sqrt{2}}\,\mathrm{e}^{-\mathrm{i}E_1^s t/\hbar}\,\left[\varphi_1^s(x) + \varphi_1^a(x)\,\mathrm{e}^{-\mathrm{i}t\,\delta E/\hbar}\right]$ où $\tau = \dfrac{2\pi\hbar}{\delta E}$ et $f = 2,38.10^{10}$ Hz

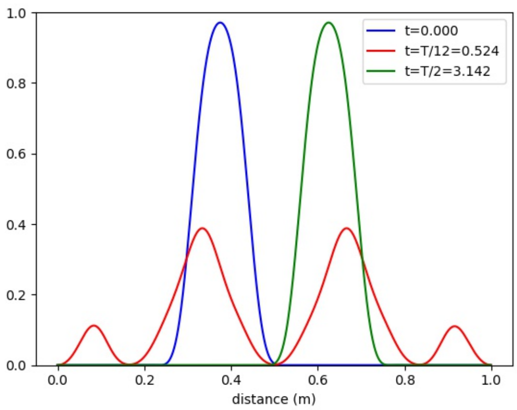
   d- $\psi(x, t_0 = \dfrac{\tau}{2}) = \dfrac{1}{\sqrt{2}}\,\left[\varphi_1^s(x) - \varphi_1^a(x)\right]$

   e- $f = \dfrac{\delta E}{2\pi\hbar}$ et $f' \simeq 0,56\times f$

Exercice 4

1. $A_n = \langle u_n | \varphi(x) \rangle = \displaystyle\int_{-\infty}^{+\infty} u_n^*(x)\,\varphi(x)\,\mathrm{d}x \Rightarrow A_{n\neq 4} = \sqrt{\frac{L}{2}} \times \frac{8}{\pi} \times \frac{\sin\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{4}\right)}{n^2 - 16}$ et $A_4 = \sqrt{\dfrac{L}{32}}$
2. $\psi(x,t) = \displaystyle\sum_{n=1}^{\infty} A_n \sqrt{\frac{2}{L}} \sin\left(n\pi\,\frac{x}{L}\right) \mathrm{e}^{-in^2\omega_0 t}$ où $\omega_0 = \dfrac{\pi^2\hbar}{2mL^2}$
3. Nouveau script :

```python
from math import*
import matplotlib.pyplot as plt
import matplotlib.animation as animation

N,L,w0=20,1,1#Nombre d'harmoniques (N), largeur du puits (L), pulsation w0
T=2*pi/w0# Période associée à w0

#Dates des représentations graphiques P2,P3=12,2 t1=0 t2=T/P2 t3=T/P3

def A(n):#Amplitude de chaque harmonique
    if n==4:
        return sqrt(L/32)
    else:
        return sqrt(L/2)*8/(pi*(n**2-16))*(sin(n*pi/2)+sin(n*pi/4))

def psi2(x,t):#Densité de probabilité $P(x,t)=|psi(x,t)|^2$
    Y=0
    for n in range(1,N+1):
        Y=Y+A(n)*sqrt(2/L)*sin(n*pi*x/L)*e**(-1j*n**2*w0*t)
    return abs(Y)**2

x=[i*L/1000 for i in range(1000)]#Intervalle de représentation de P(x,t)
y1,y2,y3=[],[],[]#Initialisation des listes de P(x,t1) ; P(x,t2) ; P(x,t3)
for X in x:#Accroissement des 3 listes
    y1.append(psi2(X,t1))
    y2.append(psi2(X,t2))
    y3.append(psi2(X,t3))

#Représentations graphiques
plt.plot(x,y1,"blue",label="t={:.3f}".format(t1))
plt.plot(x,y2,"red",label="t=T/{}={:.3f}".format(P2,t2))
plt.plot(x,y3,"green",label="t=T/{}={:.3f}".format(P3,t3))
plt.ylim(0,1)
plt.xlabel("distance (m)")
plt.legend(loc='upper right')
plt.show()
```



Physique statistique

Statique des fluides

Exercice 1

$N \simeq 10$ chevaux

Exercice 2

1. $P = P_0\, \mathrm{e}^{-\alpha z}$ où $\alpha = 1,18.10^{-4}$ m
2. $F_z = M_0 g\, \mathrm{e}^{-\alpha z} - Mg \Rightarrow M < M_0 = \dfrac{4\pi r^3 M_a\, P_0}{3RT} = 629$ kg
3. $z_1 = \dfrac{1}{\alpha} \ln\left(\dfrac{M_0}{M}\right) = 400$ m
4. $z_2 = \dfrac{1}{\alpha} \ln\left(\dfrac{20\, M_0}{19\, M}\right) = 835$ m
5. $\dfrac{\mathrm{d}\mathcal{E}_p}{\mathrm{d}z} = Mg - M_0 g\, \mathrm{e}^{-\alpha z} = 0$
6. $T = \dfrac{2\pi}{\sqrt{\alpha g}} = 185$ s

Exercice 3

1. $P(z) = P_0 + \mu g z$
2. $\vec{F} = \mu g R h^2 \sin\left(\dfrac{\alpha}{2}\right) \vec{u}_z$

Exercice 4

1. $\alpha = \dfrac{Mg}{Ra} = 5,27$
2. $P \simeq 0,56\, P_0$ pour $z = 4\,807$ m
3. $\mu = \mu_0 \left(\dfrac{P}{P_0}\right)^{\frac{\alpha-1}{\alpha}}$ avec $\mu_0 = \dfrac{P_0 M}{RT_0}$

Exercice 5

$h \geqslant \left(\dfrac{3m}{\pi\mu}\right)^{1/3}$

Distributions discrètes d'états

Exercice 1

1. $N_{1,2} = \dfrac{N}{Z}\,\mathrm{e}^{-\beta\,\varepsilon_{1,2}}$
2. $N_1 - N_2 = N \times \dfrac{\mathrm{e}^{-\beta\varepsilon_1} - \mathrm{e}^{-\beta\varepsilon_2}}{\mathrm{e}^{-\beta\varepsilon_1} + \mathrm{e}^{-\beta\varepsilon_2}}$
3. $\theta = \dfrac{\varepsilon}{kT}$
4. a- $T \ll \theta \Rightarrow N_1 \simeq N$ et $N_2 \simeq 0$

   b- $T \gg \theta \Rightarrow N_1 = N_2 = \dfrac{N}{2}$
5. $U = N\varepsilon_1 + \dfrac{N\varepsilon}{1+\mathrm{e}^{\beta\varepsilon}}$
6. $C_{vm} = R \times \dfrac{(\beta\varepsilon)^2\,\mathrm{e}^{\beta\varepsilon}}{(1+\mathrm{e}^{\beta\varepsilon})^2} \Rightarrow \begin{cases} C_{vm} \to 0 \text{ lorsque } T \to 0 \\ C_{vm} \to 0 \text{ lorsque } T \to \infty \end{cases}$
7. a- $N_1 = N_2 = \dfrac{N}{2} = 3,01.10^{23}$ et $N_1 - N_2 = \dfrac{N}{2} \times \beta\varepsilon = 1,16.10^{20}$

   b- $N_1 \simeq N$ et $N_2 \simeq N\,\mathrm{e}^{-38,6} \simeq 10^7$ d'où $N_1 - N_2 \simeq N$

Exercice 2

$Z = 2\mathrm{e}^{-\beta\varepsilon_2}\,[1+\cosh(\beta\,\Delta\varepsilon)] \Rightarrow U = N\varepsilon_2 - N\,\Delta\varepsilon\,\tanh\left(\dfrac{\beta\,\Delta\varepsilon}{2}\right)$ et $C_v = \dfrac{N\,(\Delta\varepsilon)^2}{2kT^2} \times \dfrac{1}{\cosh^2(\beta\,\Delta\varepsilon/2)}$

Exercice 3

1. $N_i^0 = \dfrac{N}{Z}\,\mathrm{e}^{-\beta\varepsilon_i}$ et $Z = \displaystyle\sum_i \mathrm{e}^{-\beta\varepsilon_i}$
2. a- $Z = \dfrac{\mathrm{e}^{-3\beta h\nu/2}}{(1-\mathrm{e}^{-\beta h\nu})^3}$

   b- $Z = \displaystyle\sum_n g_n\,\mathrm{e}^{-\beta\left(n+\frac{3}{2}\right)h\nu}$ où $g_n = \dfrac{(n+1)(n+2)}{2}$
3. $U = \dfrac{3}{2}\,Nh\nu + \dfrac{3Nh\nu}{\mathrm{e}^{\beta h\nu}-1}$
5. $C_{vm} = 3R\,(\beta h\nu)^2\,\dfrac{\mathrm{e}^{\beta h\nu}}{(\mathrm{e}^{\beta h\nu}-1)^2}$ et $C_{vm} \to 3R$
6. • $Z = \mathrm{e}^{-3h\nu/2kT}\displaystyle\int_0^\infty \mathrm{e}^{-\beta h\nu\,n}\mathrm{d}n \Rightarrow Z = \dfrac{\mathrm{e}^{-3\beta h\nu/2}}{(\beta h\nu)^3}$

   • $U_m = \dfrac{3\mathcal{N}h\nu}{2} + 3\mathcal{N}kT \Rightarrow C_{Vm} = 3R$

Exercice 4

2. $p^{\pm} = \dfrac{\mathrm{e}^{\pm\beta\varepsilon_0}}{\mathrm{e}^{\beta\varepsilon_0}+\mathrm{e}^{-\beta\varepsilon_0}}$
3. $\langle\varepsilon_k\rangle = \varepsilon_0\dfrac{\mathrm{e}^{-\beta\varepsilon_0}-\mathrm{e}^{\beta\varepsilon_0}}{\mathrm{e}^{-\beta\varepsilon_0}+\mathrm{e}^{\beta\varepsilon_0}} = -\varepsilon_0 \times \tanh(\beta\varepsilon_0)$
4. $\langle\vec{\mu}_k\rangle = \mu_0\,\vec{u}\,\tanh(\beta\varepsilon_0)$
5. $\left\langle\vec{\mathcal{M}}\right\rangle = N\mu_0\,\vec{u}\,\tanh(\beta\varepsilon_0)$
6. $\left\langle\vec{\mathcal{M}}\right\rangle \simeq C\,\dfrac{\vec{B}_0}{T}$ où $C = \dfrac{N\mu_0^2}{k}$

Physique statistique

Distributions continues d'états

Exercice 1

1. $\delta\mathcal{P} = \dfrac{\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi}{4\pi}\, a\, \dfrac{\mathrm{e}^{a\,\cos\theta}}{\sinh a}$ avec $a = \beta\mu\varepsilon_0$
2. $M = N\mu\left(\dfrac{1}{\tanh(a)} - \dfrac{1}{a}\right)$
3. • $T \to 0 \Rightarrow M \simeq N\mu$
   • $T \gg \dfrac{\mu B_0}{k} \Rightarrow M \simeq \dfrac{N\mu^2}{3k} \times \dfrac{B_0}{T}$
4.

Exercice 2

1. $Z = \dfrac{S}{\Delta}\,\dfrac{kT}{mg}\left(1 - \mathrm{e}^{-\beta mgz}\right) \times (2\pi mkT)^{3/2}$
2. a- $\mathrm{d}^2N = \dfrac{N}{S}\,\dfrac{mg}{kT}\,\dfrac{\mathrm{e}^{-\beta mgz}}{1-\mathrm{e}^{-\beta mgz}}\,\mathrm{d}^3r \Rightarrow n(z) = \dfrac{\mathrm{d}^2N}{\mathrm{d}^3r}$

   b- $\delta N_{z,z+\mathrm{d}z} = \dfrac{Nmg}{kT}\,\dfrac{\mathrm{e}^{-\beta mgz}\,\mathrm{d}z}{1-\mathrm{e}^{-\beta mgz}}$ indépendant de z si $L \ll \dfrac{kT}{mg}$

   c- $\dfrac{n_1}{n_2} = \mathrm{e}^{\beta mg(z_1-z_2)} \simeq 1 - \beta mg\,\Delta z$

   d- $\dfrac{n(L)}{n(0)} = 1 - 1,1.10^{-4}$
3. $Z = \dfrac{S}{s}\,\dfrac{kT}{mg}\,(2\pi mkT)^{3/2} \Rightarrow n(z) = \dfrac{N}{S}\,\dfrac{mg}{kT}\,\mathrm{e}^{\beta mgz}$
4. $pV = NkT$ et $p = p_0\,\mathrm{e}^{-mgz/kT}$ avec $p_0 = \dfrac{Nmg}{S}$

Exercice 3

1. $\delta N(r, \mathrm{d}r) = N \times \dfrac{r\,\mathrm{d}r\,\mathrm{e}^{\beta m\omega^2r^2/2}}{\mathrm{e}^{\beta m\omega^2H^2/2} - 1} \times \beta m\omega^2$
2. $n(r, \mathrm{d}r) = \dfrac{Nm\omega^2}{2\pi kTH}\,\dfrac{\mathrm{e}^{\beta m\omega^2r^2/2}}{\mathrm{e}^{\beta m\omega^2H^2/2} - 1}$
3. $p = p_0\,\mathrm{e}^{M\omega^2r^2/2RT}$
4. $\dfrac{P(R)}{P(0)} = 1 + 2,2.10^{-2}$

Exercice 4

1. a- $\mathcal{P}(\theta,\, \mathrm{d}\theta) = \dfrac{\sin\theta\,\mathrm{d}\theta}{2}$

   b- $\delta\mathcal{P}(\vec{p}, \theta) = \dfrac{1}{Z}\,\mathrm{e}^{x\,\cos\theta}\,\sin\theta\,\mathrm{d}\theta$ où $Z = \dfrac{2}{x}\,\sinh(x)$
2. $P = N\,\langle p\rangle = Np \times \left(\dfrac{\cosh(x)}{\sinh(x)} - \dfrac{1}{x}\right)$
3. $\lim\limits_{x\to\infty} \mathcal{L}(x) = 1$ et $\mathcal{L}(x \to 0) \sim \dfrac{x}{3} \Rightarrow \vec{P} \sim \dfrac{Np^2E}{3kT}\,\vec{u}_z$

Exercice 5

1. **Premièr modèle**

   b- $N_{\text{chocs}} = \dfrac{1}{6} \times n\,\delta\tau = \dfrac{1}{6} \times \dfrac{N}{V} \times Sv\,\mathrm{d}t$

   c- $\vec{F}_{/S} = pS\,\vec{u}_z$ où $p = \dfrac{1}{3}\,nmv^2$

   d- $\mathrm{d}U = C_{Vm}\,\mathrm{d}T$ avec $C_{Vm} = \dfrac{3}{2}\,R$

2. **Deuxième modèle**

   a- $\vec{F}_{1/S} = \dfrac{2mv\,\cos\theta}{\mathrm{d}t}\,\vec{u}_z$

   b- $\mathcal{P}(\delta\tau) = \dfrac{v\,\mathrm{d}t\,\cos\theta\,S}{V}$

   c- $f(v) = A\,\mathrm{e}^{-mv^2/2kT} \times v^2$

   d- $u^2 = \langle v^2 \rangle = 4\pi \displaystyle\int_0^\infty v^2\,f(v)\,\mathrm{d}v$

   e- $\left\langle \vec{F}_{/S} \right\rangle = N\,\left\langle \vec{F}_{1/S} \right\rangle = \dfrac{1}{3}\,nmu^2S\,\vec{u}_z \Rightarrow p = \dfrac{1}{3}nmu^2$ où $u^2 = \langle v^2 \rangle$

Chimie - révisions

Exercice 1

1. $\cos\left(\dfrac{\alpha}{2}\right) = \dfrac{\mu}{2\mu_0} \Rightarrow \alpha = 100,4^\circ$
2. a- $p = 0$

   b- $\cos\alpha = \dfrac{1}{3} \Rightarrow \alpha = 109,5^\circ$

Exercice 2

1. $[\mathrm{H}^+] = 2,72.10^{-8}\ \mathrm{mol\cdot L^{-1}} \Rightarrow \mathrm{pH} = 7,56$
2. $K_s = 2,24.10^{-20}$
3. $s = 2,24.10^{-6}\ \mathrm{mol\cdot L^{-1}}$

Exercice 3

- $s = 10^{-5}\ \mathrm{mol\cdot L^{-1}}$ dans l'eau pure ;
- $s = 10^{-9}\ \mathrm{mol\cdot L^{-1}}$ dans KCl ;
- $s = 4,1.10^{-6}\ \mathrm{mol\cdot L^{-1}}$ dans $BaCl_2$.

Exercice 4

1. $[\mathrm{Ag}^+] = \dfrac{1}{(4\beta)^{1/3}} = 2,5.10^{-3}\ \mathrm{mol\cdot L^{-1}}$
2. $\mathrm{Ag(NH_3)_2^+} + 2\,\mathrm{H^+} = \mathrm{Ag^+} + 2\,\mathrm{NH_4^+}$ avec $K^0 = \dfrac{1}{\beta\,K_a^2} = 10^{11,2}$
3. $[\mathrm{H}^+] = 2,4.10^{-9}\ \mathrm{mol\cdot L^{-1}} \Rightarrow \mathrm{pH} = 8,6$

Thermodynamique chimique

Enthalpie standard de réaction

Exercice 1

$\Delta_r H^0 = -1\,862 \text{ J} \cdot \text{mol}^{-1}$

Exercice 2

1. $\Delta_r H^0_{298} = -570,4 \text{ kJ} \cdot \text{mol}^{-1}$ et $\Delta_r {C_{p}}^0_1 = 63,6 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1}$
2. $\Delta_r H^0_1 = -570,4.10^3 + 63,6 \times (T - 25)$
3. $\Delta_r H^0_2 = -478,8.10^3 - 19,8 \times (T - 100)$

Exercice 3

1. $\left\{ \begin{array}{l} E_{\text{rupture}} = 4\,733 \text{ kJ} \cdot \text{mol}^{-1} \\ E_{\text{formation}} = 5\,994 \text{ kJ} \cdot \text{mol}^{-1} \end{array} \right. \Rightarrow \Delta_r H^0_2 = 4\,733 - 5\,994 = -1\,261 \Rightarrow \Delta_r H^0 = -1\,218 \text{ kJ} \cdot \text{mol}^{-1}$
2. $\Delta_r H^0 = -1\,277,6 \text{ kJ} \cdot \text{mol}^{-1}$

Effets thermiques des transformations

Exercice 1

$\Delta_r H^0 = -\dfrac{\Delta T_1}{n_a} \times \dfrac{RI^2\,\Delta t}{\Delta T_2} = 56,5 \;\text{ kJ} \cdot \text{mol}^{-1}$

Exercice 2

$C^0_{pf} = 184 + [500 + 0,90] \times 4,18 + 0,025 \times 29,4 = 2\,278 \text{ J} \cdot \text{K}^{-1} \cdot \text{mol}^{-1} \Rightarrow \Delta_r H^0 = -185 \text{ kJ} \cdot \text{mol}^{-1}$

Exercice 3

$C^0_{pf} = 330,5 \times n_0 \Rightarrow T_f = 1\,200 \text{ K}$

Exercice 4

1. $C^0_{pf} = n_0 \times 44,22 \Rightarrow T_f = 6\,697 \text{ K}$
2. $C^0_{pf} = n_0 \times 59,20 \Rightarrow T_f = 5\,078 \text{ K}$
3. $C^0_{pf} = n_0 \times 101,38 \Rightarrow T_f = 3\,089 \text{ K}$

Exercice 5

$\Delta_r H^0 = \dfrac{Q_v}{\Delta\xi} + 2\,RT = -790 \text{ kJ} \cdot \text{mol}^{-1}$

Potentiels chimiques

Exercice 1

1. a- $G = n_s\,\mu_s^* + n_\ell\,\mu_\ell^*$
   d- $\mathrm{d}G < 0$
   e- $\mu_s^* = \mu_\ell^*$
2. b- État liquide car $\mu_\ell^0(T_0) < \mu_s^0(T_0)$
   c- $T_f = 286\ \mathrm{K} = 13°\mathrm{C}$
   d- $p = P_0 + \dfrac{\mu_s^0(T_0) - \mu_\ell^0(T_0)}{V_\ell - V_s} = 4,4.10^8\ \mathrm{Pa}$

Exercice 2

1. a- $\mu_i^*(T,p) = \mu_i^0(T) + RT\,\ln a_i = \mu_i^0(T) + RT\,\ln\left(\dfrac{p}{P_0}\right)$
   b- $\mathrm{d}G = -S\,\mathrm{d}T + V\,\mathrm{d}p + \displaystyle\sum_i \mu_i\,\mathrm{d}n_i$
   c- $\Delta\mu_{GP}^* = RT\,\ln 5 = 4\,000\ \mathrm{J\cdot mol^{-1}}$
2. a- $\mu_{\text{eau}}^*(T) = \mu_{\text{eau}}^0(T) + RT\,\ln 1 \Rightarrow \Delta\mu_{\text{eau}}^* = 0$
   b- $\mu_{\text{eau}}^*(T,p_2) = \mu_{\text{eau}}^*(T,p_1) + V_m^* \times (p_2 - p_1)$
   c- $V_m^* = \dfrac{M}{\rho} = 18.10^{-3}\ \mathrm{g.mol^{-1}} \Rightarrow \Delta\mu_{\text{eau}}^* = 7,2\ \mathrm{J\cdot mol^{-1}}$

Exercice 3

$\mu_\ell(T_0,p) = \mu_s(T_0,p) \Rightarrow p = 40\ \mathrm{Pa}$

Exercice 4

1. $\mathrm{d}\mu_\alpha^* = \mathrm{d}\mu_\beta^* \Rightarrow (S_{m\alpha}^* - S_{m\beta}^*)\,\mathrm{d}T = (V_{m\alpha}^* - V_{m\beta}^*)\,\mathrm{d}p$
3. $\Delta_{\text{fus}}V^* = -1,63.10^{-6}\ \mathrm{m^3.mol^{-1}} \Rightarrow T = 272,41\ \mathrm{K}$

Enthalpie libre et entropie standard de réaction

Exercice 1

$\Delta_r G = -2,9.10^4\ \mathrm{J\cdot mol^{-1}} \Rightarrow$ le système évolue dans le sens direct.

Exercice 2

$\Delta_r G = -1\,025\ \mathrm{J\cdot mol^{-1}} \Rightarrow$ le système évolue dans le sens direct.

Exercice 3

1. a- $4\,NH_3 + 5\,O_2 = 4\,NO + 6\,H_2O$
   b- $4\,NH_3 + 3\,O_2 = 2\,N_2 + 6\,H_2O$
2. $\begin{cases} \Delta_r H_1^0 = -908\ \mathrm{kJ\cdot mol^{-1}} \\ \Delta_r H_2^0 = -1\,268\ \mathrm{kJ\cdot mol^{-1}} \end{cases}$ et $\begin{cases} \Delta_r S_1^0 = 181\ \mathrm{J\cdot K^{-1}\cdot mol^{-1}} \\ \Delta_r S_2^0 = 131\ \mathrm{J\cdot K^{-1}\cdot mol^{-1}} \end{cases}$
3. $\Delta_r G_1^0 = -1\,102\ \mathrm{kJ\cdot mol^{-1}}$ et $\Delta_r G_2^0 = -1\,408\ \mathrm{kJ\cdot mol^{-1}}$

Exercice 4

1. $\Delta_r S^0 = -7,7\,\ln T - 40,7 - 0,02 \times T$ et $\Delta_r H^0 = -495\,000 - 7,7\,T - 0,01\,T^2$

Thermodynamique chimique

Constantes d'équilibre

Exercice 1

$K^0 = 9.10^{26}$

Exercice 2

1. $K^0 = 7,364$
2.

acide	alcool	ester	eau
$2,513$ mol	$0,013$ mol	$0,487$ mol	$0,487$ mol

Exercice 3

1. $K^0 = \dfrac{\alpha^2}{1-\alpha^2}\dfrac{p}{P_0} = 0,0624$
2. $p = 2,71$ bar
3.

$COCl_2$	CO	Cl_2
$9,42.10^{-3}$ mol	$5,72.10^{-4}$ mol	$1,05.10^{-2}$ mol

4. $K^0 = 2.10^{-6}$

Exercice 4

1. $\Delta_r G^0(T) = 9,85.10^{-7}T^3 - 1,54.10^{-2}T^2 + 31,2\,T\,\ln T - 38\,200 - 100,4\,T \Rightarrow K^0 = 6,52$
2. $9,47\,\alpha^2 - 18,94\,\alpha + 8,47 = 0 \Rightarrow \alpha = 0,68$

Exercice 5

1. $Cr_2O_3 + 3\,H_2 = 2\,Cr + 3\,H_2O$
2. $\Delta_r G^0(T) = 414,3.10^3 - 140,7 \times T \Rightarrow K^0_{400} = 1,7.10^{-47}$
3. $K^0_{1\,200} = 2.10^{-11}$ et $m_{H_2} = 14,6$ tonnes

Optimisation des procédés chimiques

Exercice 1

1. Réaction exothermique $\Rightarrow$ Sens inverse
2. $K^0_{700} = 7,3.10^4$

Exercice 2

1. Déplacement dans le sens direct.
2. $4,75\,\rho^2 - 9,5\,\rho + 3,75 = 0 \Rightarrow \rho = 0,54$

Exercice 3

Déplacement de l'équilibre dans le sens direct.

Exercice 4

1. a- Augmenter la température.
   b- Limiter la taille de l'installation.
2. cas a, b, d : déplacement dans le sens direct et rien dans le cas c.

Exercice 5

2. Déplacement dans le sens direct.

Équilibres chimiques

Exercice 1

1. $K^0 = \left(\dfrac{2\alpha}{1-\alpha}\right)^2 = 64 \Rightarrow \Delta_r G^0 = -22,5\ \text{kJ} \cdot \text{mol}^{-1}$
2. Pas d'influence.

Exercice 2

1. Déplacement dans le sens direct si $x_1 < 0,5$, sinon déplacement dans le sens inverse.
2. Déplacement dans le sens inverse.

Exercice 3

1. $K_1^0 = \dfrac{\alpha\,(2-\alpha)}{(1-\alpha)^2} = \dfrac{P_0}{p}$
3. $K_1^0 = 0,29 \Rightarrow 1,29\,\alpha^2 - 2,58\,\alpha + 0,29 = 0 \Rightarrow \alpha = 0,12$
4. Déplacement dans le sens inverse.
5. Déplacement dans le sens direct.

Exercice 4

1. $\begin{cases} K_1^0 = 10^{14,94} \text{ à } T_1 = 273\ \text{K} \\ K_2^0 = 10^{13,26} \text{ à } T_2 = 323\ \text{K} \end{cases} \Rightarrow$ réaction exothermique avec $\Delta_r H^0 = -56,7\ \text{kJ} \cdot \text{mol}^{-1}$
2. $K^0 = 5,09.10^{13} \Rightarrow \text{pH} = 6,85$

Exercice 5

1. $K_1^0 = \left[\dfrac{\alpha}{2\,(1-\alpha)}\right]^2 = \dfrac{1}{64} \simeq 1,56.10^{-2}$
2.

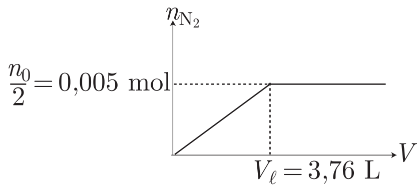
NH_3	HI	H_2	I_2
$0,50$ bar	$0,40$ bar	$0,05$ bar	$0,05$ bar

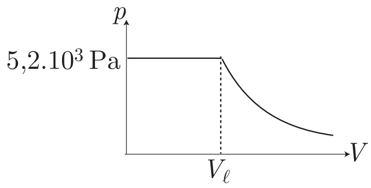
Exercice 6

1.

$CuBr_2$	CuBr	Br_2
$7,4.10^{-3}$ mol	$2,6.10^{-3}$ mol	$1,3.10^{-3}$ mol

2. .





Cinétique électrochimique

Courbes courant-potentiel

Exercice 3

2. $I = 0$ pour $0,10$ V

Exercice 4

2. $2\,IO_3^- + 12\,H^+ + 10\,e^- = I_2 + 6\,H_2O$ et $I_2 + 2\,e^- = 2\,I^-$

Exercice 6

1. À l'anode : $Cu \longrightarrow Cu^{2+} + 2\,e^-$ et à la cathode : $Cu^{2+} + 2\,e^- \longrightarrow Cu$
2. $\eta_a = 0,1$ V et $\eta_c = -0,4$ V

Exercice 7

1. Cas a : $2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4\,e^-$ et cas b : $2\,Cl^- \longrightarrow Cl_2 + +2\,e^-$
2. $\eta_a = 0,52$ V et $\eta_b = 0,1$ V

Exercice 8

$K_s = C^2 \times 10^{\frac{U_2 - U_1}{0,06}} = 1,3.10^{-13}$

La corrosion

Exercice 1

1. $2\,H^+ + Mg = Mg^{2+} + H_2 \Rightarrow K^0 = 10^{79}$

Exercice 3

2. $E^0 = -2,36$ V
3. $K_s = 10^{-11}$
6. $\Delta t = \dfrac{2m\,\mathcal{F}}{MI}$

Piles et électrolyses

Exercice 1

3. $Q = 2\mathcal{F}n_0 = 1\,930$ C $= 0,54$ A.h

Exercice 2

1. • À l'anode : $Cl^- \to Cl_2$ et $H_2O \to O_2$
   • À la cathode : $H^+ - \to H_2$ et $Sn^{2+} \to Sn$

Exercice 3

A- Étude prélimiraire

1. À l'anode : $2\,Cl^- \longrightarrow Cl_2 + 2\,e^-$ et $2\,H_2O \longrightarrow O_2 + 4\,H^+ + 4e^-$
2. $2\,H_2O = O_2 + 2\,H_2$ avec $K^0 = 10^{-82}$

B- Procédé à cellule à membrane

3. $2\,Cl^- + 2\,H_2O \longrightarrow Cl_2 + H_2 + 2\,HO^-$
5. $U = 2,9$ V

Exercice 4

1. $Pt|H_2|H^+, SO_4^{2-}||H^+, SO_4^{2-}|O_2|Pt$
2. a- $e = E_2^0 + 0,03\,\log(p_{H_2}\sqrt{p_{O_2}})$
   c- $e = 1,24$ V
   d- $e_0 = 1,23$ V
3. c- $2\,H_2 + O_2 = 2\,H_2O$ avec $\Delta_r G^0 = -4\mathcal{F}\,e_0 = -475$ kJ $\cdot$ mol^{-1}
   d- $\dfrac{de_0}{dT} = \dfrac{\Delta_r H^0 - \Delta_r G^0}{\eta\mathcal{F}T} = -8,4.10^{-4}$ V.K^{-1}

Exercice 5

1. $\eta_{Zn} = \dfrac{M_{Cu}}{M_{Zn}} \times \dfrac{\Delta m_{Zn}}{\Delta m_{Cu}} = 84\,\%$
2. $e = \dfrac{\Delta m_{Zn}}{\rho_{Zn} S} = 18,2\ \mu$m

Exercice 6

1. À l'anode : $\left\{\begin{array}{l} Mn^{2+} \to MnO_2 \\ H_2O \to O_2 \end{array}\right.$ et à la cathode : $\left\{\begin{array}{l} H^+ \to H_2 \\ Mn^{2+} \to Mn \end{array}\right.$
2. Anode : $Mn^{2+} + 2\,H_2O \longrightarrow MnO_2 + 4\,H^+$. Cathode : $2\,H^+ + 2\,e^- \longrightarrow H_2$
3. $Q = 2\,\xi_f\,\mathcal{F} = 2,2.10^6$ C $= 0,6$ kAh
4. $\Delta t = \dfrac{2\mathcal{F}\rho e}{Mj} = 1,38.10^6$ s $= 16$ j 1 h 8 min
5. $\mathcal{E}_{th} = 5,1.10^6$ J $= 1,4$ kWh