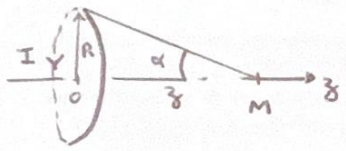
$\vec{B}$ créé par une spire - approx dipolaire

1).



$\sin\alpha = \dfrac{R}{\sqrt{R^2+z^2}}$ $\Rightarrow \boxed{\vec{B}(M) = B_0 \cdot \left(\dfrac{R}{\sqrt{R^2+z^2}}\right)^n \vec{u_z}}$

si $z \gg R$, on a $R^2+z^2 \simeq z^2$

$\Rightarrow \boxed{\vec{B}(M) \simeq B_0 \cdot \left(\dfrac{R}{|z|}\right)^n \vec{u_z}}$

2) $\boxed{\vec{m} = S I \vec{n} = -\pi R^2 I \vec{u_z}}$

3). $\vec{B}(M) = \dfrac{\mu_0}{4\pi OM^5}\left(3(\vec{m}\cdot\vec{OM})\cdot\vec{OM} - OM^2\cdot\vec{m}\right)$

$\vec{B}(M) = \dfrac{\mu_0}{4\pi|z|^5}\left(3(-\pi R^2 I \vec{u_z}\cdot z\vec{u_z})\cdot z\vec{u_z} - z^2(-\pi R^2 I \vec{u_z})\right)$

$\vec{B}(M) = \dfrac{\mu_0}{4\pi|z|^5}\left(-3\pi R^2 I z^2 \vec{u_z} + \pi R^2 I z^2 \vec{u_z}\right)$ ($z^2 = |z|^2$)

$\boxed{\vec{B}(M) = \dfrac{-2\pi R^2 I z^2 \mu_0}{4\pi|z|^5}\vec{u_z} = -\dfrac{\mu_0 R^2 I}{2|z|^3}\vec{u_z} = \underbrace{-\dfrac{\mu_0 I}{2R}}_{B_0}\cdot\dfrac{R^3}{|z|^3}\vec{u_z}}$ ($\downarrow n=3$)

Dipôle et spire chargée

1. $\vec{F} = \text{grad}(\vec{p}\cdot\vec{E}_{ext}) = \text{grad}\left(p\vec{u}_z \cdot \frac{\lambda a}{2\varepsilon_0}\cdot\frac{z}{(z^2+a^2)^{3/2}}\vec{u}_z\right)$.

$\Leftrightarrow \vec{F} = \frac{\lambda a p}{2\varepsilon_0}\cdot\frac{\partial}{\partial z}\left(\frac{z}{(z^2+a^2)^{3/2}}\right)\vec{u}_z$.

$\Leftrightarrow \vec{F} = \frac{\lambda a p}{2\varepsilon_0}\cdot\frac{(z^2+a^2)^{3/2} - \frac{3}{2}\times 2z(z^2+a^2)^{1/2}\cdot z}{(z^2+a^2)^3}\vec{u}_z$.

dipôle placé en $z=0 \Rightarrow \vec{F} = \frac{\lambda a p}{2\varepsilon_0}\cdot\frac{a^3}{a^6}\vec{u}_z$.

$\Leftrightarrow \boxed{\vec{F} = \frac{\lambda p}{2\varepsilon_0 a^2}\vec{u}_z}$. $\left(\frac{C.m^{-1}.C.m}{\varepsilon_0.m^2}\text{ ok } \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\right)$

↳ si $\lambda>0$, a tendance à déplacer $\vec{p}$ selon $+\vec{u}_z$.

2. $\vec{p} = p\vec{u}_z$

$E_p = -\vec{p}\cdot\vec{E}_{ext} = -p\vec{u}_z\cdot\frac{\lambda a}{2\varepsilon}\cdot\frac{z}{(z^2+a^2)^{3/2}}\vec{u}_z = \frac{-p\lambda a}{2\varepsilon_0}\cdot\frac{z}{(z^2+a^2)^{3/2}}$

posit. d'éq extremum de E_p $\frac{dE_p}{dz}=0$? $\rightarrow \vec{F}=\vec{0}$. $\vec{F} = -\text{grad}\,E_p$.

$(z^2+a^2)^{3/2} - 3z^2(z^2+a^2)^{1/2} = 0$.

$(z^2+a^2)^{1/2}\left[z^2+a^2-3z^2\right] = 0$.

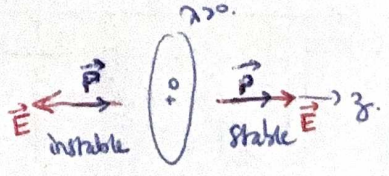
$(z^2+a^2)^{1/2}\left[-2z^2+a^2\right] = 0$.

$\neq 0$ $= 0 \Leftrightarrow \boxed{z = \pm\frac{a}{\sqrt{2}}}$

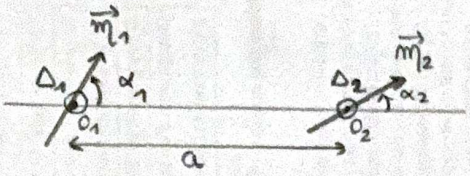
$\lambda>0$

z	$-\infty$		$-\frac{a}{\sqrt{2}}$		$\frac{a}{\sqrt{2}}$		$+\infty$
$\frac{dE_p}{dz}$		+	0	−	0	+	
E_p		↗	max instable	↘	min → stable	↗	

signe de $\frac{dE_p}{dz}$ = signe de $(2z^2-a^2)$.



Oscillateur à 2 dipôles.



1) On considère (1) dipôle actif qui créé $\vec{B}_1(M) = \frac{\mu_0}{4\pi O_1M^5}\left(3(\vec{m}_1\cdot\vec{O_1M})\vec{O_1M} - O_1M^2.\vec{m}_1\right)$

(2) dipôle passif soumis à $\vec{B}_1(O_2) = \frac{\mu_0}{4\pi O_1O_2^5}\left(3(\vec{m}_1\cdot\vec{O_1O_2}).\vec{O_1O_2} - O_1O_2^2.\vec{m}_1\right)$

L'énergie potentielle d'interaction entre ces 2 dipôles s'écrit alors

$$E_{p_{12}} = -\vec{m}_2\cdot\vec{B}_1(O_2)$$

$$\Leftrightarrow E_{p12} = -\vec{m}_2\cdot\frac{\mu_0}{4\pi a^5}\left(3(m_1.a.\cos\alpha_1).\vec{O_1O_2} - a^2.\vec{m}_1\right)$$

$$\Leftrightarrow E_{p12} = -\frac{\mu_0}{4\pi a^4}\left(3m_1\cos\alpha_1 m_2 a\cos\alpha_2 - a m_1 m_2\cos(\alpha_1-\alpha_2)\right)$$

$$\Leftrightarrow E_{p12} = -\frac{\mu_0}{4\pi a^3}\underbrace{m_1 m_2}_{m^2}\left(3\cos\alpha_1\cos\alpha_2 - \cos(\alpha_1-\alpha_2)\right)$$

$$\Leftrightarrow E_{p12} = -W_0\left(3\cos\alpha_1\cos\alpha_2 - \cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2\right)$$

$$\Leftrightarrow \boxed{E_{p12} = -W_0\left(2\cos\alpha_1\cos\alpha_2 - \sin\alpha_1\sin\alpha_2\right).}$$

2)- Etat d'éq. du syst : Etat tq E_{p12} est min.

$$dE_{p12} = \frac{\partial E_{p12}}{\partial\alpha_1}.d\alpha_1 + \frac{\partial E_{p12}}{\partial\alpha_2}.d\alpha_2$$

on cherche (α_1,α_2) tq $\frac{\partial E_{p12}}{\partial\alpha_1}=0$ et $\frac{\partial E_{p12}}{\partial\alpha_2}=0$.

$$\frac{\partial E_{p12}}{\partial\alpha_1} = -W_0\left(2\cos\alpha_2(-\sin\alpha_1) - \sin\alpha_2.\cos\alpha_1\right) = 0 \quad \left|\ \frac{\partial E_{p12}}{\partial\alpha_2}=0\right.$$

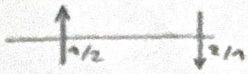
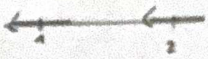
$$\Leftrightarrow -2\cos\alpha_2\sin\alpha_1 = \sin\alpha_2\cos\alpha_1 \quad \left|\ \Leftrightarrow -2\cos\alpha_1\sin\alpha_2 = \sin\alpha_1\cos\alpha_2\right.$$

$$\Leftrightarrow -2\cos\alpha_2\sin\alpha_1 = -\frac{1}{2}\sin\alpha_1\cos\alpha_2$$

$$\Leftrightarrow -\frac{3}{2}\cos\alpha_2\sin\alpha_1 = 0\ (1) \qquad \text{et} \quad -\frac{3}{2}\cos\alpha_1\sin\alpha_2 = 0\ (2)$$

$$\Leftrightarrow (1)\ \alpha_2 = \pm\frac{\pi}{2} \underset{(2)}{\Rightarrow} \alpha_1 = \pm\frac{\pi}{2}$$

$$(1)\ \alpha_1 = 0 \text{ ou } \pi \underset{(2)}{\Rightarrow} \alpha_2 = 0 \text{ ou } \pi$$

- $(\alpha_1 = \frac{\pi}{2}\ ,\ \alpha_2 = \frac{\pi}{2})$ $E_{p12} = -W_0 \times (0-1) = W_0$. (max local) → instable 
- $(\alpha_{1/2} = \frac{\pi}{2}\ ,\ \alpha_{2/1} = -\frac{\pi}{2})$ $E_{p12} = -W_0 \times (0+1) = -W_0$ (min local) → stable 
- $(\alpha_1 = 0\ ,\ \alpha_2 = 0)$ $E_{p12} = -W_0 (2-0) = -2W_0$ (min) → stable 
- $(\alpha_1 = \pi\ ,\ \alpha_2 = \pi)$ $E_{p12} = -W_0 (2(-1).(-1) - 0) = -2W_0$ (min) → stable 
- $(\alpha_{1/2} = 0\ ,\ \alpha_{2/1} = \pi)$ $E_{p12} = -W_0 (2(-1)\cdot 1 - 0) = 2W_0$ (max) → instable 

3). syst {dipôle 2} étudié ds $\mathcal{R}_{terrestre}$ galiléen
↳ solide en rotat° autour de (Δ_2) axe fixe ds $\mathcal{R}_{terrestre}$

Bilan action magnétique exercée par (1).

TPC $\frac{dE_c}{dt} = P_{ext}$ avec $E_c = \frac{1}{2} J . \dot{\alpha}_2^2$

$$P_{ext} = \frac{\delta W_{ext}}{dt} = \frac{-dE_{p12}}{dt} = W_0 \left(2\cos\alpha_1 \dot{\alpha}_2 (-\sin\alpha_2) - \sin\alpha_1 \dot{\alpha}_2 \cos\alpha_2\right)$$

d'où $\frac{1}{2} J . 2 . \dot{\alpha}_2 \ddot{\alpha}_2 = -\dot{\alpha}_2 . W_0 . (2\cos\alpha_1 \sin\alpha_2 + \sin\alpha_1 \cos\alpha_2)$

$$J . \ddot{\alpha}_2 + W_0 (2\cos\alpha_1 \sin\alpha_2 + \sin\alpha_1 \cos\alpha_2) = 0.$$

On considère le cas où $\alpha_1 = 0$ et α_2 oscille autour de 0

$$J \ddot{\alpha}_2 + W_0 (2\sin\alpha_2 + 0) = 0.$$

cas des petites oscillat° : $\alpha_2 \ll 1 \Rightarrow \sin\alpha_2 \simeq \alpha_2$

d'où $\ddot{\alpha}_2 + \underbrace{\frac{2W_0}{J}}_{\omega_0^2} \alpha_2 = 0$ Eqn d'un OH.

$$\boxed{\omega_0 = \sqrt{\frac{2W_0}{J}}}$$

Moteur synchrone. CCINP PSI 2008

I. 1). Tous les plans contenant $(M\vec{z})$ sont des plan d'antisym. de la distr de courants.

$\Rightarrow \vec{B}(M) \in$ à tous ces plans

$\Leftrightarrow \vec{B}(M) \parallel \vec{e_z}$.

II. A. 1. $\vec{B_S} = \vec{B_1} + \vec{B_2} + \vec{B_3}$: Th. de superpositi...

$$\vec{B_S} = K i_1(t)\vec{e_x} + K i_2(t)\left(\cos\left(\frac{2\pi}{3}\right)\vec{e_x} + \sin\left(\frac{2\pi}{3}\right)\vec{e_y}\right)$$
$$+ K i_3(t)\left(\cos\left(\frac{2\pi}{3}\right)\vec{e_x} - \sin\left(\frac{2\pi}{3}\right)\vec{e_y}\right).$$



$$= \begin{pmatrix} K i_1(t) - \frac{1}{2}K i_2(t) - \frac{1}{2}K i_3(t) \\ \frac{\sqrt{3}}{2}K i_2(t) - \frac{\sqrt{3}}{2}K i_3(t) \\ 0 \end{pmatrix}$$

$$= K I_m \begin{pmatrix} \cos(\omega_s t + \varphi) - \frac{1}{2}\cos(\omega_s t + \varphi - \frac{2\pi}{3}) - \frac{1}{2}\cos(\omega_s t + \varphi + \frac{2\pi}{3}) \\ \frac{\sqrt{3}}{2}\cos(\omega_s t + \varphi - \frac{2\pi}{3}) - \frac{\sqrt{3}}{2}\cos(\omega_s t + \varphi + \frac{2\pi}{3}) \\ 0 \end{pmatrix}$$

$$= K I_m \begin{pmatrix} \cos(\omega_s t + \varphi) - \frac{2}{2}\left[\cos(\omega_s t + \varphi)\cdot\cos\left(\frac{2\pi}{3}\right)\right] \\ -\frac{\sqrt{3}}{2} \times 2 \sin(\omega_s t + \varphi)\sin\left(-\frac{2\pi}{3}\right) \\ 0 \end{pmatrix}$$

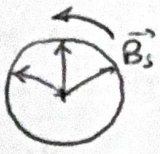
$$= K I_m \begin{pmatrix} \cos(\omega_s t + \varphi) + \frac{1}{2}\cos(\omega_s t + \varphi) \\ -\sqrt{3}\sin(\omega_s t + \varphi)\cdot\left(-\frac{\sqrt{3}}{2}\right) \\ 0 \end{pmatrix}$$

$$= K I_m \begin{pmatrix} \frac{3}{2}\cos(\omega_s t + \varphi) \\ \frac{3}{2}\sin(\omega_s t + \varphi) \\ 0 \end{pmatrix}$$

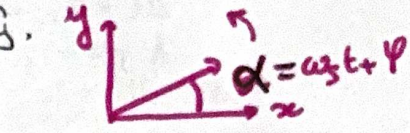
$\Leftrightarrow \vec{B_S} = \frac{3}{2} K I_m \left(\cos(\omega_s t + \varphi)\vec{e_x} + \sin(\omega_s t + \varphi)\vec{e_y}\right)$.

II.A.2. $\|\vec{B_s}\| = \frac{3}{2}|k| I_m (\cos^2 + \sin^2)$

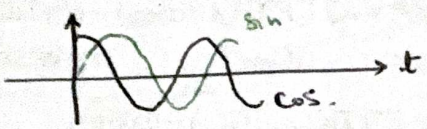
$\Leftrightarrow \|\vec{B_s}\| = \frac{3}{2}|k| I_m = \text{cste}$.

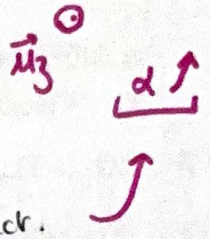


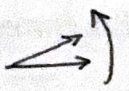
$\vec{u} = \frac{\vec{B_s}}{\|\vec{B_s}\|} = \cos(\omega_s t + \varphi)\vec{e_x} + \sin(\omega_s t + \varphi)\vec{e_y}$.



$\vec{u}$ dépend du tps → $\vec{B_s}$ de direc° variable







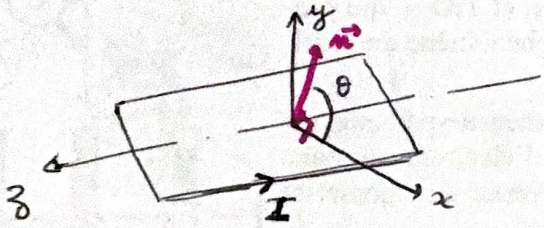
tourne autour de (Oz) ds le sens direct.

II.A.3. $\|\vec{B_s}\| = \frac{3}{2} k I_m = \frac{3}{2} k \sqrt{2}. I_{eff} = 1{,}59\ T$.

vitesse de rotate $= \omega_s = 2\pi f_s = 314\ rad.s^{-1} = \frac{314}{2\pi} \times 60 = 3{,}0.10^3\ tr/min$.

↳ $\dot{\alpha} =$

II.B.1). $\vec{u}(t) = \cos(\omega_s t + \varphi)\vec{e_x} + \sin(\omega_s t + \varphi)\vec{e_y}$.



$\vec{\Gamma} = \Gamma \vec{e_z}$. $\vec{m} = I\rho S \vec{n} = I\rho 2 r_0 H \vec{n}$ mom^t magn. de la bobine.

$\vec{\Gamma} = \vec{m} \wedge \vec{B_s} = 2\rho I r_0 H \vec{n} \wedge B_s \vec{u}(t)$.

$\Leftrightarrow \vec{\Gamma} = \underbrace{2\rho I r_0 H B_s \sin(\omega_s t + \varphi - \theta)}_{\Gamma(t)} \vec{e_z}$. avec $\theta = \theta_0 + \Omega t$

II.B.2). $\Gamma_{syn} = \langle \Gamma(t) \rangle = 2\rho I r_0 H B_s \langle \sin((\omega_s - \Omega)t + \varphi - \theta_0) \rangle$

si $\omega_s - \Omega \neq 0 \rightarrow \Gamma_{syn} = 0$.

sinon i.e $\omega_s = \Omega$ → $\Gamma_{syn} = 2\rho I r_0 H B_s \sin(\varphi - \theta_0)$.

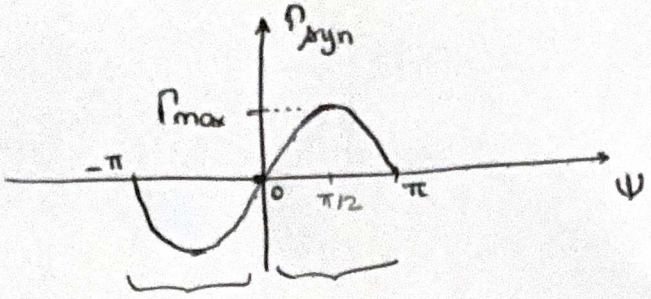
↳ synchrone : rotor tourne à la vitesse angulaire de rotate de $\vec{B}$.

au démarrage $\Omega = 0$, $\omega_s = 2\pi f_s \neq 0$ } → $\langle P \rangle = 0$ → moteur ne peut pas démarrer seul.

II.B.3. • $\Gamma_{syn} = 2p\,I\,r_0\,H B_S \sin\Psi$

• $\Gamma_{max} = 2p\,I\,r_0\,H B_S$

atteint pour $\Psi = \frac{\pi}{2}$ rad



fct générateur $\Gamma_{syn} < 0$ — fct moteur $\Gamma_{syn} > 0$

Γ_{syn} : pt des acc magn. sur rotor

car $\mathcal{P} = \Gamma_{syn} \cdot \Omega$; $\Omega > 0$ car rotat° autour de (Oz) ds sens direct.

• $\Gamma_{syn} = \Gamma_{max} \Leftrightarrow \Psi = \varphi - \theta_0 = \frac{\pi}{2}$ rad

$$\Phi_{mag} = p \iint_{rotor} \vec{B}_S \cdot d\vec{S}\vec{n} = p \iint_{rotor} B_S \vec{u} \cdot d\vec{S}\vec{n} = p B_S \iint_{rotor} dS \cdot \cos(\omega_s t + \varphi - \theta)$$

↳ cf fig 3.

$$\Leftrightarrow \Phi_{mag} = p B_S \cos(\omega_s t + \varphi - \theta_0 - \Omega t) \iint_{rotor} dS$$

or $\omega_s = \Omega$ sinon $\langle \Gamma_{syn} \rangle = 0$

et $\varphi - \theta_0 = \frac{\pi}{2}$ rad

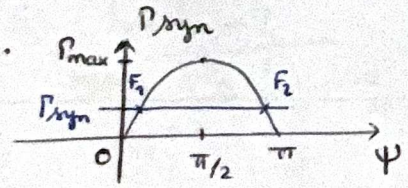
↓

situat° telle que $\forall t\ \vec{B}_S \perp \vec{n}$

$$\Leftrightarrow \Phi_{mag} = p B_S \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} \cdot S \Rightarrow \Phi_{mag} = 0.$$

⇒ pas de phénomènes inductifs

II.B.4.



• si $\Gamma_{syn} < \Gamma_{max}$: 2 valeurs possibles pour Ψ

① point F_1 : $\Psi < \frac{\pi}{2}$

Ei : $\Gamma = \Gamma_{syn}$ $\Omega = \omega_s$

perturbat° $\Omega \nearrow$

$\Rightarrow \Psi = (\omega_s - \Omega)t + \varphi - \theta_0 \searrow$ $\Rightarrow$ (courbe en F_1) $\Gamma_{syn} \searrow \Rightarrow \Omega \searrow$

rotor soumis à un couple ⊕ faible ⇒ ralentit

ainsi l'↗° de Ω est compensée.

fct stable ← perturbat° $\Omega \searrow$ ⇒ $\Psi \nearrow$ ⇒ (courbe en F_1) $\Gamma_{syn} \nearrow \Rightarrow \Omega \nearrow$ rotor soumis à un couple ⊕ élevé ⇒ accélère.

ainsi la ↘° de Ω est compensée.

② point F_2 : $\Psi > \frac{\pi}{2}$ selon un raisonnemt similaire : si $\Omega \nearrow \Rightarrow \Psi \searrow \Rightarrow$ (Γ_{syn} courbe en F_2) $\Gamma_{syn} \nearrow \Rightarrow \Omega \nearrow$

↳ fct instable

si $\Omega \searrow \Rightarrow \Psi \nearrow \Rightarrow \Gamma_{syn} \searrow \Rightarrow \Omega \searrow$

• si $\Gamma_{syn} = \Gamma_{max}$ (courbe en $\Psi = \pi/2$)

si $\Omega \nearrow \Rightarrow \Psi \searrow \Rightarrow \Gamma_{syn} \searrow \Rightarrow \Omega \searrow$ → ↗° de Ω compensée

si $\Omega \searrow \Rightarrow \Psi \searrow \Rightarrow \Gamma_{syn} \searrow \Rightarrow \Omega \searrow$ → ↘° de Ω renforcée