

Combinaison de plongée

plongeur sans feu

$$\frac{dU}{dt} = \frac{\delta W}{dt} + \delta Q$$

$$m c_{\text{corps}} \frac{dT}{dt} = 0$$

$$\delta Q = P_{\text{corps}} dt = \Phi_{\text{sortant}} dt$$

$$\delta Q = \left(P_{\text{corps}} - \frac{T - T_{\text{ext}}}{R_{\text{eq}}} \right) dt$$

$$m c_{\text{corps}} \frac{dT}{dt} = P_{\text{corps}} + \frac{T_{\text{ext}}}{R_{\text{eq}}} - \frac{T}{R_{\text{eq}}}$$

$$\frac{dT}{dt} + \frac{T}{R_{\text{eq}} m c_{\text{corps}}} = \frac{P_{\text{corps}}}{m c_{\text{corps}}} + \frac{T_{\text{ext}}}{R_{\text{eq}} m c_{\text{corps}}}$$

$$\tau = R_{\text{eq}} \cdot m \cdot c_{\text{corps}}$$

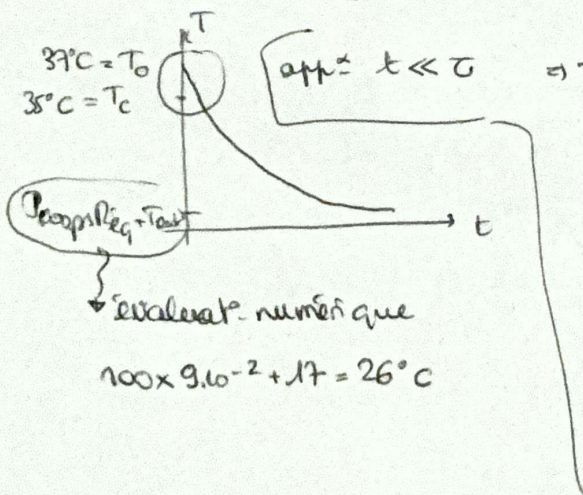
$$T(t) = T_h(t) + T_p(t)$$

$$T_h(t) = A e^{-t/\tau}$$

$$T_p(t) = \frac{P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}}}{R_{\text{eq}}}$$

$$A + P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}} = T_0$$

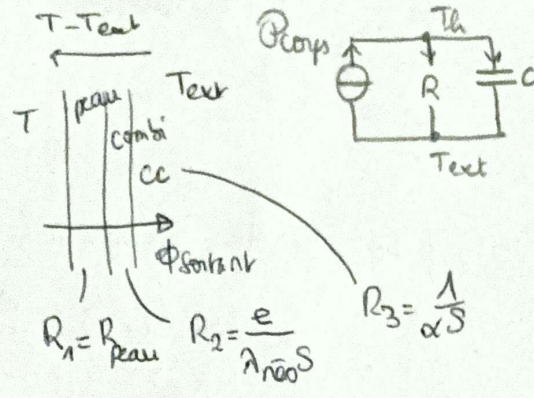
$$T(t) = (T_0 - P_{\text{corps}} R_{\text{eq}} - T_{\text{ext}}) e^{-t/\tau} + P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}}$$



$$\tau = R_{\text{eq}} \cdot m \cdot c_{\text{corps}}$$

$$\tau = 9.10^{-2} \times 80 \times 35.10^3 = 2.5.10^4 \text{ s}$$

ou sans approx



$$R_{\text{eq}} = R_1 + R_2 + R_3$$

$$T - T_{\text{ext}} = R_{\text{eq}} \cdot \Phi_{\text{sortant}}$$

sans combi
 $e=0$ $R_2=0$

$$R_{\text{eq}} = 3.10^{-2} + \frac{1}{10 \times 1.7} = 9.10^{-2}$$

$$S = 2\pi R h = 1.7 \text{ m}^2$$

15 cm 1.8 m.

$$\Rightarrow T(t) \approx (T_0 - P_{\text{corps}} R_{\text{eq}} - T_{\text{ext}}) \left(1 - \frac{t}{\tau}\right) + P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}}$$

$$T(t) \approx T_0 \left(1 - \frac{t}{\tau}\right) + (P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}}) \frac{t}{\tau}$$

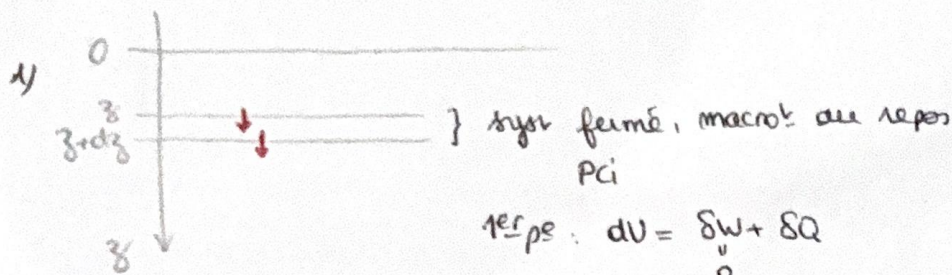
$$T_c - T_0 \approx (P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}} - T_0) \frac{t_f}{\tau}$$

$$t_f \approx \tau \cdot \frac{T_c - T_0}{P_{\text{corps}} R_{\text{eq}} + T_{\text{ext}} - T_0}$$

$$t_f \approx 2.5.10^4 \cdot \frac{35 - 37}{26 - 37} = 4.5.10^3 \text{ s} \ll \tau \approx 0.6 \text{ h}$$

$$t_f \approx 1.3 \text{ h}$$

Âge de la Terre selon Lord Kelvin (TDMP).



1^{er} pr : $du = \delta w + \delta q$

$$\rho \cdot S dz \cdot c \frac{\partial T}{\partial t} dt = \Phi(z, t) \cdot dt - \Phi(z+dz, t) \cdot dt$$

$$= - \frac{\partial \Phi}{\partial z} \cdot dz \cdot dt$$

avec $\Phi = \int_S \vec{j}_H \cdot d\vec{S} = \int_S j_H(z, t) \cdot \vec{u}_z \cdot dS = j_H(z, t) \cdot S$

$$\Rightarrow \rho \cancel{S} dz c \frac{\partial T}{\partial t} dt = - \frac{\partial j_H}{\partial z} \cdot \cancel{S} dz dt$$

loi de Fourier $\vec{j}_H = -\lambda \vec{\text{grad}} T = -\lambda \cdot \frac{\partial T}{\partial z} \vec{u}_z$

$$\Rightarrow \rho c \frac{\partial T}{\partial t} = + \lambda \frac{\partial^2 T}{\partial z^2}$$

$$\Leftrightarrow \frac{\partial T}{\partial t} = \underbrace{\left(\frac{\lambda}{\rho c} \right)}_{D} \cdot \frac{\partial^2 T}{\partial z^2} \Rightarrow \frac{\partial}{\partial t} \left(\underbrace{-\lambda \frac{\partial T}{\partial z}}_{j_H} \right) = D \cdot \frac{\partial^2}{\partial z^2} \left(\underbrace{-\lambda \frac{\partial T}{\partial z}}_{j_H} \right)$$

E. de Schwarz

2. $\vec{j}_H = -\lambda \vec{\text{grad}} T = -\lambda \frac{\partial T}{\partial z} \vec{u}_z$

$$|j_H| = \lambda \cdot \left| \frac{\partial T}{\partial z} \right|$$

$j_H(z=0, t \rightarrow 0) \rightarrow +\infty$
car T discontinue en $z=0$
à $t=0$.

$j_H(z=0, t \rightarrow +\infty) \rightarrow 0$
car éq. thermique entre Terre et son environnement
 $T_{00} = T_s$
↳ Rhéostat

$j_H(z > 0, t \rightarrow 0) = 0$
car T uniforme en $z > 0$
" " à $t=0$
 T_0

$j_H(z > 0, t \rightarrow +\infty) \rightarrow 0$
car T uniforme en $z > 0$
" " à $t \rightarrow +\infty$
 T_s

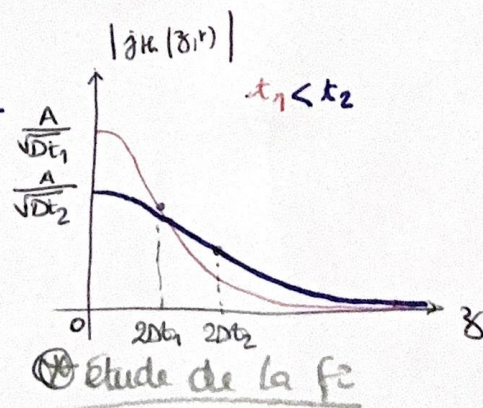
3) $j_H(z, t) = - \frac{A}{\sqrt{Dt}} \exp\left(-\frac{z^2}{4Dt}\right)$

$\lim_{z \rightarrow 0} j_H(z, t) = - \frac{A}{\sqrt{Dt}}$

$\rightarrow +\infty$ pour $t \rightarrow 0$ ci + c
 $\rightarrow 0$ pour $t \rightarrow +\infty$ c

$\lim_{t \rightarrow 0} j_H(z > 0, t) = 0$ car $\exp\left(-\frac{z^2}{4Dt}\right)$ domine

$\lim_{t \rightarrow \infty} j_H(z > 0, t) = 0$



③ on a $\lim_{z \rightarrow 0} j_H(z, t) = -\frac{A}{\sqrt{Dt}}$ décroît lorsque $t \uparrow$

on a $\frac{\partial j_H}{\partial z} = -\frac{A}{\sqrt{Dt}} \cdot -\frac{2z}{2\sqrt{Dt}} \cdot \exp\left(-\frac{z^2}{4Dt}\right)$

$\frac{\partial^2 j_H}{\partial z^2} = \frac{A}{2Dt\sqrt{Dt}} \exp\left(-\frac{z^2}{4Dt}\right) \left[1 - \frac{2z^2}{4Dt}\right]$

$\rightarrow \frac{\partial^2 j_H}{\partial z^2} = 0 \Leftrightarrow 4Dt = 2z^2 \Leftrightarrow z = \sqrt{2Dt}$ croît lorsque $t \uparrow$
point d'inflex.

4). $A = a (T_0 - T_s)^\alpha \cdot \lambda^\beta \cdot \rho^\gamma \cdot c^\delta$ $[a] = 1$

avec $\left[\frac{A}{\sqrt{Dt}}\right] = [j_H] \Rightarrow [A] = [\text{puissance} \cdot L^{-2} \cdot \sqrt{L^2 \cdot T^{-1} \cdot T}] = [\text{puissance} \cdot L^{-1}] = [\text{force} \cdot T^{-1}]$
 $\downarrow$
 $\text{force} \times L \cdot T^{-1}$
 $\Leftrightarrow [A] = M \cdot L \cdot T^{-3}$

$[T^\alpha \cdot (\text{puissance} \cdot T^{-1} \cdot L^{-1})^\beta \cdot (M \cdot L^{-3})^\gamma \cdot (\text{énergie} \cdot T^{-1} \cdot M^{-1})^\delta] = M \cdot L \cdot T^{-3}$
 $\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$
 $M^{\beta+\gamma} L^{\beta-3\gamma+2\delta} T^{-\beta-3\gamma-2\delta} = M \cdot L \cdot T^{-3}$

$\begin{cases} \alpha - \beta - \delta = 0 \\ \beta + \gamma = 1 \\ \beta - 3\gamma + 2\delta = 1 \\ +3\beta + 2\delta = +3 \end{cases} \Leftrightarrow \begin{cases} \alpha = \beta + \delta & (1) \\ \gamma = 1 - \beta & (2) \\ \beta - 3(1 - \beta) + 2\delta = 1 & (3) \\ \delta = \frac{3 - 3\beta}{2} & (4) \end{cases}$

(3) $\Rightarrow \beta(1 + 3 - 3) = 1 + 3 - 3 = 1 \Leftrightarrow \beta = 1$

$\begin{cases} \gamma = 1 - \beta = 0 \\ \delta = \frac{3 - 3\beta}{2} = \frac{3 - 3}{2} = 0 \\ \alpha = \beta + \delta = 1 + 0 = 1 \end{cases}$

bilan $\begin{cases} \alpha = \beta = 1 \\ \gamma = \delta = 0 \end{cases}$

$\Rightarrow A = a(T_0 - T_s) \cdot \lambda$

5). On a $\vec{j}_H = -\lambda \vec{q} \text{ grad } T = -\lambda \frac{\partial T}{\partial z} \vec{u}_3$

$\Leftrightarrow \frac{\partial T}{\partial z} = -\frac{\vec{j}_H \cdot \vec{u}_3}{\lambda} = -\frac{j_H}{\lambda}$

en surface, on a $\frac{\partial T}{\partial z}(z=0, t) = -\frac{1}{\lambda} \cdot -\frac{A}{\sqrt{Dt}} \cdot 1 = \frac{a(T_0 - T_s)\lambda}{\lambda\sqrt{Dt}} = \frac{T_0 - T_s}{\sqrt{\pi Dt}}$

Connaissant $\frac{\partial T}{\partial z}(z=0, t)$, on peut en déduire t :

$\pi Dt = \left(\frac{T_0 - T_s}{\frac{\partial T}{\partial z}(z=0, t)}\right)^2 \Leftrightarrow t = \frac{1}{\pi D} \cdot \left(\frac{T_0 - T_s}{\frac{\partial T}{\partial z}(z=0, t)}\right)^2$

AN: $t = \frac{1}{\pi \times 1.10^{-6}} \cdot \left(\frac{1500}{30.10^{-3}}\right)^2 \approx 80.10^{14} \text{ s} \approx 25.10^7 \text{ ans} = 25 \text{ millions d'années}$

6). Âge de la Terre $\approx 4,5$ milliards d'années (d'ap. géologie) $\gg$ âge estimé par Lk

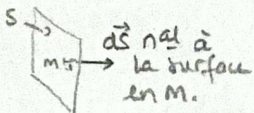
erreur due à la correction du manteau
à la pte de roches radioactives $\rightarrow$ terme source

$\hookrightarrow$ radiact. découverte fin XIX^e $\rightarrow$ ap. Lk milieu XIX^e

III. Fonction d'erreur de Gauss : erf(x) (d'ap CCMP MP-MPI 2023).

23) Prise en compte de la matière radioactive au sein de la Terre | Prise en compte convect?
 → terme source : pc volumique > 0.

24) $\vec{j}_Q$ tq $\Phi = \int_S \vec{j}_Q \cdot d\vec{s}$
 ↑
 flux thermique traversant S
 = $\rho \frac{Q}{s}$
 ds le sens de $d\vec{s}$



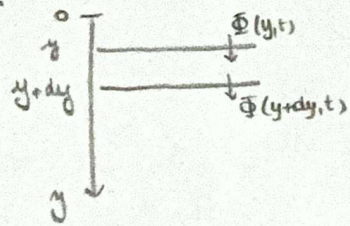
$d\vec{s}$ n^{al} à la surface en M.

$\|\vec{j}_Q\|$ en $W \cdot m^{-2}$
 ↳ pc surfacique

loi de Fourier : $\vec{j}_Q = -K \vec{q} \frac{dT}{dy}$
 $\downarrow$ $\downarrow$
 $W \cdot m^{-2}$ $K \cdot m^{-1}$

cdt? d'application : valable pour des variat^s spatiales de T pas trop élevées.
 (app^s linéaire).
 et pour un milieu isotrope

$[K] = \frac{W \cdot m^{-1} \cdot K^{-1}}{L \cdot \theta}$
 ↳ température

25) 

$\delta Q = [\Phi(y,t) - \Phi(y+dy,t)] \cdot dt = - \frac{\partial \Phi}{\partial y} dy dt$
 avec $\Phi(y,t) = \int_S \vec{j}_Q \cdot d\vec{s} = \int_S \underbrace{\vec{j}_Q \cdot \vec{u}_y}_{\text{uniforme sur S}} \cdot dS \vec{u}_y = j_Q \cdot S$

⇒ $\delta Q = - \frac{\partial j_Q}{\partial y} dy dt S$

tranche mésoscopique car équilibre thermod. local seulement.
 ↳ de sol. ↳ T dépend de y

sys fermé macro! au repos qui subit une transp infinitésimale entre t et t+dt :

1^{re} pc : $dU = \underbrace{\delta W}_{=0} + \delta Q$ ⇒ $dU = - \frac{\partial j_Q}{\partial y} dy dt S$

or sys assimilé à une pci $dU = C (T(y,t+dt) - T(y,t)) = C \cdot \frac{\partial T}{\partial t} dt = c \cdot \rho S dy \frac{\partial T}{\partial t} dt$
 ainsi $c \rho S dy \frac{\partial T}{\partial t} dt = - \frac{\partial j_Q}{\partial y} dy dt S = + K \frac{\partial^2 T}{\partial y^2} dy dt S$ avec loi de Fourier 1D.

$$\Rightarrow c \rho \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial y^2} \Rightarrow \left\| \frac{\partial T}{\partial t} = \left(\frac{K}{c \rho} \right) \cdot \frac{\partial^2 T}{\partial y^2} \right. \text{ avec } [D] = \frac{[\partial T / \partial t]}{[\partial^2 T / \partial y^2]} = \frac{L^2}{T}$$

$$\text{on pose } \frac{T^*}{L} = D \cdot \frac{T^*}{L^2} \Rightarrow L = \sqrt{Dt}$$

26) $(T_0 - T_1) \Theta(y, t) + T_1 = T(y, t)$

$$\Rightarrow \left\{ \begin{array}{l} \frac{\partial T}{\partial t} = (T_0 - T_1) \frac{\partial \Theta}{\partial t} \\ \frac{\partial T}{\partial y} = (T_0 - T_1) \frac{\partial \Theta}{\partial y} \end{array} \right. \text{ et } \left\| \frac{\partial^2 T}{\partial y^2} = (T_0 - T_1) \cdot \frac{\partial^2 \Theta}{\partial y^2} \right\} \Rightarrow \left\| \frac{(T_0 - T_1) \frac{\partial \Theta}{\partial t}}{(T_0 - T_1) \frac{\partial \Theta}{\partial y}} = D \cdot \frac{(T_0 - T_1) \frac{\partial^2 \Theta}{\partial y^2}}{(T_0 - T_1) \frac{\partial \Theta}{\partial y}} \right.$$

$T(y > 0, t = 0) = T_1 \Rightarrow \Theta(y > 0, t = 0) = \frac{T_1 - T_1}{T_0 - T_1} = 0$

$T(y = 0, t > 0) = T_0 \Rightarrow \Theta(y = 0, t > 0) = \frac{T_0 - T_1}{T_0 - T_1} = 1$

$T(y \rightarrow +\infty, t > 0) = T_1 \Rightarrow \Theta(y \rightarrow +\infty, t > 0) = 0$

27) $\eta = \frac{y}{2\sqrt{Dt}}$ on a $\left\| \frac{\partial \eta}{\partial y} = \frac{1}{2\sqrt{Dt}} \right.$ et $\left\| \frac{\partial \eta}{\partial t} = \frac{y}{2\sqrt{Dt}} \cdot \left(-\frac{1}{2}\right) t^{-3/2} \right.$

on a $\left\| \frac{\partial \Theta}{\partial t} = \frac{d\Theta}{d\eta} \cdot \frac{\partial \eta}{\partial t} = \frac{d\Theta}{d\eta} \cdot \left(-\frac{y}{4\sqrt{Dt}}\right) \right.$

$\left\| \frac{\partial \Theta}{\partial y} = \frac{d\Theta}{d\eta} \cdot \frac{\partial \eta}{\partial y} \right.$ et $\left\| \frac{\partial^2 \Theta}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{d\Theta}{d\eta} \cdot \frac{\partial \eta}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{d\Theta}{d\eta} \right) \cdot \frac{\partial \eta}{\partial y} \right.$
indpt de y
 $= \frac{d^2 \Theta}{d\eta^2} \cdot \left(\frac{\partial \eta}{\partial y} \right)^2$
 $= \frac{d^2 \Theta}{d\eta^2} \cdot \frac{1}{4Dt}$

d'où $-\frac{y}{4\sqrt{Dt}} t^{-3/2} \cdot \frac{d\Theta}{d\eta} = \cancel{D} \cdot \frac{1}{\cancel{4Dt}} \frac{d^2 \Theta}{d\eta^2}$

$\frac{d^2 \Theta}{d\eta^2} + \frac{y t^{-3/2}}{\sqrt{Dt}} t \cdot \frac{d\Theta}{d\eta} = 0 \Rightarrow \left\| \frac{d^2 \Theta}{d\eta^2} + \left(\frac{y}{\sqrt{Dt}} \right) \cdot \frac{d\Theta}{d\eta} = 0 \right.$

28) on introduit $\psi(\eta) = \frac{d\Theta}{d\eta}$, on a $\frac{d\psi}{d\eta} = \frac{d^2 \Theta}{d\eta^2}$

$\Rightarrow \psi$ vérifie $\left\| \frac{d\psi}{d\eta} + 2\eta \psi = 0 \right.$ $\Rightarrow \psi(\eta) = \lambda_i \exp\left(-\int \frac{2\eta'}{1} d\eta'\right) = \lambda_i \exp(-\eta^2)$

EDLH₁ à coeff qui dépendent de η à déterminer avec les ci/cl.
 ainsi $\left\| \frac{d\Theta}{d\eta} = \lambda_i e^{-\eta^2} \right.$ $\Rightarrow \int_{\Theta(\eta_1)}^{\Theta(\eta)} d\Theta = \lambda_i \int_{\eta_1}^{\eta} e^{-\eta'^2} d\eta' \Rightarrow \Theta(\eta) - \Theta(\eta_1) = \lambda_i \int_{\eta_1}^{\eta} e^{-\eta'^2} d\eta'$

d'ap Q26 on a $T(y=0, t>0) = T_0 \Rightarrow \theta(\eta=0) = 1$
 et $T(y \rightarrow +\infty, t>0) = T_1 \Rightarrow \theta(\eta \rightarrow +\infty) = 0.$

(10)

on choisit $\eta_1 = 0 \Rightarrow \theta(\eta) = \underbrace{\theta(\eta_1)}_{=1} + \lambda_1 \int_0^\eta e^{-\eta'^2} d\eta'$

et $\lim_{\eta \rightarrow +\infty} \theta(\eta) = 1 + \lambda_1 \underbrace{\int_0^{+\infty} e^{-\eta'^2} d\eta'}_{\frac{\sqrt{\pi}}{2}} = 1 + \lambda_1 \frac{\sqrt{\pi}}{2} = 0.$
 $\Rightarrow \lambda_1 = -\frac{2}{\sqrt{\pi}}$

ainsi $\theta(\eta) = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\eta'^2} d\eta' = 1 - \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-z^2} dz$

$T(y, t) = T_1 + (T_0 - T_1) \theta(y, t)$

$\Rightarrow T(y, t) = T_1 + (T_0 - T_1) \cdot \left[1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{y}{2\sqrt{Dt}}} e^{-z^2} dz \right] = T_1 + T_0 \cdot \frac{1}{2} + (T_1 - T_0) \cdot \frac{2}{\sqrt{\pi}} \int_0^{\frac{y}{2\sqrt{Dt}}} e^{-z^2} dz$

29) Q27 $\frac{d^2\theta}{d\eta^2} + 2\eta \frac{d\theta}{d\eta} = 0$ avec $\theta(y, t) = \frac{T(y, t) - T_0}{T_f - T_0}$

on a à nouveau $\theta(\eta) = \theta(\eta_1) + \lambda_1' \int_{\eta_1}^\eta e^{-z^2} dz$

ici on a $T(y=0, t>0) = T_0 \Rightarrow \theta(\eta=0) = 0 \rightarrow$ on choisit aussi $\eta_1 = 0$

$\Rightarrow \theta(\eta) = 0 + \lambda_1' \int_0^\eta e^{-z^2} dz = \lambda_1' \cdot \frac{\sqrt{\pi}}{2} \text{erf}(\eta)$

et $T(y_s, t>0) = T_f \Rightarrow \theta(\eta = \frac{y_s}{2\sqrt{Dt}} = \eta_s) = 1$

$\Rightarrow \theta(\eta_s) = \lambda_1' \cdot \frac{\sqrt{\pi}}{2} \text{erf}(\eta_s) = 1 \Rightarrow \lambda_1' = \frac{1}{\frac{\sqrt{\pi}}{2} \text{erf}(\eta_s)}$

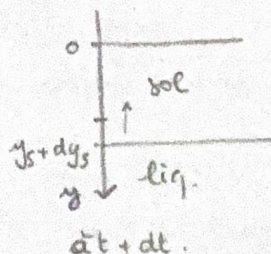
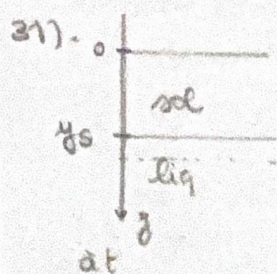
d'où $\theta(\eta) = \frac{1}{\frac{\sqrt{\pi}}{2} \text{erf}(\eta_s)} \cdot \frac{\sqrt{\pi}}{2} \text{erf}(\eta) = \frac{\text{erf}(\eta)}{\text{erf}(\eta_s)} \underset{\eta_s = \lambda}{=} \frac{\text{erf}(\eta)}{\text{erf}(\lambda)}$

30) solidification d'une tranche de laue d'épaisseur dy_s entre t et $t+dt$

$\delta Q_{\text{reque}} = -\rho S dy_s \Delta h_{\text{sol} \rightarrow \text{liq}}(T_f)$ car solidification isotherme à $T_f \Rightarrow$ isobare à $P_{\text{sol}}(T_f)$
 par sys $\underbrace{\frac{dV}{dm}}_{\text{masse infinitésimale solidifiée}}$ $\xrightarrow{1^{\text{er}} p^e} dH = \frac{\delta Q_{\text{reque}}}{\rho S dy_s} + \frac{\delta w'}{\rho S dy_s} = 0$

$\Rightarrow \delta Q_{\text{libérée}} = \rho S dy_s \Delta h_{\text{sol} \rightarrow \text{liq}}$

continuité du flux



$\delta Q_{\text{libérée}} = - \left[\iint \vec{q}_A(y_s, t) \cdot dS \vec{u}_y \right] \cdot dt$
 $\downarrow$ Fourier

$\Rightarrow \rho S dy_s \Delta h_{\text{sol} \rightarrow \text{liq}} = +K \cdot \frac{\partial T}{\partial y}(y_s) \cdot S \cdot dt$

$\Rightarrow \rho \cdot \Delta h_{\text{sol} \rightarrow \text{liq}}(T_f) \cdot \frac{dy_s}{dt} = K \cdot \frac{\partial T}{\partial y}(y_s)$

32) on a $\eta_s = \frac{y_s(t)}{2\sqrt{Dt}} = \lambda$

$$\Rightarrow y_s(t) = 2\lambda\sqrt{Dt}$$

$$\Rightarrow \left\| \frac{dy_s}{dt} = \frac{\lambda\sqrt{D}}{\sqrt{t}} \right.$$

et $\theta = \frac{T-T_0}{T_f-T_0} \Rightarrow \theta \cdot (T_f-T_0) + T_0 = T$

$$\Rightarrow \left\| \frac{\partial T}{\partial y} = (T_f-T_0) \cdot \frac{\partial \theta}{\partial y} = (T_f-T_0) \cdot \frac{\partial \theta}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = (T_f-T_0) \cdot \frac{1}{2\sqrt{Dt}} \cdot \frac{\partial \theta}{\partial \eta} \right.$$

l'éq. Q31 devient: $\rho \Delta h_{\text{sol-liq}}(T_f) \cdot \frac{\lambda\sqrt{D}}{\sqrt{t}} = K \cdot (T_f-T_0) \cdot \frac{1}{2\sqrt{Dt}} \cdot \frac{\partial \theta}{\partial \eta}(\eta_s)$

et $\frac{\partial \theta}{\partial \eta} = \frac{\partial}{\partial \eta} \left(\lambda_1 \cdot \int_0^\eta e^{-z^2} dz \right) = \lambda_1 e^{-\eta^2}$

$$\left\| \frac{\partial \theta}{\partial \eta}(\eta_s) = \lambda_1 \cdot e^{-\eta_s^2} = \frac{2}{\sqrt{\pi} \operatorname{erf}(\eta_s)} \cdot e^{-\eta_s^2} \right.$$

soit $\rho \cdot \Delta h_{\text{sol-liq}}(T_f) \cdot \lambda\sqrt{D} = K(T_f-T_0) \cdot \frac{1}{2\sqrt{D}} \cdot \frac{2e^{-\eta_s^2}}{\sqrt{\pi} \operatorname{erf}(\eta_s)}$

$$\frac{\rho D \sqrt{\pi}}{K(T_f-T_0)} \Delta h_{\text{sol-liq}}(T_f) = \frac{e^{-\eta_s^2}}{\lambda \cdot \operatorname{erf}(\eta_s)}$$

avec $D = \frac{K}{\rho c} \Leftrightarrow \frac{\rho D}{K} = \frac{1}{c}$

$$\Rightarrow \left\| \frac{\sqrt{\pi}}{c(T_f-T_0)} \Delta h_{\text{sol-liq}}(T_f) = \frac{e^{-\eta_s^2}}{\lambda \cdot \operatorname{erf}(\eta_s)} = \frac{e^{-\lambda^2}}{\lambda \cdot \operatorname{erf}(\lambda)} \right.$$

$\eta_s = \lambda$

33). $f: \lambda \mapsto \frac{f(\lambda)}{\lambda \cdot \operatorname{erf}(\lambda)} = \frac{e^{-\lambda^2}}{\lambda \cdot \operatorname{erf}(\lambda)}$ strict monotone sur $[0,6;3]$

et $\lambda = \eta_s = \frac{y_s(t)}{2\sqrt{Dt}} > 0$

on peut résoudre numériq. l'éq. de la Q32 avec un algo de dichotomie
en recherchant λ dans l'intervalle $[0,6;3]$

tel que $\left| \frac{e^{-\lambda^2}}{\lambda \cdot \operatorname{erf}(\lambda)} - 0,708 \right| = \frac{1,77 \times 4\%}{1 \times 10^4} = 0,708 \in [0;1,5]$, on pose $g(\lambda) = f(\lambda) - 0,708$
on a $g(0,6) > 0$ et $g(3) < 0$ g str. monotone sur $[0,6;3]$

$\lambda \approx 0,8$ d'ap lecture graphique.

intervalle $[a,b]$ initialisé à $[0,6;3]$

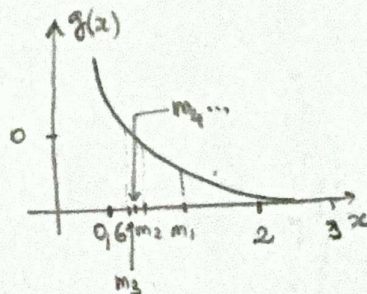
on calcule $m_1 = \frac{a+b}{2}$ et $g(m_1)$

Si $g(a) \times g(m_1) < 0$ on pose $b = m_1$

sinon $a = m_1$

on recommence avec $m_2 = \frac{a+b}{2}$

on réduit l'intervalle de valeurs auquel $\lambda \in$



34). $\lambda \approx 0,8$

on a $\frac{y_s(t)}{2\sqrt{Dt}} = \lambda \approx 0,8$

$$y_s(t) = 1,6\sqrt{Dt} \quad \text{ou } \frac{365}{2}$$

pour $t = 6 \text{ mois} = \underbrace{6}_{\text{ans}} \times \underbrace{30,5}_{\text{jours}} \times \underbrace{24}_{\text{h}} \times \underbrace{3600}_{\text{s}}$ et $D = 7 \cdot 10^{-7} \text{ m}^2 \cdot \text{s}^{-1}$

$$y_s(t) = 1,6 \cdot \sqrt{7 \cdot 10^{-7} \times 6 \times 30,5 \times 24 \times 3600}$$

$$= 1,6 \times 60 \times 65 \times 5 \times \sqrt{305 \cdot 10^{-6}}$$

$$= \frac{1,6 \times 60}{9,3} \times \underbrace{6,5 \times 5}_{8,5} \times \underbrace{1,7 \times 10^{-3}}_{10^{-2}} \text{ m.}$$

$$= 514 \cdot 10^{-2} \text{ m}$$

$$y_s(t) \approx 5,1 \text{ m}$$

d'ap. fig 8 on a $y_s(6 \text{ mois} = \frac{1 \text{ an}}{2}) \approx 5 \text{ m}$ pour Makapuhi
6 m pour Kilauae Iki

CCI valeur issue de la modélisation cohérente avec l'xp en particulier Makapuhi.